Influence of Torsional-Bending Coupling on Transverse Vibration of Piston Engine

Zbigniew DĄBROWSKI
Institute of Machinery Design Fundamentals
Faculty of Automotive and Construction Machinery Design
Warsaw University of Technology
ul. Narbutta 84, Warsaw, zdabrow@simr.pw.edu.pl

Bogumił CHILIŃSKI
Institute of Machinery Design Fundamentals
Faculty of Automotive and Construction Machinery Design
Warsaw University of Technology
ul. Narbutta 84, Warsaw, bogumil.chilinski@gmail.com

Abstract
The article presents the analysis of the influence of bending-torsional coupling of vibrations in the crankshaft on transverse vibrations of the engine body. In practice, there is used a simplified model, wherein transverse and torsional oscillations are analyzed independently. With the use of the model of deformable crankshaft, the authors show the influence of bending-torsional coupling on the frequency structure of transverse vibrations. The introduction presents the problem of vibrations in combustion engines and their modelling. Further, there is presented the elastic model of the crankshaft, together with the applied assumptions and equations of motion describing vibrations in one cylinder combustion engine. Next chapter shows numerical simulation results with their initial analysis. The whole paper is summarized with conclusions about calculations and the possibility to use the results in practice.

Keywords: Bending-torsional vibrations of the crankshaft, modeling of crank system, analytical solutions, numerical simulations.

1. Introduction
Dynamics of crank system is a very important technical problem. Basic parameters of the engine and its work are directly related to this system. In the case of motion with a constant velocity of the crankshaft, it is easy to determine the forces and displacements which appear in the crankshaft. The dynamics of crank system in unsteady motion requires many studies [1-4].

Due to the complex geometry of the rotor and "complicated" construction of the crank mechanism there exists a coupling between vibrations occurring in the engine [5-8]. In practice this phenomenon considerably hinders the analysis, due to the coupling of individual degrees of freedom. Therefore, in order to make calculations there is used the most commonly applied simplification based on rejecting any dependencies connecting bending and torsional vibrations in engines. Such an approach is used in preliminary design calculations. However, in the case of problems connected with an operation it may be insufficient [9-13]. Moreover, in practice, measured vibrations are different from theoretical model results.
This issue is important because there may appear new critical areas due to the coupling of bending-torsional vibrations. What is more, torsional vibrations affect significantly transverse displacements. This motion influences the vibrations of the whole body. In practice there appear a shift and modulations of particular frequencies of eigen vibrations of uncoupled system. This shows the presence of nonlinear or parametric effects in the considered object [14-16].

The authors propose to apply this phenomenon to analyze torsional vibrations of the engine based on the spectra of transverse displacements of the body. This problem is important because the measurement of angular vibrations of the crankshaft of a combustion engine is more difficult than the measurements of transverse vibrations.

2. The dynamic model of piston engine with an elastic crankshaft

Due to the complex geometric and material structure it is convenient to replace the continuous mass system, which is the crankshaft, with a discrete model. In such cases, the masses are usually reduced to selected constructional nodes, whereas the remaining part of the object is treated as a massless deformable structure.

Of course, the model of the system of point masses is a significant simplification of the continuous system, which is characterized by infinite (but countable) set of eigen values. The number of eigen frequencies in the case of discrete systems is the finite number. Therefore, it is not possible to replace "fully" the continuous system with a model of point masses. However, it is possible to make an equivalent reduction in a selected frequency band, for example, in the range of low frequencies. In practice, such a simplification does not lead to serious errors. At the same time, it must be emphasized that this method significantly simplifies the calculations.

Single crank of the crankshaft of the piston engine is presented schematically in Figure 1.

