Natural Frequencies of Layered Elongated Cylindrical Panels for Geometrically Nonlinear Deformation at Discrete Consideration of Components

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Abstract

The proposed and verified the technique of finding a finite number of first natural frequencies for geometrically nonlinear vibrations of layered elongated cylindrical panels at discrete consideration of components. The influence of the radius of curvature on the natural frequencies of three- and five-layered panels is investigated. The dependence between the volume of filler three-layer panels and the lowest natural frequency has been established.

Keywords: elongated layered panel, nonlinear vibrations, perturbations method, natural frequencies

1. Introduction

The flexible layered cylindrical panels constitute a significant part of various structures and hardware. The specificity of the functional purpose of components of layers causes a sharp difference in their physical and mechanical properties and thickness, causing the need for discrete consideration of the thickness of the structure, of the above mentioned objects, as the averaged approach can lead to significant errors when assessing the ability to support or determine their amplitude and frequency characteristics.

Effects of intensive dynamic (including cyclic) loads are usually the cause of geometrically non-linear stress-strain state. Therefore, there is a need for the development and verification of the methods for determining the parameters of free vibrations of geometrically nonlinear deformation of layered cylindrical panels for consideration of discrete components.

Free vibrations of the shell structural elements are studied using numerical and experimental methods [1–3] or only pliability to transversal shear [4]. Some analytical results for pliability to transversal compression are given in [8].

In this paper proposed the technique and with its using the investigated the free geometrically nonlinear vibrations of layered cylindrical panels with into account all the physical and mechanical properties of components in the spatial statement of the problem.

2. The problem statement for a particular component of a layered panel

A curved anisotropic elastic layer with thickness h we assume in a natural mixed system of coordinates $\alpha_1, \alpha_2, \alpha_3$ on the median surface. This surface is formed by the motion of the line $\alpha_1 = 0; \alpha_3 = 0$ on the segment of arbitrary guiding. We consider that the layer is significantly larger along the axis α_2 to the length of the section arc $\alpha_2 = 0$ of the median surface $\alpha_3 = 0$. So we have an elongated panel. If the conditions of fixing the ends of the panel $\alpha_1 = \pm\alpha_1^0$ and the initial conditions are independent of the coordinate α_2, then through a little influence of conditions of fixing the edges $\alpha_2 = \pm\alpha_2^0$, the functions, that determine the characteristics of geometrically nonlinear vibration processes in the plane of the median section, are dependent from α_1, α_3. To find these functions we have [9]:

– motion equations

$$div\,\hat{S} = \rho \frac{\partial^2 U}{\partial t^2}\,; \tag{1}$$

– elasticity relations

$$\hat{\Sigma} = \widetilde{A} \otimes \hat{\varepsilon}\,; \tag{2}$$

– deformation relation between the strain tensor components $\hat{\varepsilon}$ and the components of the elastic displacement vector $\vec{U} = u_i \vec{e}_i \vec{e}_j$

$$\varepsilon_{ij} = \frac{1}{2}(\nabla_i u_j + \nabla_j u_i + \nabla_i u^k \nabla_j u_k)\,; \tag{3}$$

– relation between the components S^{ij} of the nonsymmetrical Kirchhoff stress tensor $\hat{S}$ and the components σ^{ik} of the symmetric Piola stress tensor $\hat{\Sigma}$

$$S^{ij} = \sum_k \sigma^{ik}(\delta_k^j + \nabla_k u^j)\,. \tag{4}$$

In equations (1) and (2) $\widetilde{A}$ is the tensor of elastic properties of anisotropic layer, and ρ is its density.

Boundary conditions on the front surface of the panel $\alpha_3 = \pm h/2$ in the case of its belonging to the layered structure are shown below, and initial conditions have the form

$$u_i(\alpha_1, \alpha_3, t)\big|_{t=t_0} = v_i^0(\alpha_1, \alpha_3), \quad \left.\frac{\partial u_i(\alpha_1, \alpha_3, t)}{\partial t}\right|_{t=t_0} = v_i^1(\alpha_1, \alpha_3), \quad i = 1, 3, \tag{5}$$

$$\left|v_3^0(\alpha_1, \alpha_3)\right| >> \left|v_1^0(\alpha_1, \alpha_3)\right|, \quad (\alpha_1, \alpha_3) \in \Omega = [-\alpha_1^0, \alpha_1^0] \times [-h/2, h/2]. \tag{6}$$

3. The layered panels

Assume that a panel consists of N layers (see Fig. 1). Each k-th layer is considered as a separate thin panel with its own mechanical and material characteristics. Hooke's law is different for each layer:

$$\sigma^{(k)} = [Q^k]\varepsilon^k, \quad k = 1,\ldots,N, \tag{7}$$

where $[Q^k]$ is tensor of elastic properties of anisotropic k-th layer.

