Transverse and Longitudinal Damped Vibrations of Hydraulic Cylinder in a Mining Prop

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Abstract

This study presents the influence of different kinds of damping on transverse and longitudinal vibrations of hydraulic cylinder in a mining prop. The dissipation of vibration energy in the model is caused by simultaneous internal damping of viscoelastic material of beams that model the system, external viscous damping and constructional damping. Constructional damping (modelled by the rotational viscous dampers) occurs as a result of movement resistance in the cylinder supports. The eigenvalues of the system with respect to changes in system geometry with two values of load and for a selected and variable damping coefficient values were calculated.

Keywords: damped vibration, hydraulic cylinder, transverse vibration, longitudinal vibration

1. Introduction

A hydraulic cylinder as an object of research studies on dynamics of mechanical systems has been extensively investigated in the number of studies. Most of the published studies focused on the interactions between the cylinder tube and piston rod. Results of the investigations of the dynamic response of the model of a cylinder to axial impulse were presented in paper [1]. The work [2] presents an analysis of the effect of initial inaccuracy of connection between the piston and cylinder tube on critical loading force in the cylinder. Many authors analysed the effect of sealing or the medium on the cylinder's dynamics and dynamic stability of cylinder. In study [3] calculations of free vibration frequencies were extended with the investigations of the dynamic stability of the cylinder by means of determination of geometrical parameters and load at the time of losing the stability were presented. In paper [4] the problem of the stability and free vibrations of a slender system in the form of a hydraulic cylinder subjected to Euler's load was carried out. The studies [5] and [6] present the effect of internal damping on vibrations of a support beam with a mass attached to a free end of the beam and on stability of a support column loaded with a follower force, respectively. The influence of small internal and external damping on stability of non-conservative beam systems is described in paper [7]. Equally interesting publication concerning the effect of external damping on vibration of beams with stepped cross-section is the study [8]. The effect of structural damping of fixations on free vibration of the linear Bernoulli-Euler beam was presented in the study [9].

In study [10] dissipation of vibration energy in the model of hydraulic cylinder – boom crane system occurs as a result of simultaneous internal damping of the viscoelastic material of the beam used in the model and the constructional damping in the supports of the cylinder and crane boom. The constructional damping of supports was modelled using rotational viscous dampers. The problem to be considered in the study [11] is the natural vibration of the system consisting of two clamped-free rods carrying tip masses to which several double spring-mass systems are attached across the span. The study is concerned with longitudinal vibrations of this mechanical system and the major contribution of this study is to derive a general formulation for the exact solution of the system described by using the Green's function method.

This study analyses the simultaneous effect of the constructional damping, internal damping, external damping and the influence of changes in system geometry on the transverse and longitudinal vibrations of hydraulic cylinder in a mining prop. The results obtained in the study were presented in 2D figures and spatial presentations.

2. Mathematical Model

A scheme of the considered system is presented in Fig. 1. The model of a hydraulic cylinder is composed of four beams. Two of them model a cylinder tube (l_{11}, l_{12}) and two - piston rod (l_{21}, l_{22}) in the cylinder. The liquid in the cylinder was adopted as the medium of load transfer between the piston and the cylinder along the length filled with liquid. The liquid rigidity in the cylinder was modelled by the translational spring. Stiffness coefficient of spring was denoted by k_S.

In adopted model dissipation of vibration energy was caused by simultaneous internal damping, external damping and constructional damping. Internal damping of the viscoelastic material for individual parts of hydraulic cylinder was characterized by Young's modulus E_{mn} and viscosity coefficients E^*_{mn}. External damping of medium surrounding the system were denoted by coefficient c_e. Constructional damping occurs as a result of movement resistance in the piston and the cylinder supports and it was modelled by the rotational viscous dampers. Damping coefficients of rotational viscous dampers were denoted by c_R.

The boundary problem connected to the free vibrations of the considered non-conservative (due to damping) system was formulated on the basis of Hamilton's principle in the following form:

$$\delta \int_{t_1}^{t_2} (T - V)dt + \int_{t_1}^{t_2} \delta W_N \, dt = 0 \qquad (1)$$

where: T – kinetic energy, V – potential energy, δW_N – virtual work of non-conservative forces originating from damping.

