Energy Analysis of a Mechanical System with a Dynamic Vibration Absorber

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Abstract

The study relates to the phenomenon of power distribution in mechanical systems equipped with a dynamic vibration absorber. It is one of the methods of eliminating vibrations in a mechanical system, which stabilises its operation. This solution helps to reduce dynamic stress in subsystems of a vehicle's suspension or stabilise the motion of flying machines, such as helicopters. The article describes the phenomenon of power distribution of structural forces, which has not been described so far. The phenomenon reveals the power distribution in a dynamic structure of a system of interest and can be used to determine the rate of energy flow as a function of the dynamic state resulting from the selection of dynamic parameters of the vibration absorber. The energy analysis applied in the study is based on an energy-based optimization method of adjusting the dynamic vibration absorber to the main mechanical system without changing its dynamic parameters, as is the case, for example, in turbine rotor balancing.

Keywords: energy flow, dynamics of machines, elimination of energy flow

1. Introduction

The phenomenon of power distribution of structural forces in mechanical systems with a dynamic vibration absorber has not been recognised so far [5]. It is a holistic approach, which makes it possible to control and optimize energy flow in order to ensure effective stabilisation of the main mechanical system thanks to the influence of the dynamic vibration absorber. The analysis of power distribution can be used in mechanical and biomechanical systems to optimize the structural design, to evaluate the amount of energy absorbed by particular elements and, globally, by entire systems, and as a diagnostic tool at every life stage of these systems [2, 3, 4].

2. The physical model of the dynamics of the system of interest

Dynamic analysis of a mechanical system with a dynamic vibration absorber requires a physical model with two degrees of freedom. The first point of reduction is mass M, which models the mass of the main mechanical system, which is to be stabilized, while the second point of reduction corresponds to mass "m" of the dynamic vibration absorber, connected with mass M through a damping-energy dissipating element. Vibrations are generated by the driving force $F(t)$, which excites mass M. The physical model of such a mechanical system is shown in Figure 1. The purpose of the absorber is to minimize the vibration amplitude of the main subsystem. The tuning parameters of the absorber are determined by the dynamics of the system of interest. For this purpose a dynamic mathematical model of the system has been formulated.

3. The mathematical model of the dynamics of the system of interest

The mathematical model was derived using Lagrange equations of the second kind given by [1]:

$$\frac{d}{dt}\left(\frac{\partial E}{\partial \dot{q}_j}\right)-\frac{\partial E}{\partial q_j}=Q_j-\frac{\partial V}{\partial q_j}-\frac{\partial \Phi}{\partial \dot{q}_j}; \quad j=1, 2,\ldots s; \tag{1}$$

where:

s – the number of degrees of freedom,

ϕ – the power of forces of energy dissipation,

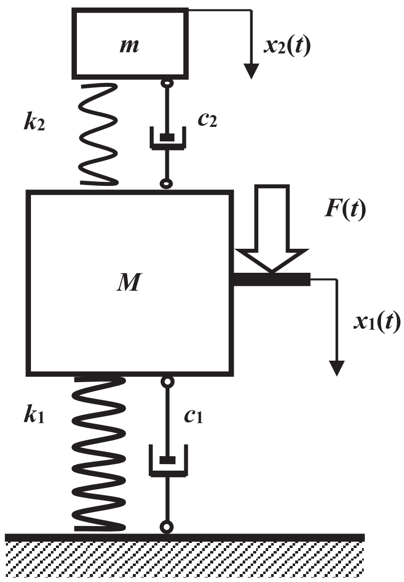
E – kinetic energy of the mechanical system,

V – potential energy of the mechanical system,

Q_j – generalised active forces,

q_j – generalised coordinates,

$\dot{q}_j$ – generalised velocities.



M – reduced mass of the main system

k_1 – reduced coefficient of elasticity of the main system

c_1 – reduced damping coefficient of the main system

$F(t)$ – the driving force with a variable frequency

m – reduced mass of the dynamic vibration absorber

k_2 – reduced coefficient of elasticity of the dynamic vibration absorber

c_2 – reduced damping coefficient of the dynamic vibration absorber

Figure 1. The physical model of a mechanical system with a dynamic vibration absorber

As mentioned earlier, the mechanical system of interest has two degrees of freedom, hence $s = 2$. The following generalised coordinates have been assumed:
$q_1 = x_1(t)$ – the location coordinate of mass M of the stabilized mechanical system,
$q_2 = x_2(t)$ – the location coordinate of mass m of the attached dynamic vibration absorber.