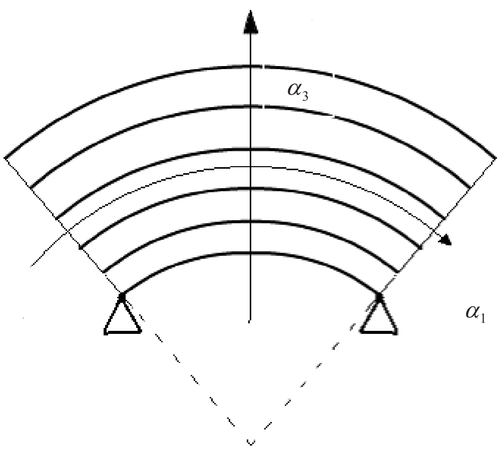


Figure 1. Layered cylindrical panel with hinges fixed on the elongated edges

Assuming that the value of α_3 coordinate at the top of k-th layer is h_k, and $h_0 = -h/2$, the equations (1) for a layered structure are written as

$$\sum_{i=1}^{3} \nabla_i S^{(k)ij} = \rho \frac{\partial^2 u_j^{(k)}}{\partial t^2}, \tag{8}$$

$$(\alpha_1, \alpha_3) \in \Omega = [-\alpha_1^0, \alpha_1^0] \times [h_{k-1}, h_k], \quad k = 1,\ldots,N.$$

The contact conditions between the layers are

$$u_i^{(k-1)}(\alpha_1, h_{k-1}, t) = u_i^{(k)}(\alpha_1, h_k, t), \quad i = 1, 2, 3, \tag{9}$$

$$S^{(k-1)3i}(\alpha_1, h_{k-1}, t) = S^{(k)3i}(\alpha_1, h_k, t), \quad |\alpha_1| \le \alpha_1^0, \quad k = 2,\ldots,N, \tag{10}$$

and on the lower and upper facial surfaces of the layered structure we have

$$S^{(m)31}(\alpha_1, h_m, t) = S^{(m)33}(\alpha_1, h_m, t) = 0, \quad |\alpha_1| \le \alpha_1^0, \quad m = 0, N. \tag{11}$$

At the elongated ends of the panel $\alpha_1 = \pm\alpha_1^0$ under the conditions of the fixing the hinge on the lower surface of the front $\alpha_2 = -h/2$ the boundary conditions have the form

$$S^{(k)1i}(a, \alpha_3, t) = 0, \quad k = 1, N, \tag{12}$$

$$u_i^{(N)}(a, \pm h/2, t) = 0, \quad |\alpha_3| \le h/2, \quad i = 1, 3, \quad a = \pm\alpha_1. \tag{13}$$

4. Approximations

Assuming that each *k*-th layer is thin, quadratic approximations along α_3 coordinate are used for components of elastic displacement vector u_1 and u_3 [10]:

$$u_i^{(k)}(\alpha_1, \alpha_3) = \sum_{j=0}^{2} u_{ij}^{(k)}(\alpha_1) p_j(\alpha_3), \quad i = 1, 3, \tag{14}$$

where

$$p_0(\alpha_3) = \frac{1}{2} - \frac{2\alpha_3 - h_{k-1} - h_k}{2(h_k - h_{k-1})},$$

$$p_1(\alpha_3) = \frac{1}{2} + \frac{2\alpha_3 - h_{k-1} - h_k}{2(h_k - h_{k-1})}, \quad p_2(\alpha_3) = 1 - \left(\frac{2\alpha_3 - h_{k-1} - h_k}{h_k - h_{k-1}}\right)^2, \quad \alpha_3 \in [h_{k-1}, h_k].$$

For finding the unknown coefficients $u_{ij}^{(k)}(\alpha_1)$ in (14), approximation by the tangential coordinate α_1 was used on one-dimensional isoperimetric linear finite elements [10]:

$$u_{ij}^{(k)(e)} = \sum_{j,m}^{2} u_{ijm}^{(k)(e)}(\alpha_1)\varphi_m^{(e)}(\xi), \quad \xi = \frac{2\alpha_1}{\alpha_{12}^{(e)} - \alpha_{11}^{(e)}} - 1, \tag{15}$$

where e is the number of finite elements of *k*-th layer; $u_{ijm}^{(k)(e)} = u_{ij}^{(k)}(\alpha_{1m}^{(e)})$, $m = 1, 2$ are the values on nodes $\alpha_{1m}^{(e)}(\alpha_1)$ of finite element; $\varphi_1^{(e)}(\xi) = \frac{1}{2}(1-\xi)$; $\varphi_2^{(e)}(\xi) = \frac{1}{2}(1+\xi)$.