Figure 1. The model of the crankshaft of one piston. 1 - 2 flywheel - crankshaft, 3 - pulley

In constructions of real combustion engines, very rigid crankshafts are used. Basically due to the precision required from crank mechanisms. Even small changes in the angular position of the crank may affect the process of combustion in a given system,
which directly influences its dynamics. In addition, in vibrating systems there is a risk of resonance with a basic harmonic of extortion which comes from gas forces [17-19]. In this case, oversizing of the crankshaft allows to move the frequency of eigen vibrations into the area of higher components of drive moment.

Due to high rigidity of crank system, it can be assumed that with a good approximation, deformations occurring in the crank systems are very small. This allows to use the model of linear-elastic system for calculations [20,21]. Figure 2 shows the displacement of the crank described in the moving coordinate system.

Figure 2. Displacement of crank of the crankshaft

In Figures 1 and 2 there are used the following generalized coordinates describing the dynamics of the analyzed model of the crank:

\[ \phi \] – rotation angle of the flywheel of the engine,
\[ \varphi \] – rotation angle of the disc of torsional vibration damper,
\[ h \] – horizontal deformation of the crank,
\[ v \] – vertical deformation of the crank.

Generalized forces in selected constructional bands may be determined on the basis of the equations:
\[ F = K \cdot u \] (1)

where:

- \( K \) – stiffness matrix,
- \( F \) – generalized force vector,
- \( u \) – displacement generalized vector.

Due to the symmetry conditions and the load system, the stiffness matrix has a simplified form:

\[
K = \begin{bmatrix}
    k_{nn} & 0 & 0 \\
    0 & k_{tt} & -k_{t0} \\
    0 & -k_{t0} & k_{00}
\end{bmatrix}
\] (2)

It is possible to find motion equations for the system presented in Figure 1 with the use of any formalism of analytical mechanics. Due to the linearity of the model and holonomic constraints appearing in the system, there are used Lagrange equations of second kind. On this basis, the following dynamic model is determined:

\[
(I_d + m_w R^2) \ddot{\phi} + m_w R \dddot{u}_r + k_{t0} u_t + k_{00}(\phi - \varphi) = M_0
\] (3)

\[
m_w \dddot{u}_r + m_w R \dddot{\phi} + k_{t0} u_t - k_{00}(\varphi - \phi) = P_z
\] (4)

\[
I_{kr} \ddot{\varphi} + [ -k_{t0} u_t + k_{00}(\varphi - \phi) ] = 0
\] (5)

\[
m_w \dddot{u}_r - m_w R \dddot{\phi}^2 + k_r u_r = P_r
\] (6)

3. Simulation analysis of transverse vibrations of the crankshaft

The series of numerical simulations was carried out for a proposed system of equations. Transverse vibrations of the crankshaft without the coupling of bending and torsional vibrations presented in plot 3 are taken as a point of reference.
In the case when the coupling is taken into account, the spectral structure of transverse vibrations is much more complex. The spectrum of displacement of transverse vibrations in a moving coordinate system is shown in plot 4. It is possible to observe additional frequencies connected with torsional vibrations.

Figure 3. Bending vibrations of the crankshaft of the system without coupling

Figure 4. Bending vibrations of the crankshaft system with a coupling
4. Conclusion

The phenomenon of coupling of bending and torsional vibrations in vibrating systems is usually omitted in model calculations. Such calculations are justified at the design stage, when it is necessary to pre-define the basic dimensions of the system for further designing process. However, the dynamics of motion of the real crankshaft system is much more complex. As a result, the authors proposed a model which takes into account more phenomena and allows for more detailed analysis of vibrations occurring in combustion engines.

The proposed system of dynamics equations in moving coordinate system is possible to be solved analytically. Part of the equations is uncoupled and linear.

The simulations clearly show the impact of taking into account the coupling on transverse displacements of the crankshaft. The frequencies of torsional vibrations are transferred to bending oscillations. This allows to draw conclusions about the frequencies occurring in the spectral structure of angular vibrations only on the basis of the measurements of body vibration [22-24]. The proposed model can be used successfully in the diagnostics of combustion engines [25-27].

References