The mathematical model of forces acting in the system consists of two differential equations of forces given by (2):

$$\begin{aligned} &M\ddot{x}_1(t) + (c_1 + c_2)\dot{x}(t) + (k_1 + k_2)x(t) - k_2 x_2(t) - c_2\dot{x}(t) = F_0 \sin[2\pi f(t)t]; \\ &m\ddot{x}_2(t) + c_2\dot{x}_2(t) + k_2 x_2(t) - c_2\dot{x}_1(t) - k_2 x_1(t) = 0 \end{aligned} \tag{2}$$

The first equation describes forces acting in a stabilized mechanical system, the second one describes forces acting in the additional system of the dynamic absorber attached to the main system. When dynamic forces acting in the mechanical system are known, it is possible to formulate an energy model. The model was formulated by applying the First Principle of Power Distribution in a Mechanical System (PPDiMS) [2, 3].

4. The energy model of power distribution in a mechanical system with a dynamic vibration absorber

The above-mentioned principle can be used to derive equations of power distribution in the mechanical system. The energy model of the system of interest consists of two equations of power given by:

$$\begin{aligned} &M\ddot{x}_1(t)\dot{x}_1(t) + (c_1 + c_2)\dot{x}_1^2(t) + (k_1 + k_2)x(t)\dot{x}_1(t) - k_2 x_2(t)\dot{x}_1 - c_2\dot{x}_2(t)\dot{x}_1(t) = \\ &= F_0\dot{x}_1(t)\sin[2\pi f(t)t]; \\ &m\ddot{x}_2(t)\dot{x}_2 + c_2\dot{x}_2^2(t) + k_2 x_2(t)\dot{x}_2(t) - c_2\dot{x}_1(t)\dot{x}_2(t) - k_2 x_1(t)\dot{x}_2(t) = 0 \end{aligned} \tag{3}$$

The first equation describes how the powers of all structural forces change over time, that is: the power of inertial forces, the power of dissipative forces, the power of elastic forces and the power of the driving force, which excites the motion of the mechanical system.The equation also accounts for the powers of the elastic and dissipative coupling with the vibration absorber.

The second equation describes power distribution at the reduction point connected with the mass of the dynamic absorber and the power of forces involved in the elastic and dissipative coupling with the main system.

The equation of energy flow can be derived from the First Principle of Energy Flow in a Mechanical System based on integral equations given by (3).
Given the energy models of the system of interest, one can solve the energy model and determine power distribution and energy flow in its dynamic structure for specific data.

$$\int_0^{t_S}\left[M\ddot{x}_1(t)\dot{x}_1(t)\right]dt+\int_0^{t_S}\left[(c_1+c_2)\dot{x}_1^2(t)\right]dt+\int_0^{t_S}\left[(k_1+k_2)x(t)\dot{x}_1(t)\right]dt=$$
$$=\int_0^{t_S}\left[k_2x_2(t)\dot{x}_1\right]dt+\int_0^{t_S}\left[c_2\dot{x}_2(t)\dot{x}_1(t)\right]dt+\int_0^{t_S}\left[F_0\dot{x}_1(t)\sin(2\pi f_w t)\right]dt;$$
$$\int_0^{t_S}\left[m\ddot{x}_2(t)\dot{x}_2(t)\right]dt+\int_0^{t_S}\left[c_2\dot{x}_2^2(t)\right]dt+\int_0^{t_S}\left[k_2x_2(t)\dot{x}_2(t)\right]dt= \tag{3}$$
$$\int_0^{t_S}\left[c_2\dot{x}_1(t)\dot{x}_2(t)\right]dt+\int_0^{t_S}\left[k_2x_1(t)\dot{x}_2(t)\right]dt;$$

5. The solution of the energy model of power distribution in a mechanical system with a dynamic vibration absorber

The above models were solved usingnumerical simulation implemented in the MATLAB/simulink environment. An original simulation programme called SPED was developed for this purpose. The programme makes use of the Elementary Processor of Energy Flow MWD, which implements two principles: the First Principle of Power Distribution in a Mechanical System (PPDiMS) and the First Principle of Energy Flow in a Mechanical System (FPEFiMS) [2, 3, 4].

