Evaluation of a Simple Method of Identification of Dynamic Parameters of a Single-Degree-of-Freedom System

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Abstract

The paper presents a simple method of identification of dynamic parameters of single-degree-of-freedom systems based on an impulse test. Numerical simulations consisting in generating a model impulse excitation and the response of a model system with specified dynamic parameters to this excitation have been carried out to present the applicability range of the method. The influence on parameter estimation errors at various set values of these parameters and procedure parameters has been investigated.

Keywords: impulse test, identification of dynamic parameters, single-degree-of-freedom system

1. Introduction

To identify selected or all dynamic parameters, such as damping, stiffness and mass of a vibrating linear single-degree-of-freedom system it is possible to use many methods, from the simplest ones based on free vibrations and investigation of the influence of the test mass to harmonic or impulse tests [1, 2].

In the paper a method based on approximation of an experimentally determined frequency response function (FRF) is described. This function can be described for a single-degree-of-freedom systems as [3]:

$$H(j\omega) = \frac{X(j\omega)}{F(j\omega)} = \frac{\frac{1}{m}}{-\omega^2 + j\omega\frac{c}{m} + \frac{k}{m}} = \frac{\frac{1}{m}}{-\omega^2 + 2j\omega h + \omega^2{}_o}, \tag{1}$$

where: $H(j\omega)$ is the dynamic compliance , m – mass, k – stiffness coefficient, c – damping coefficient, $X(j\omega)$ is the system response in frequency domain, $F(j\omega)$ is the system excitation in frequency domain, j – imaginary unit, $\omega = 2\pi f$, $2h = c/m$, and ω_o is the frequency of free undamped vibrations.

The used identification methods were based on empirical determination of characteristics H_1 or H_2 according to the following definition:

$$H(j\omega) = H_1(j\omega) = \frac{G_{FX}(j\omega)}{G_{FF}(j\omega)}, \tag{2}$$

$$H(j\omega) = H_2(j\omega) = \frac{G_{XX}(j\omega)}{G_{XF}(j\omega)}, \tag{3}$$

where: $G_{FX}(j\omega)$ – cross power spectral density of the excitation and response signals, $G_{FF}(j\omega)$ – auto power spectral density of the excitation signal, $G_{XF}(j\omega)$ – cross power spectral density of the response and excitation signals, and $G_{XX}(j\omega)$ – auto power spectral density of the response signal. In case of absence of any disturbances both characteristics give the same results. In the event, however, where there are disturbances in the response signal, better results are obtained with characteristic H_1, and in case of disturbances in the excitation signal better results gives characteristic H_2.

The auto spectral density of signal x is estimated as [2]:

$$G_{xx}(\omega_k) = \frac{2\Delta t}{N}\left|X(\omega_k)\right|^2, \tag{4}$$

where: $X(\omega)$ – complex spectrum of the signal, Δt – sampling period, N – number of samples, k – consecutive number of a spectral line, and $\left|X(\omega_k)\right|^2 = X_{\mathrm{Re}}{}^2(\omega_k) + X_{\mathrm{Im}}{}^2(\omega_k)$.

By analogy, cross power spectral density of signals x and y may be expressed as:

$$G_{xy}(\omega_k) = \frac{2\Delta t}{N}\left[X^*(\omega_k)Y(\omega_k)\right]. \tag{5}$$

where: $X^*(\omega_k)$ - complex conjugate of a complex number.

Determination of a spectrum consists in computation of a discrete Fourier transform (most often with use of FFT algorithm):

$$X(k\Delta f) = \frac{1}{N}\sum_{k=0}^{N-1} x(n\Delta t)e^{-j2\pi nk/N}, \tag{6}$$

where: $x(n\Delta t)$ – signal sample, n – consecutive number of the signal sample.

The essential part of the algorithm in use is approximation of the experimentally determined characteristic H_1 or H_2 by means of the least square method. One can easily see that formula (1), taking the absolute value of the characteristic into account, can be written as:

$$\left|H\right| = \frac{1}{m}\left|A\right|, \tag{7}$$

where: $A = \dfrac{\omega_o^2 - \omega^2 - 2j\omega h}{\left(\omega_0^2 - \omega^2\right)^2 + 4\omega^2 h^2}$, $\mathrm{Im}(A) = -2h\omega / B$, $\mathrm{Re}(A) = (\omega_0^2 - \omega^2)/B$

and $B = (\omega_0^2 - \omega^2)^2 + 4h^2\omega^2$, $A = j\cdot\mathrm{Im}(A) + \mathrm{Re}(A)$, $|A| = \sqrt{\mathrm{Im}^2(A) + \mathrm{Re}^2(A)}$

