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Transverse Vibration Analysis of a Compound Plate with Using Cyclic Symmetry Modeling

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Abstract

In the paper the transversal vibration of a representative annular plate with complex geometry is studied on the basis of the numerical method and simulation. The research is focused on preparing the numerical model by using the cyclic symmetry modeling approach. The obtained results are discussed and compared with the experimental data. FE models are formulated by using ANSYS code.

Keywords: transversal vibration, cyclic model, annular plate

1. Introduction

Problems of transverse vibration of annular plates have been the subject of many recent investigations [1, 4]. This is due to the fact that some rotating systems can be treated as annular plates, where both their shape and dimensions are affected by the design of these systems. In papers [1, 4] the authors analyse free transverse vibration of toothed gears by using the finite element (FE) technique. In papers [3, 4] the cyclic symmetry modeling is included in the solving process of the vibration problems of compound systems. In the above presented article free transverse vibration of a compound annular plate is analysed by using the FE technique and experimental investigation.

2. Formulation of the problem

The objective of this paper is to present the methods of FE modeling of the compound annular plates transverse vibration and analyse their usefulness in the representation of the vibration process. For that purpose, a set of two compound circular plates has been analysed. The analysed systems have the geometry as it is displayed in Figure 1. Primary geometrical dimensions of the systems (diameters: d_z, d_w, d_1; thickness: l_r, l_w) are shown in Table 1.

Table 1. Parameters characterizing the analysed plates

No of models	d_z [m]	d_w [m]	d_1 [m]	l_r [m]	l_w [m]	E [Pa]	ρ [kg/m^3]	ν
1	0.191	0.159	0.02	0.008	0.002	$2.1\cdot10^{11}$	$7.85\cdot10^{3}$	0.28
2	0.203	0.147						

In these Table, E is Young's modulus of elasticity, ρ is the mass density and ν is the Poisson ratio, respectively.

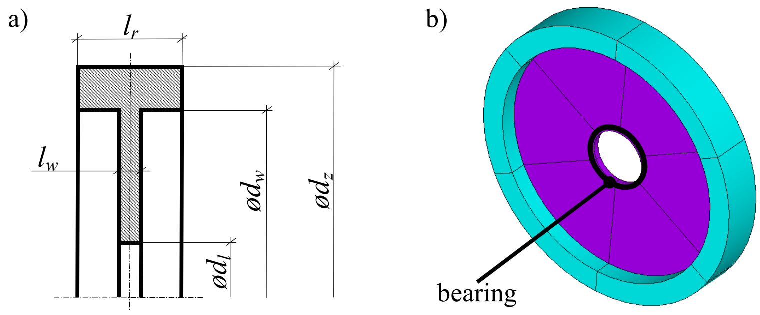

Figure 1. Geometrical models of the systems

For each case the problem of free vibration is solved by the finite element method. After elaboration of discrete models of the structures to be analysed, the differential equations of motion of the each analysed system can be written in the form [2]

$$\mathbf{M}\,\ddot{\mathbf{u}} + \mathbf{K}\mathbf{u} = 0 \tag{1}$$

where **M** is the global mass matrix, **K** is the global stiffness matrix, and **u** is the nodal displacement vector. Both global mass and stiffness matrices are obtained from the element matrices that are given by [2]

$$\mathbf{M}^{(e)} = \int_{V^{(e)}} \rho^{(e)} \mathbf{N}^T \mathbf{N}\, dV^{(e)}, \quad \mathbf{K}^{(e)} = \int_{V^{(e)}} \mathbf{B}^T \mathbf{E}\, \mathbf{B}\, dV^{(e)} \tag{2}$$

where $\rho^{(e)}$ is density of the element, **N** is the matrix of the element shape functions, **B** is the element shape function derivatives matrix, **E** is the material stiffness matrix, and $V^{(e)}$ is volume of the element. The natural frequencies of the system are obtained by solving the eigenvalue problem

$$\left(\mathbf{K} - \omega^2 \mathbf{M}\right)\overline{\mathbf{u}} = 0 \tag{3}$$