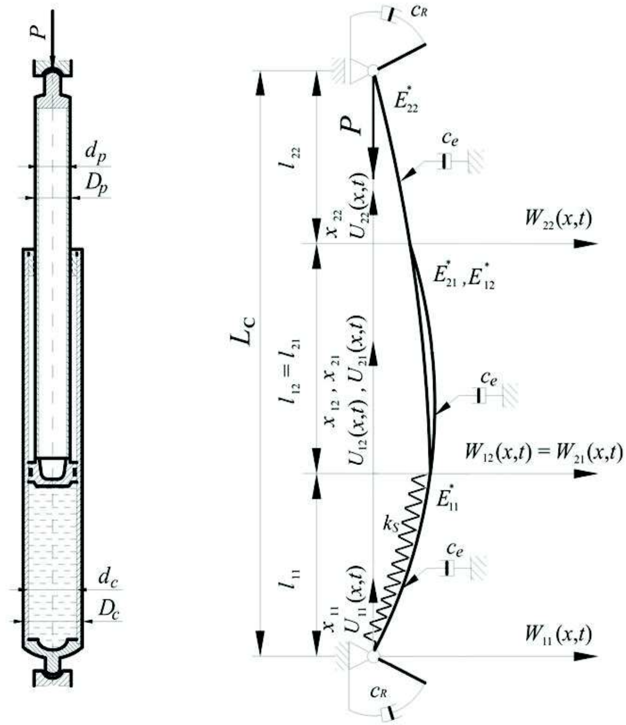

Figure 1. Diagram and beam model of a hydraulic cylinder with damping

The vibration equations for individual beams are known and have the following form:

$$J_{mn}\left(E_{mn}+E_{mn}^{*}\frac{\partial}{\partial t}\right)\frac{\partial^4 W_{mn}(x_{mn},t)}{\partial x_{mn}^4}+P\frac{\partial^2 W_{mn}(x_{mn},t)}{\partial x_{mn}^2}+ \\ +\rho_{mn}A_{mn}\frac{\partial^2 W_{mn}(x_{mn},t)}{\partial t^2}+c_e\frac{\partial W_{mn}(x_{mn},t)}{\partial t}=0 \tag{2}$$

where:

$$-A_{mn}\left(E_{mn}+E_{mn}^{*}\frac{\partial}{\partial t}\right)\frac{\partial^2 U_{mn}(x_{mn},t)}{\partial x_{mn}^2}+\rho_{mn}A_{mn}\frac{\partial^2 U_{mn}(x_{mn},t)}{\partial t^2}=0 \tag{3}$$

where: $m,n = 1,2$ ($c_e = 0$ for $m = 2$ and $n = 1$)
$W_{mn}(x_{mn}, t)$ – transverse displacement of beams that model cylinder and piston rod
$U_{mn}(x_{mn}, t)$ – longitudinal displacement of beams that model cylinder and piston rod
E_{mn} – Young's modulus for individual beams,
E^{*}_{mn} – material viscosity coefficient,
J_{mn} – moment of inertia in beam cross-sections,
A_{mn} – cross-sectional areas of the beams,

ρ_{mn} – beam material density,
c_e – viscous damping coefficient,
P – cylinder loading force (at the length l_{12} of the cylinder tube coverage with the piston rod in the cylinder P=0)
x_{mn} – spatial coordinates, t – time

Solutions of equations (2) and (3) are in the form:

$$W_{mn}(x_{mn},t) = w_{mn}(x_{mn})e^{i\omega^* t} \tag{4}$$

$$U_{mn}(x_{mn},t) = u_{mn}(x_{mn})e^{i\omega^* t} \tag{5}$$

where: ω^* – the complex eigenvalue of the system, $i = \sqrt{-1}$
Substitution of (4) and (5) into (2) and (3) leads to, respectively:

$$w_{mn}^{IV}(x_{mn}) + \beta_{mn}^2 w_{mn}^{II}(x_{mn}) - \gamma_{mn} w_{mn}(x_{mn}) = 0 \tag{6}$$

$$u_{mn}^{II}(x_{mn}) + \alpha_{mn}^2 u_{mn}(x_{mn}) = 0 \tag{7}$$

where:

$$\begin{gathered}\gamma_{mn} = \frac{\rho_{mn} A_{mn}}{J_{mn}(E_{mn} + E_{mn}^* i\omega^*)}\left(\omega^{*2} - \frac{c_e}{\rho_{mn} A_{mn}} i\omega^*\right), \\ \alpha_{mn} = \frac{\rho_{mn}\omega^{*2}}{(E_{mn} + E_{mn}^* i\omega^*)},\ \beta_{mn} = \sqrt{\frac{P}{(E_{mn} + iE_{mn}^*\omega^*)J_{mn}}}\end{gathered} \tag{8}$$