Example analytical calculations were done for the following data:

M = 10 kg, k_2 = 3948 N/m, c_2 = 1.257 Ns/m, m = 1 kg,
k_1 = 3.948E+004 N/m, c_1 = 252.6 Ns/m, $F(t)$ = 100 sin [2πf(t)t]

The SPED programme enables a synchronous solution of the mathematical model of the system's motion, power distribution and energy flow in the mechanical system.

Figure 2 shows the results of the simulation of the dynamics of the system of interest, comparing values of acceleration, velocity and displacement of the reduction points of the dynamic vibration absorber and the main system. Response characteristics were obtained by inducing the motion of the main system through a sinusoidal driving force with amplitude of 100 N and with a frequency varying at the rate of 1 Hz/s.

Analysis of all kinematic quantities indicates mutual interactions between the subsystems. The effect of the main subsystem on the dynamic vibration absorber is evident for all characteristics once the driving frequency reaches the resonant frequency of the main system and is manifested by extended characteristics of all kinematic quantities. The strong effect of the dynamic vibration absorber is especially evident in the characteristics of the main subsystem. One significant change is manifested by reduced values of all kinematic quantities for a frequency of 10 Hz, which the absorber was tuned to. It is precisely the purpose of the dynamic vibration absorber, which ensures stabilization of the main subsystem's motion by reducing its vibration amplitudes.

To facilitate comparative analysis of the motion of the main(stabilized) subsystem, Figure 3 shows dimensionless dynamic characteristics of vibration amplitudes relative to the static deflection of the main subsystem. The horizontal line at a height of 1 divides the chart into two sections: the area of amplified vibrations of the main subsystem for values greater than 1 and the area of vibration elimination for values less than 1.

$$x_{stat} = \frac{F_{z0}}{k_1} \quad [m] \tag{4}$$

where: F_{z0} – reduced amplitude of the driving force inducing the motion of the main subsystem, k_1 – reduced coefficient of elasticity of the main system.

Results of the dynamic analysis for the dynamic vibration absorber

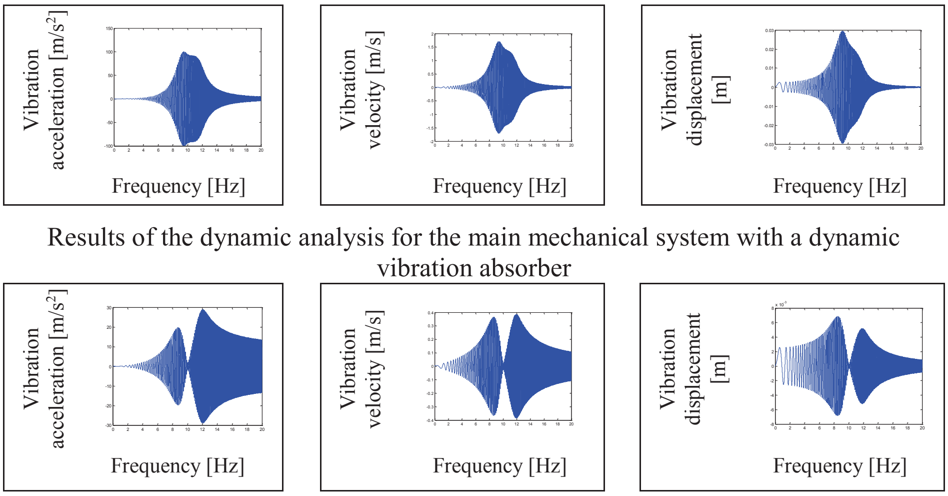


Figure 2. Results of the dynamic analysis of a mechanical system (stabilized) with a dynamic vibration absorber during a harmonic test with a driving force $F(t) = 100 \sin [2\pi F(t)]$ with a constant rate of frequency switching $f = 1$ Hz/s in the range 0-20 Hz.