where ω is natural frequency and $\overline{\mathbf{u}}$ is corresponding mode shape vector, which is determined by the relation (3). The number of eigenpairs ($\omega_i, \overline{\mathbf{u}}_i$) corresponds to the number of degrees of freedom of the system. The block Lanczos method is applied to solve the eigenvalue problem (3). Because of the discretization process, the FE models of the considered systems are treated as approximations of the exact systems. The error between the objects and the FE models is defined by

$$\varepsilon = \left(\omega^f - \omega^e\right) / \omega^e \times 100[\%] \tag{4}$$

where ω^f is the natural frequency from the FE solution, while ω^e is the natural frequency of the exact system. Equation (4) is the so–called frequency error [2]. For the investigation presented in this paper the needed accurate values of the natural frequencies are achieved by the realization of experimental investigation.

3. Numerical analysis

In this section, the FE models of the systems under consideration are prepared and natural frequencies of the transverse vibration are determined. In accordance with the circular and annular plate vibration theory [2] the particular natural frequencies of vibration are denoted as ω_{mn} where m refers to the number of nodal circles and n is the number of nodal diameters. For each system, three FE models are realized. The first FE model is prepared as follows. Each geometrical model of these systems is meshed by using standard procedures of the ANSYS software. The 3–D solid mesh is prepared and the ten node tetrahedral element (solid187) with three degrees of freedom in each node is employed to realize each model. During the mesh generation process, it is assumed that the maximum length of each element's side needs to be no more than 2 [mm]. The bigger FE model refers to the second object and includes 143760 solid elements. The smaller FE model includes 97404 solid elements. For all models discussed here, calculations were continued until the natural frequency ω_{16} was determined. Tables 2 and 5 display the natural frequencies obtained by using the discussed FE models.

Table 2. Natural frequencies of the first FE model related to the object no 1 ω_{mn} [Hz]

		n						
		0	1	2	3	4	5	6
m	1	236.5	149.4	643.6	1770	3340	5292	7523
	2	1932	2240	2899	3931	5327	7009	
	3	4531	5167	6580				

The second FE model of each system is prepared by using cyclic symmetry feature of the analysed systems. Geometrical model of each system consists of six sectors (see Fig. 1b) which have the cyclic symmetry feature. One of these segments is meshed by using standard procedures of the ANSYS software and the cyclic symmetry boundary conditions are included. The mesh generation and the computation process are conducted under the same conditions as for the previously discussed FE model cases (the full model cases). The bigger cyclic FE model refers to the second object and includes 23960 solid elements. The smaller cyclic FE model includes 16234 solid elements. Thus, the solution of free vibration is obtained on the basis of these single symmetric sectors. Tables 3 and 6 display the natural frequencies obtained by using the discussed cyclic symmetry FE models (the first cyclic model cases).

Table 3. Natural frequencies of the second FE model related to the object no 1 ω_{mn} [Hz]

		n						
		0	1	2	3	4	5	6
m	1	236.4	149.4	643.6	1770	3340	5292	7523
	2	1935	2240	2898	3930	5327	7009	
	3	4530	5166	6579				

The third FE model of each system is prepared in the same manner as the second FE model cases, but the maximum length of each element's side is different. For these cases the maximum length of each element's side is no more than 1.5 [mm]. So, the bigger cyclic FE model, which refers to the second object, includes 57834 solid elements and the smaller cyclic FE model includes 38131 solid elements, respectively. Tables 4 and 7 show the natural frequencies obtained by using the above cyclic symmetry FE models (the second cyclic model cases).

Table 4. Natural frequencies of the third FE model related to the object no 1 ω_{mn} [Hz]

		n						
		0	1	2	3	4	5	6
m	1	235.8	148.9	643.4	1770	3340	5291	7519
	2	1928	2233	2890	3922	5317	6997	
	3	4520	5157	6566				

Table 5. Natural frequencies of the first FE model related to the object no 2 ω_{mn} [Hz]

		n						
		0	1	2	3	4	5	6
m	1	213.5	123.4	658.9	1804	3398	5403	7781
	2	1916	2458	3337	4507	6059		
	3	4109	4680	6321				