Using the least square method one can determine the reciprocal of mass $\varphi = 1/m$ fitting model (7) to the experimentally obtained characteristic:

$$\varphi = (\mathbf{A}^{\mathbf{T}}\mathbf{A})^{-1}\mathbf{A}^{\mathbf{T}}\mathbf{H}, \tag{8}$$

where:

$$\mathbf{H} = \begin{bmatrix} |H_1| \\ |H_2| \\ \cdots \\ |H_{N/2}| \end{bmatrix} \qquad \mathbf{A} = \begin{bmatrix} |A_1| \\ |A_2| \\ \cdots \\ |A_{N/2}| \end{bmatrix}$$

H_k – vector of discrete values of characteristic H obtained experimentally, and $A_k = A(\omega_k)$. By means of the above we can determine the unknown parameter of mass. In the first step, however, vector **A** should be known. Elements of this vector may only be determined when parameters h and ω_0 of the investigated system are known. We will further assume that the damping is not so high.

Using the method of half power and the assumption of system linearity (the assumption of symmetry of the characteristic around the eigenfrequency) one can estimate coefficient h [3]:

$$h = \frac{\Delta\omega}{2} = \frac{\omega_2 - \omega_1}{2} \tag{9}$$

where: ω_1 and ω_2 are determined in such a way that the following condition is fulfilled:

$$|H(j\omega_1)| = |H(j\omega_2)| = \frac{|H(j\omega_o)|}{\sqrt{2}} \tag{10}$$

As the obtained characteristic H is discrete, it is worthwhile using here interpolation to determine ω_1 and ω_2 more accurately.

In accepting some error by assumption of small damping one can determine ω_0 by finding the frequency, which corresponds to the local minimum of the imaginary part of the FRF characteristic.

The use of the least square method (8) enables to fit the model function $|H|$ and to identify the mass parameter. This in turn, with the assumption of ω_o, enables to estimate stiffness parameter k and c.

2. Testing of procedures

It should be pointed out that in practice the errors of parameter estimation may depend on many factors. The errors will stem from the assumption of small damping in the system, the approximation of power spectral densities, the matching of the model characteristic, and from parameters of signal sampling.

To estimate the applicability range of the discussed simple method of parameter estimation some numerical simulations were performed. The simulations consisted in generation of a response of a system with defined parameters to the excitation in the form of an impulse, which duration was equal to the simulated sampling period and which had an assumed amplitude. The response signal in frequency domain was determined from equation (1). In the next step the response signal in time domain was obtained by means of the inverse Fourier transform. The generated signals created an input to the previously described procedure. Such an approach made it possible to simulate to a certain degree a situation of a real measurement. Obviously in a real experiment one should expect much more sources of errors, but identification of those which stem from the procedure itself seems valuable.

As simulation parameters were used both dynamic parameters of the system: c, k, m and sampling frequency f_p and the number of samples used for the analysis N. Both the latter ones were expressed by resolution:

$$\Delta f = \frac{f_p}{N} \tag{11}$$

whereby the sampling frequency ensured the actual analysis band, encompassing with a large excess the natural frequency of the simulated system.

Examples of the excitation and system response are shown in figure (1).

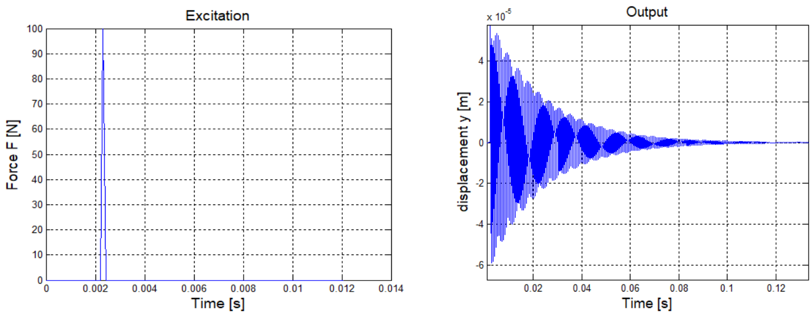

Figure1. Example of the simulated excitation and the response of the modelled system

A very important problem in the algorithm is the quality of fitting the model characteristic (1) to the obtained simulated $|H|$. Figure 2 shows an example of fitting (solid line) to a numerically determined characteristic (points). Figure 3 shows errors of identification of the mass parameter using the presented procedure at constant assumed stiffness k = 1MN/m and constant resolution df = 0.255Hz (11).