Boundary conditions:

$$\begin{gathered}
w_{11}(0) = w_{12}^{II}(l_{12}) = w_{21}^{II}(0) = w_{22}(l_{22}) = 0,\ w_{11}^{I}(l_{11}) = w_{12}^{I}(0), \\
w_{21}^{I}(l_{21}) = w_{22}^{I}(0),\ \ w_{11}(l_{11}) = w_{12}(0) = w_{21}(0), \\
w_{12}(l_{12}) = w_{21}(l_{21}) = w_{22}(0),\ \ E_{11}J_{11}w_{11}^{II}(0) = c_R i\omega^* w_{11}^{I}(0), \\
(E_{12} + E_{12}^* i\omega^*)J_{12}w_{12}^{II}(0) = (E_{11} + E_{11}^* i\omega^*)J_{11}w_{11}^{II}(l_{11}), \\
(E_{22} + E_{22}^* i\omega^*)J_{22}w_{22}^{II}(0) = (E_{21} + E_{21}^* i\omega^*)J_{21}w_{21}^{II}(l_{21}), \\
E_{22}J_{22}w_{22}^{II}(l_{22}) = -c_R i\omega^* w_{22}^{I}(l_{22}),\ \ (E_{11} + E_{11}^* i\omega^*)J_{11}w_{11}^{III}(l_{11}) + \\
+ Pw_{11}^{I}(l_{11}) - (E_{12} + E_{12}^* i\omega^*)J_{12}w_{12}^{III}(0) + \\
- (E_{21} + E_{21}^* i\omega^*)J_{21}w_{21}^{III}(0) - Pw_{21}^{I}(0) = 0, \\
(E_{12} + E_{12}^* i\omega^*)J_{12}w_{12}^{III}(l_{12}) + (E_{21} + E_{21}^* i\omega^*)J_{21}w_{21}^{III}(l_{21}) + \\
- (E_{22} + E_{22}^* i\omega^*)J_{22}w_{22}^{III}(0) = 0,\ \ u_{21}(l_{21}) = u_{22}(0),\ \ u_{11}(l_{11}) = u_{12}(0), \\
u_{11}(0) = u_{21}(l_{21}) = 0,\ \ (E_{21} + E_{21}^* i\omega^*)A_{21}u_{21}^{I}(l_{21}) = (E_{22} + E_{22}^* i\omega^*)A_{22}u_{22}^{I}(0), \\
(E_{21} + E_{21}^* i\omega^*)A_{21}u_{21}^{I}(0) = -k_S u_{21}(0),\ \ (E_{22} + E_{22}^* i\omega^*)A_{22}u_{22}^{I}(l_{22}) = P, \\
(E_{12} + E_{12}^* i\omega^*)A_{12}u_{12}^{I}(0) = (E_{11} + E_{11}^* i\omega^*)A_{11}u_{11}^{I}(l_{11})
\end{gathered} \tag{9}$$

The solution of equations (6) and (7) are expressed in the form of functions:

$$w_{mn}(x) = C_{1mn}e^{\lambda_{mn}x} + C_{2mn}e^{-\lambda_{mn}x} + C_{3mn}e^{i\bar{\lambda}_{mn}x} + C_{4mn}e^{-i\bar{\lambda}_{mn}x} \tag{10}$$

$$u_{mn}(x) = D_{1mn}e^{i\bar{\delta}_{mn}x} + D_{2mn}e^{-i\bar{\delta}_{mn}x} \tag{11}$$

where:

$$\lambda_{mn} = \sqrt{-\frac{\beta_{mn}^2}{2} + \sqrt{\frac{\beta_{mn}^4}{4} + \gamma_{mn}}}\ ,\ \bar{\lambda}_{mn} = \sqrt{\frac{\beta_{mn}^2}{2} + \sqrt{\frac{\beta_{mn}^4}{4} + \gamma_{mn}}}\ ,\ \bar{\delta}_{mn} = \sqrt{\alpha_{mn}} \tag{11}$$

The boundary problem is solved numerically for the eigenvalues ω^*. Depending on the solution adopted, the roots ω^* are complex numbers (that represent the damped vibration frequencies $Re(\omega^*)$ and damping $Im(\omega^*)$ in the considered system) and they may accept positive or negative value. In this paper, presentation of the results was based on positive values of the real and imaginary parts of solutions.