5. The discretized problem

Considered above differential formulation of the problem of geometrically nonlinear free vibrations for single layer is equivalent to the problem of minimizing the functional L [10]:

$$L = -\int_{\Omega} \sum_i \sum_j u_i \frac{\partial S^{ij}}{\partial x_j} d\Omega - \int_{\Omega} \rho \frac{\partial^2 U^T}{\partial t^2} \cdot U d\Omega =$$

$$= -\int_{\Omega} \sum_i \sum_j S^{ij} \frac{\partial u_i}{\partial x_j} d\Omega - \int_{\Omega} \rho \frac{\partial^2 U^T}{\partial t^2} \cdot U d\Omega \to \min . \qquad (16)$$

Boundary conditions (11), (12) and contact conditions (9), (10) are a natural for the variation formulation of the problem (16) [10], but conditions (13) must be take into account during solving.

In a case layered panel we obtain:

$$L = \sum_{k=1}^{K} \left(-\int_{\Omega_k} \sum_i \sum_j S_k{}^{ij} \frac{\partial u_i}{\partial x_j} d\Omega - \int_{\Omega_k} \rho_k \frac{\partial^2 U^T}{\partial t^2} \cdot U d\Omega \right) \to \min. \qquad (17)$$

After substituting (14), (15), and using (4) into (8) in (17) and composing results together we obtain:

$$L^{\Delta} = \{u\}^T K_L \{u\} + \{u\}^T K_{NL}(u)\{u\} + \{u\}^T M\{\ddot{u}\} \to \min , \qquad (18)$$

where $\{u\} = \{u\}(t)$ – vector of values of the coefficients $u_{ijm}^{(k)(e)}$ at nodes on the finite-element of *k*-th layer; K_L – linear, and K_{NL} – nonlinear components of stiffness matrix; M – matrix of mass [5]. Stiffness and mass matrices composed from *M* matrices for each layer.

For solving discretized problem (18) perturbation method is used, that is described in [5, 6].

6. Numerical results

6.1. Verification of the proposed technique

Consider a cylindrical five-layer panel, the edges of which are fixed by hinges at the bottom of the front plane (see Fig. 1.) with geometrical $l = 1$ m; $h = 0,01$ m and physical-mechanical characteristics:

$$E_1 = 40E_2, \quad G_{12} = G_{13} = 0{,}6E_2, \quad G_{23} = 0{,}5E_2, \quad \nu_1 = 0{,}25.$$

For the analysis of reliability of the results we applied the proposed technique to the problem, the solutions of which are known [4]. Consider a cylindrical panel with radius curvature $K = 0$. For finding the values of natural frequencies apply partition at 50 finite elements by coordinate α_1.

In Table 1 compared the values ω_{NL} / ω_L obtained at the amplitudes $\frac{w_{max}}{h}$ for free vibrations of five-layered panel with the results from the work [4].

Table 1.

$\frac{w_{max}}{h}$	ω_{NL}/ω_L	
	[4]	Proposed technique
0,2	1,0313	1,0401
0,4	1,1198	1,1214
0,6	1,2536	1,2695
0,8	1,4199	1,4418
1,0	1,6086	1,6588
1,2	1,8127	1,8627

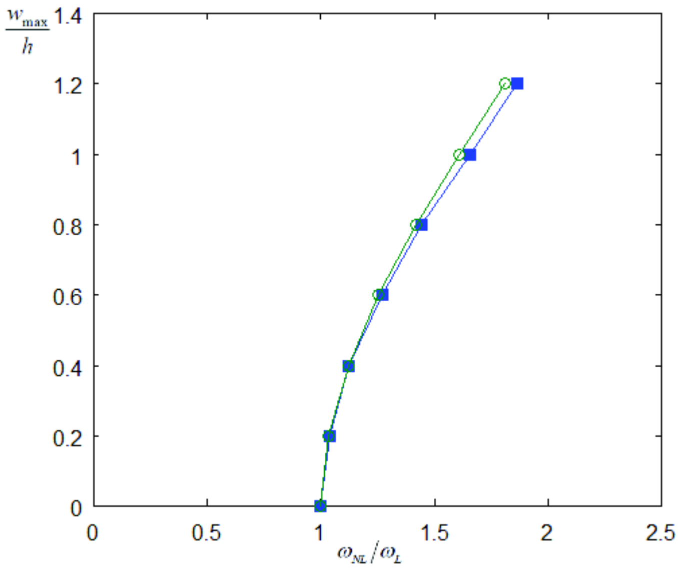


Figure 2. Comparison of amplitude-frequency characteristics obtained using the method of perturbation and results of work [4]

Fig. 2 shows the skeletal curves [11], constructed using the proposed technique (■) and the results given in the work [4] (o).