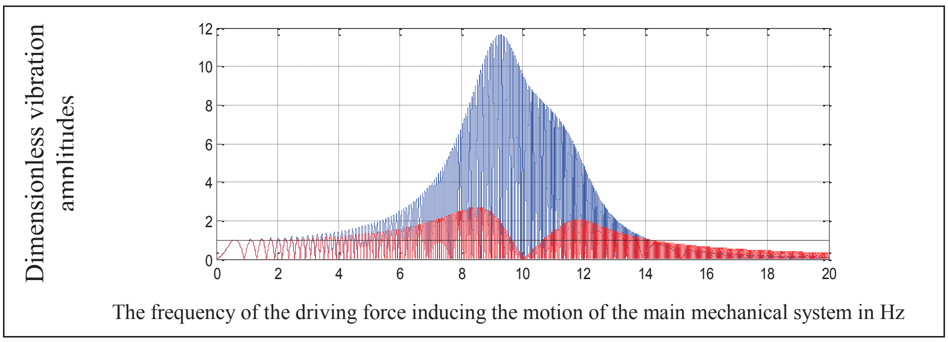


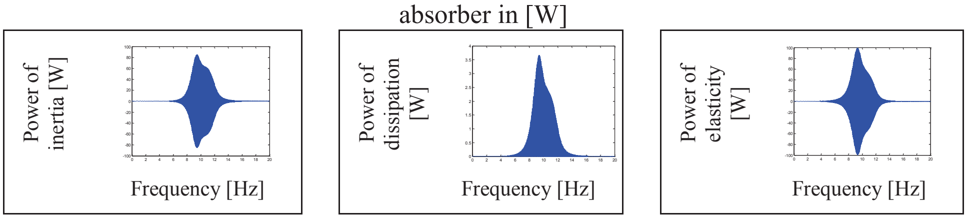
Figure 3. Dimensionless characteristics of amplitude and frequency of the mechanical system with a dynamic vibration absorber

Analysis of Figure 3 indicates that the frequency band where vibration elimination occurs in the main subsystem is very narrow and its middle lies at 10 Hz, which is the frequency the absorber was tuned to.The vibrations of the main subsystem were reduced by 89.3% relative to the static deflection that would be produced if a static force with an amplitude equal to that of the driving force was applied to it.This means that the dynamic coefficient for a frequency of 10 Hz amounts to **0.107**. It is a well-known fact that an absorber eliminates the amplitude of vibrations at a specific frequency, which makes it a selective absorber. This limits the application of the absorber to machines and devices that operate at constant (stabilized) frequency.

6. Amplitude and frequency characteristics of powers of structural forces in a mechanical system with a dynamic vibration absorber

The above properties of a dynamic vibration absorber were also confirmed by a novel dynamic analysis in the domain of power distribution of structural forces acting at reduction points.Figure 4 shows instantaneous powers of inertial, dissipative and elastic forces as functions of frequency in the range 0-20 Hz. In other words, these are amplitude and frequency characteristics of powers for the above mentioned structural forces.

The distribution of instantenous power at the reduction point of the dynamic vibration absorber in [W]



The distribution of instantenous power at the reduction point of the main mechanical system

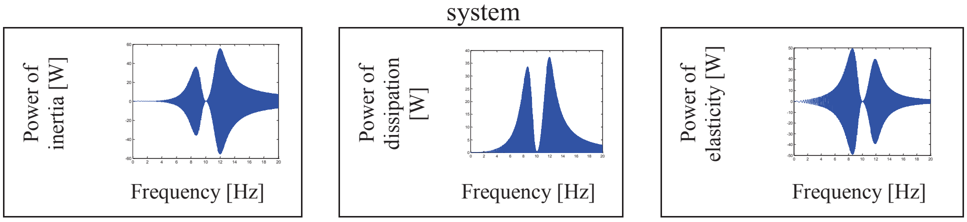


Figure 4. The distribution of instantaneous power in the dynamic structure of the(stabilized)mechanical system with a dynamic vibration absorber during a harmonic test with a constant rate of switching frequency f = 1 Hz/s

The figures indicate that the power of structural forces in the main subsystem for the frequency of 10 Hz (effective operation of the absorber) is close to 0 and amounts to: Nbg(10 Hz) = 0.2 [W], Nstg(10 Hz) = 0.125 [W] and Nspg(10 Hz) = 0.114 [W]. This means that the rate of energy flow is very low and suspension elements of the main subsystem are exposed to little dynamic load.Fatigue depends on the amount of energy transferred through structural elements of the suspension of the main (stabilized)subsystem.