As it can be seen from the presented sample analyses (figure 3), with the increase in the mass the values of errors increase unacceptably. For small masses, corresponding to frequencies f0 higher than 700 Hz (for the assumed parameters) the errors are smaller than 5%, for lower frequencies, however, the estimation accuracy depends strongly on the value of the damping coefficient – the higher the better. In relation to the assumption of small damping this seems to be surprising. This, however, results from the fact, that at the defined resolution of the spectral analysis the determination of damping by means of the full width at half maximum (fwhm) method (formulae (9) and (10)) will be relatively more accurate for higher values of resonance breadth than for the lower ones. High inaccuracy of estimation of h influences the identification of the values of remaining parameters significantly. Moreover, it should be pointed out that in the presented simulations the requisite of a small damping value is fulfilled anyway even for the highest values of c, because the dimensionless damping coefficient does not exceed here the value of 0.01. Figure 3 indicates also that generally with the increase in the damping the approximation error of characteristic $|H|$ measured with error MSE decreases. For very small damping the characteristic is very smooth, which causes some problems at fitting. It should be noted, that for very small damping determination of the maximum value of the characteristic may be very difficult and the results should be treated as approximate.

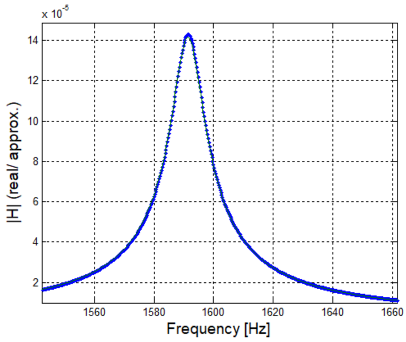

Figure 2. Example of an obtained characteristic |H| (points) and its approximation (solid line)

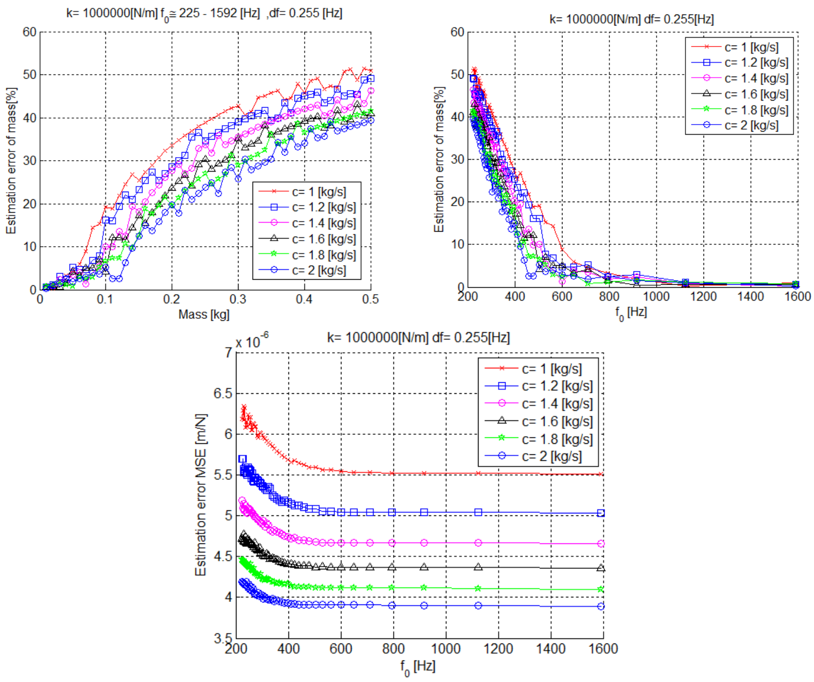

Figure 3. Sample result of simulation for different masses of the system and estimation of mass for resolution of 0.255Hz

Next figure shows the improvement of the accuracy of the estimation of mass with the increase in resolution adjusted by means of the number of samples. Unfortunately in practice increasing the number of samples may cause some problems related to the analysis of the response signal, which may disappear in a relatively short period of time in comparison to the analysis time. This will cause, that the noise recorded after disappearance of vibrations will be analyzed as well.
Examples of another simulations are shown in figure 5, where the estimation error of the stiffness parameter and the MSE error of the fitting of model characteristic |*H*| to the simulated data are shown. As it can be seen in the first two graphs the estimation error of stiffness is, for the assumed constant mass, acceptable and depends on the assumed stiffness of the model. Whereas with the increase in stiffness, or the value of f0, the MSE error decreases significantly. Hence, at the identification of mass of the system the MSE error may provide an indication as to the estimation accuracy of parameter *m*, but in case of identification of *k* such an indication does not exist. This stems from the fact, that the stiffness parameter is not identified by means of equation (8).