Also, the influence of the radius of curvature K on the free vibrations of the panel is investigation. Fig. 3 shows the dependence of the lowest natural frequency of the radius of curvature of five-layered panels from carbon fiber.

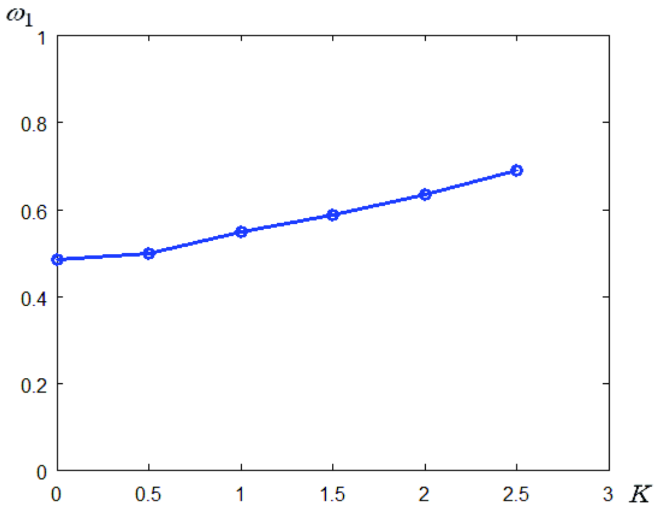


Figure 3. Dependence of the lowest natural frequency of the radius of curvature of the cylindrical panels

The maximum relative error in the Table 1 does not exceed 3%, which shows the effectiveness of the proposed technique. Comparative analysis of the graphs in Fig. 2 shows the reliability of the results obtained using proposed technique. Also established, that the main amplitude of natural vibrations increases with increasing radius curvature of the panel.

6.2. Three-layered panel

We considered a layered plate-strip with elongated edges that are fixed with stationary hinges on the lower plane (see Fig. 4). Geometrical characteristics of plane are $l = 1\ m$, $h = 0{,}1\ m$. It consists of three layers with following characteristics:

1) Rubber – $E = 0.1 \cdot 10^9\ N/m^2$, $\nu = 0{,}49$;
2) Steel – $E = 210 \cdot 10^9\ N/m^2$, $\nu = 0{,}3$.

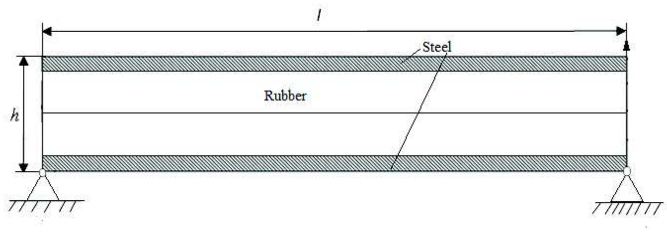


Figure 4. Panel with three layers

In Table 2 first five natural frequencies is shown for panel consisting of three layers where steel layers have thickness 0.01*m* and rubber has thickness 0.08*m*.

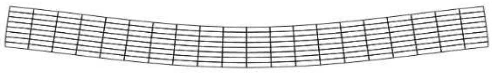
[1] Table 2.

n	ω_n
1	283000
2	1019000
3	1457300
4	1839600
5	2615200

In Table 3 dependency between first natural frequencies and thickness of middle layer (rubber layer) thickness is shown.

[2] Table 3.

$\frac{h_{rubber}}{h}$	ω_1
0.9	225650
0.8	283000
0.7	372770
0.6	490850
0.5	635100



a)

b)

Figure 5. View panels in different modes: a) – the first mode; b) – second

In the Fig. 5 we show the vibrations of the structure for first and second modes of the panel consisting of three layers where the steel layers have the thickness 0.01*m* and the rubber has the thickness 0.08*m*.

[3] Table 4.

K	ω_1
0	283000
0.5	254200
1	232000
2	218700

In Table 4 dependency between the radius of curvature and first natural frequency of the panel that consists of three layers where the steel layers have thickness 0.01*m* and the rubber has thickness 0.08*m* is shown.

For considered above panel we can make next conclusions:

1. the more matrix (rubber) component are included in the panel, the less is the first natural frequency;
2. the more radius curvature is the panel, the less is the first natural frequency of it.

7. Conclusion

We can make a conclusion that the method proposed in this paper is suitable for the layered panel because it provides logical results (Fig.5). Also this method can use at arbitrary amount of layers in the panel.

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