Table 6. Natural frequencies of the second FE model related to the object no 2 ω_{mn} [Hz]

		n						
		0	1	2	3	4	5	6
m	1	213.5	123.3	658.9	1804	3398	5403	7781
	2	1916	2457	3336	4507	6059		
	3	4108	4679	6321				

Table 7. Natural frequencies of the third FE model related to the object no 2 ω_{mn} [Hz]

		n						
		0	1	2	3	4	5	6
m	1	212.8	122.9	658.8	1804	3398	5403	7781
	2	1913	2452	3326	4496	6046		
	3	4092	4666	6312				

4. Experimental studies

In this section the results related to the experimental verification of the considered numerical models are discussed. LMS measurement environment is used in the experimental study. The measuring set contained the PCB model 086C03 type modal hammer, accelerometer PCB model 353B18, LMS SCADA data acquisition system, and SCM-V4E type measuring module supported by LMS Test.Lab software. The experimental study is conducted to identify natural frequencies and corresponding mode shapes related to the transverse vibration of the considered objects. The values of the excited natural frequencies are shown in Table 8 (for the first object) and in Table 12 (for the second object), respectively.

Table 8. Natural frequencies of the first object ω_{mn} [Hz] (experimental investigation)

		n						
		0	1	2	3	4	5	6
m	1	263.8	141.9	575.6	1697	3272	5233	7463
	2	1847	2247	2948	3976	5318	6941	
	3	4397	5001	6453				

In Tables 9–11 the values of the frequency error related to the FE models of the first object are displayed. In each FE model case, only for two natural frequencies (ω_{10} and ω_{12}) the frequency error is above 10 [%]. In Tables 13–15 the values of the frequency error related to the FE models of the second object are displayed. In this object, only nine natural frequencies were excited. In each FE model case, for two natural frequencies the frequency error is above 10 [%].

Table 9. Frequency error related to the first FE model of the first object ε_{mn} [%]

		n						
		0	1	2	3	4	5	6
m	1	-10.35	5.29	11.81	4.3	2.08	1.13	0.8
	2	4.6	-0.31	-1.66	-1.13	0.17	0.98	
	3	3.05	3.32	1.97				

Table 10. Frequency error related to the second FE model of the first object ε_{mn} [%]

		n						
		0	1	2	3	4	5	6
m	1	-10.39	5.29	11.81	4.3	2.08	1.13	0.8
	2	4.76	-0.31	-1.7	-1.16	0.17	0.98	
	3	3.02	3.3	1.95				

Table 11. Frequency error related to the third FE model of the first object ε_{mn} [%]

		n						
		0	1	2	3	4	5	6
m	1	-10.61	4.93	11.78	4.3	2.08	1.11	0.75
	2	4.39	-0.62	-1.97	-1.36	-0.02	0.81	
	3	2.8	3.12	1.75				

Table 12. Natural frequencies of the object no 2 ω_{mn} [Hz] (experimental investigation)

		n						
		0	1	2	3	4	5	6
m	1	221.3	106.3	596.3	1740		5330	7709
	2		2444	3254	4389			
	3							

Table 13. Frequency error related to the first FE model of the second object ε_{mn} [%]

		n						
		0	1	2	3	4	5	6
m	1	-3.53	16.09	10.5	3.68		1.37	0.93
	2		0.57	2.55	2.69			
	3							

Table 14. Frequency error related to the second FE model of the second object ε_{mn} [%]

		n						
		0	1	2	3	4	5	6
m	1	-3.53	16	10.5	3.68		1.37	0.93
	2		0.53	2.52	2.69			
	3							

Table 15. Frequency error related to the third FE model of the second object ε_{mn} [%]

		n						
		0	1	2	3	4	5	6
m	1	-3.84	15.62	10.48	3.68		1.37	0.93
	2		0.33	2.21	2.44			
	3							

5. Conclusions

The present paper deals with free transverse vibration of a compound annular plate. Three FE models are proposed. For all the FE model cases discussed, comparable results have been obtained. The most attractive is the second FE model case, which includes cyclic symmetry features. Moreover, this model includes a substantially lower number of finite elements compared to the remainder of models. It is worth pointing out that in the preferred FE model case the maximum length of each element's side equal to the lesser plate thickness was assumed. At this stage of the research, it seems that further investigation needs to be continued.

References

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