3. Numerical Calculation Results

Calculations were carried out for a cylinder used in a mining prop. Computations were carried out for the data contained in Table 1. Dimensionless damping parameters: η for internal damping, μ for constructional damping, and v for external damping were placed below the table.

Table 1. Geometrical and material data adopted in the study

Quantity	Symbol	Unit	Value
Cylinder tube - external diameter	$D_{11} = D_{12}$	mm	290
Cylinder tube - internal diameter	$d_{11} = d_{12}$	mm	250
Piston rod - external diameter	$D_{21} = D_{22}$	mm	160
Piston rod - internal diameter	$d_{21} = d_{22}$	mm	120
Cylinder tube and piston rod density	ρ_{mn}	kg/m^3	7.86e3
Young's modulus	E_{mn}	Pa	21e11

Damping parameters:

$$\eta = \frac{E_{mn}^*}{hE_{mn}}\ ,\ \nu = \frac{c_e L_C^{\ 3}}{d}\ ,\ \mu = \frac{C_R}{d}\ ,\ p = \frac{P}{P_C}\ ,$$

$$h^2 = L_C^{\ 4}\frac{\sum_{m,n=1}^{2}\rho_{mn}A_{mn}}{\sum_{m,n=1}^{2}E_{mn}J_{mn}}\ ,\ d = L_C\sqrt{\sum_{m,n=1}^{2}\rho_{mn}A_{mn}E_{mn}J_{mn}}\ , \tag{11}$$

where: P_C – the critical load of the cylinder extended to L_C=4m and $L_C = l_{11} + l_{12} + l_{22}$.

The results of the calculations are presented in Figures 2 to 5. The system was loaded with the longitudinal force P (p=0 and p=0.3). The dependency of the eigenvalues (real parts $Re(\omega_1^*)$ and imaginary parts $Im(\omega_1^*)$) on coefficients of constructional damping μ, external damping v, internal damping η and total length of cylinder that ranged from L_C=2.6m to L_C=4m was also determined. The relationships between the first eigenvalue of cylinder and changes its total length L_C and coefficient of constructional damping μ at p=0.3 without internal and external damping in the system are presented in the form of spatial diagrams in Figure 2.

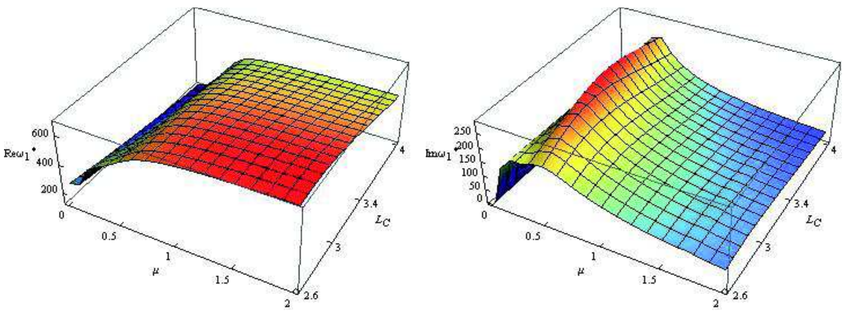

Figure 2. The dependency of the first eigenvalue ($Re(\omega^*)$ and $Im(\omega^*)$) for the cylinder on total length L_C and constructional damping μ at η=0, v =0 and p=0.3

As can be seen in the figure above, the higher value of $Im(\omega_n^*)$ then the more the amplitudes of a particular (n) mode of vibration are damped. Figure 3 presents the maximum values of $Im(\omega^*_{max})$ for the first mode of vibration in the examined system depending on the hydraulic cylinder length L_C for two values of loading.