7. The effectiveness of eliminating energy flow in a stabilized subsystem

The effectiveness of eliminating energy flow in a stabilized subsystem by means of a dynamic vibration absorber can be expressed in the form of a dimensionless characteristic of elasticity, which relates the power of elasticity at both reduction points to the maximum power at the frequency for which the power of elasticity in the main subsystem is the smallest – Fig. 5.

Figure 5 shows the factor by which the power of elasticity is reduced when the absorber reaches the point of its effective operation; the factor reduction is expressed as a ratio of maximum power of energy characteristics obtained in both systems to the maximum power of elasticity in the main subsystem observed at the driving frequency, i.e. at the point of elimination. The chart shows a high degree of power reduction, which confirms the specific effect in which the subsystem of the dynamic absorber affects the main subsystem (stabilized) in the domain of power.A properly tuned dynamic vibration absorber effectively eliminates energy flow in elastic elements of the suspension of the main subsystem. A comparison of both characteristics of instantaneous powers of elasticity clearly reveals that this kind of power is neutralized by the dynamic absorber. In the frequency band where elimination occurs, instantaneous elastic power reaches a maximum value, which is 872 times greater than the peak power obtained for instantaneous elastic power in the main subsystem (stabilized).

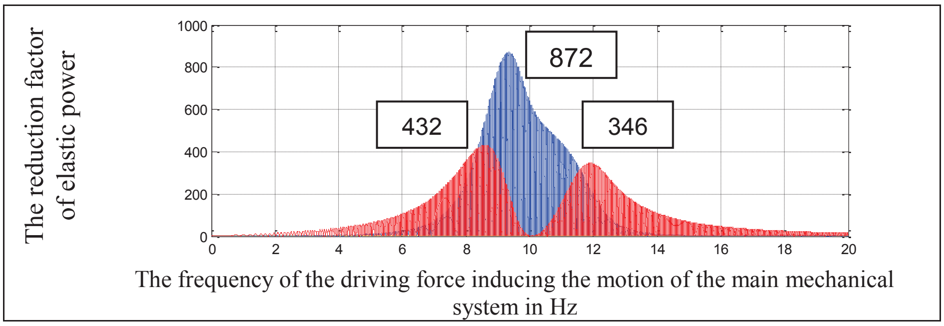


Figure 5. The reduction factor of the power of elastic forces in a mechanical system with a dynamic vibration absorber expressed as a function of the ratio of the driving frequency to the maximum instantaneous power obtained for optimal parameters of the absorber

It can also be concluded that in the design of a dynamic vibration absorber one should ensure that its elastic element is not exposed to stress exceeding permissible values.

Elastic power can be regarded as a measure of fatigue load exerted on suspension (elastic) structures of the main subsystem. A dynamic absorber contributes to increasing the durability and reliability of the suspension of the main (stabilized)subsystem.

8. Conclusions

Based on the results of energy analysis of the mechanical system with a dynamic vibration absorber, one can formulate a few important conclusions.

1. The dynamic analysis conducted in the study explains the phenomenon of power distribution and energy flow in a mechanical system with a vibration absorber.
2. The energy analysis has demonstrated a considerable reduction in the flow of all kinds of energy in the stabilized subsystem for the selected frequency which the vibration absorber was tuned to.
3. The optimal energy flow in the main (stabilized) subsystem depends on its damping ratio.
4. The dynamic absorber absorbs energy introduced into the system by the driving force in the optimal range of vibration elimination and has a strong effect on the main (stabilized) subsystem by reducing the flow of energy transferred to it.
5. The elimination of the flow of elastic energy in the main subsystem and in the absorber, which was computed in relation to the maximum instantaneous elastic power for the optimal frequency of vibration elimination, amounted to, respectively: in the main system – 432, and in the subsystem of the absorber – 872.

Acknowledgments

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