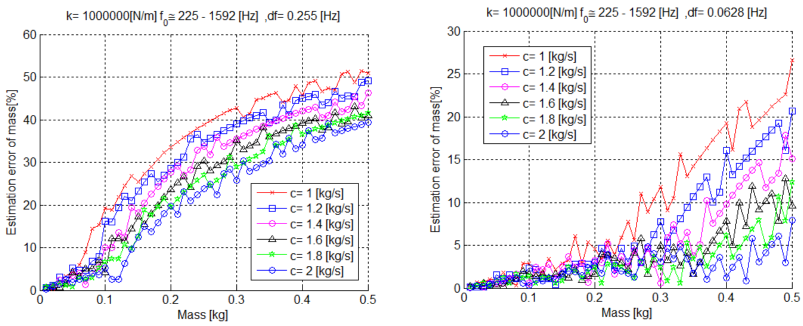

Figure 4. Sample simulation results for different masses of the system and estimation of mass for resolutions of 0.255Hz and 0.0628Hz

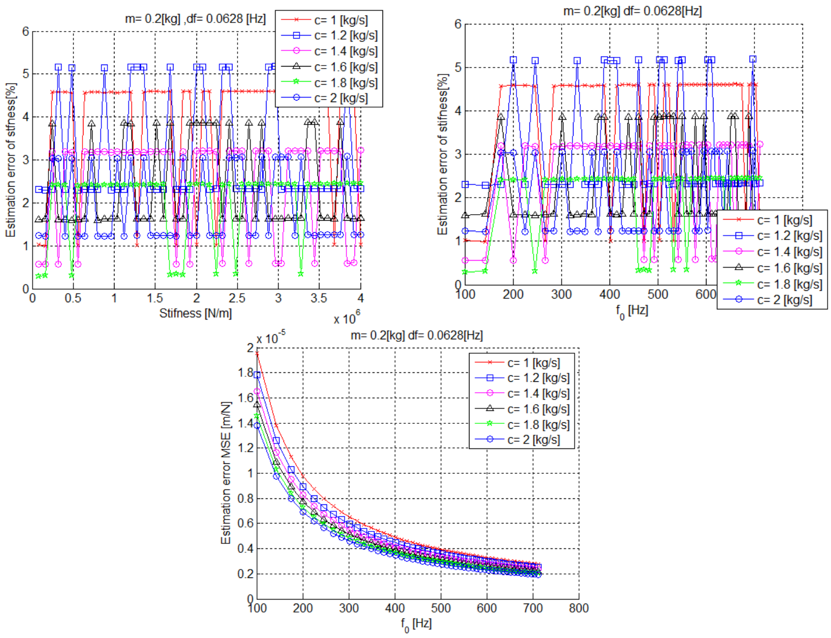

Figure 5. Sample simulation results for different stiffness and estimation of stiffness

Figure 6 shows a simulation concerning a change in system damping at a specified stiffness and mass. The damping properties are presented in categories of coefficient *c* and dimensionless damping coefficient ξ. As it can be seen, too small values of parameter *c* make the proper identification of this quantity difficult. For the values of the dimensionless damping coefficient lower than 0.005 (at the specified resolution) the errors of estimation can be higher than 5%.

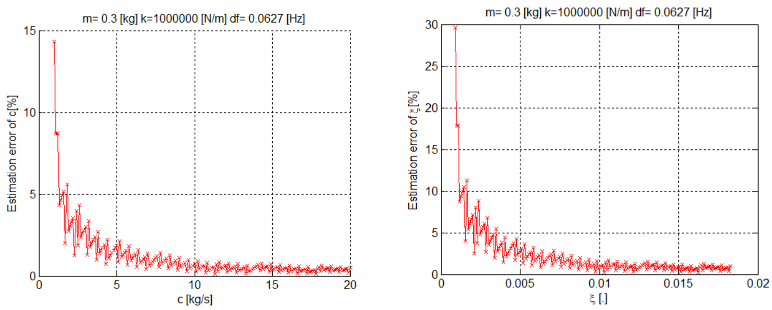

Figure 6. Sample results of simulation for different damping of the system and estimation of damping

3. Conclusions

The presented simple method of identification of dynamic parameters gives satisfactory results only within a certain range of these parameters. What is crucial here is the damping in the system. In the in simulations investigated range of values the method performs better for higher values of damping rather than for the lower ones, which is caused by errors in approximation of very narrow characteristics a-f and by too big errors in determination of full width at half maximum of the resonance in such cases. The situation can be improved by increasing the resolution of the analysis, but in practice such a possibility is limited. The correctness of identification of parameters cannot be evaluated directly by the analysis of quality of fitting of the model to characteristic $|H|$. It should also be noted that in the adopted method the errors in determination of parameters are of a different nature for different physical quantities. Assuming the remaining parameters to be constant – for the mass the error increases with its increase, i.e. decreases with the increase in f0. For the stiffness it does not depend monotonically neither from its value nor from f0. For the damping it decreases with its increase (for the investigated range of parameters).

Generally, as a result of the performed simulations and analyses it can be said that for damping values $\xi > 0{,}01$, free vibration frequencies f0 > 600Hz and for resolution $\Delta f \leq 0.255$ Hz the obtained estimated values of dynamic parameters do not differ from the real ones by more than 10%. In case of actual measurements one can expect higher error values, which is due to measurement uncertainties of the measured quantities, which were not taken into account in the presented simulations.

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