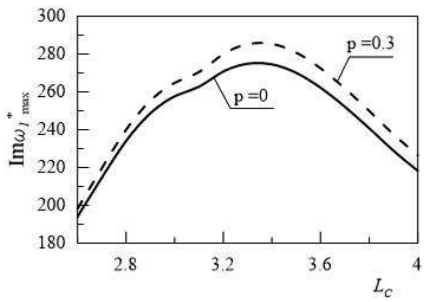

Figure 3. The relationships between the maximum values of $Im(\omega^*_{max})$ for the first mode of vibration in the cylinder and the extension total length L_C (for η=0 and v=0)

Next investigations focused on consideration of effect of different kind of damping on cylinder vibration. The dependency of real and imaginary parts of the first eigenvalue of the hydraulic cylinder on extension total length L_C for selected values of damping (η=0.02, v=0.5, μ=0.5) and for two values of loading are presented in Figure 4.

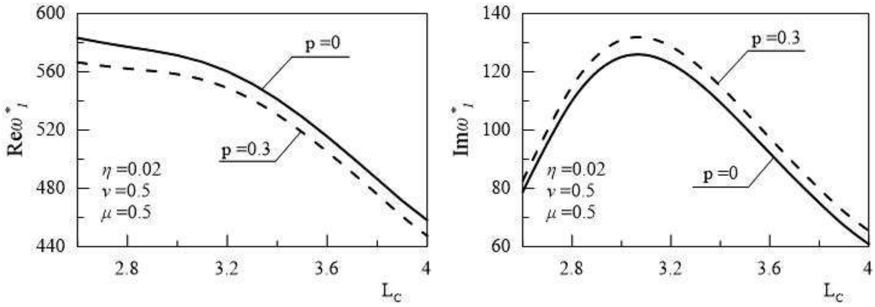

Figure 4. The dependency of the first eigenvalue (*Re*(ω^*) and *Im*(ω^*)) for the cylinder on extension total length L_C

The next figure (Figure 5) presents the change in the first eigenvalue of the hydraulic cylinder depending on the external damping ν and internal damping η without loading and loaded with the force p=0.3 for selected length of cylinder L_C=3m. The investigations were carried out for optimal constructional damping value μ=0.5.

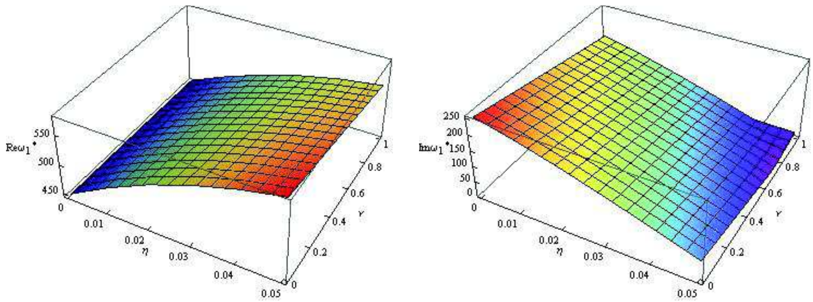

Figure 5. The dependency of the first eigenvalue (*Re*(ω^*) and *Im*(ω^*)) for the cylinder on internal damping η and external damping ν at μ=0.5 and L_C =3

4. Conclusions

This study presents a beam model of a hydraulic cylinder based on the system used in mining props. The computations for the model of transverse and longitudinal vibrations in a hydraulic cylinder with damping were carried out. The model of damping took into consideration the internal damping of the beams that modelled a cylinder tube and a piston rod, external damping and constructional damping that modelled motion resistance in the supports. Substantial changes can be observed in the damped frequencies *Re*(ω_1^*) and in degree of amplitude decay *Im*(ω_1^*) in the case of changes the length of hydraulic cylinder L_C and coefficient of constructional damping μ (Figure 2). An increase in constructional damping causes the increase in the values of degree of amplitude decay *Im*(ω_1^*) to maximum values, followed by *Im*(ω_1^*)$\rightarrow$0 where $\mu\rightarrow\infty$. These substantial changes in both *Re*(ω*) and *Im*(ω*) are caused by considerable intervention in the conditions of system fixation (in extreme cases, the fixation points are changed from joint mountings into rigid mountings). The length of hydraulic cylinder extension for which the degree of vibration amplitude decay is the highest allows for determination of optimum lengths of the hydraulic cylinder with respect to minimum vibration amplitudes in the system (Figure 3). It can be concluded based on the calculations that introduction of the internal and external damping causes only insignificant changes in the first eigenvalue (Figure 5). The results presented in the study help determine the geometric parameters and values of the coefficients that characterize damping of the system for which the maximum degree of amplitude decay is maintained.

Acknowledgments

This research was supported by the Ministry of Science and Higher Education in 2016, Warsaw, Poland.

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