Vibrations in Physical Systems Vol.26 (2014)

Uncertainty in Vibroacoustic Investigations – Research Challenges

Wojciech BATKO
AGH University of Science and Technology
Faculty of Mechanical Engineering and Robotics
Department of Mechanics and Vibroacoustics
Al. Mickiewicza 30, 30-059 Kraków, Poland
batko@uci.agh.edu.pl

Abstract

The paper formulates the need of the modification of currently applied solutions of uncertainty assessments in vibroacoustic investigations, including the ones which use the convention rules developed by 7 international metrological organisations and described in the 'Guide to the Uncertainty'. Reservations versus currently used solutions are given in the hereby paper. It directs attention toward assumptions limiting the likelihood assessment of uncertainties of the obtained acoustic results.

It draws the possible ways of the problem solutions and related to them methods. It presents the need of connecting belonging to them algorithms with not classic statistical methods, allowing taking into account departures from generally used assumptions in presently applied solutions of uncertainty estimations of acoustic investigations results. The paper presents new research trends related to the uncertainty assessment of the environment acoustic hazards control.

The presented results and their conclusions can constitute the base for wider verification of the correctness of the currently applied procedures of acoustic measurements assessment and related to them estimations of errors levels of the uncertainty estimation of acoustic investigations results.

1. Introduction

The basic task of vibroacoustic measurements is obtaining reliable information on the vibroacoustic effect being investigated, since only likelihood results enable taking proper decisions. The problem of the uncertainty assessment of effects identified in vibroacoustic experiments is inseparably related to the uncertainty of measurements. It requires the validation of measurement procedures, the analysis of sources of possible random errors and the way of their working out in dependence of the probability distribution of their occurrence. The attention is focused on the determination methods of the standard uncertainty at direct and indirect measurements, on the uncertainty budget analysis and on determining conditions of selecting the expansion factor *k,* which is necessary in the expanded uncertainty assessment.

The unification of principles of calculations and uncertainty expressing was developed by the Joint Committee for Guides in Metrology (JCGM); under the aegis of the International Office of Measures (BIPM). These principles are contained in the 'Guide to the Expression of Uncertainty in Measurement (**GUM)** [2]. This Guide determines the methods of: drawing up measured data, principles of expressing uncertainty of measurements, and also defines the basic terminology. They are contained in nine documents under the common title: '*Evaluation of measurement data'* [1]. The document is accepted and recognised by the European co-operation for Accreditation as the basic pattern for the uncertainty determination in the certified research laboratories EA in every field of their activity, it means also in units dealing with vibroacoustic investigations. Whenever possible it is required, that the certified laboratories act according to the GUM when determining uncertainties related to quantitative results. The recommendations are also contained in legal acts and standards determining principles of the estimation of environment acoustic hazards.

The basic principle applied in uncertainty calculations, according to this document, is the uncertainty division into type A and type B. The type A uncertainty is determined on the basis of statistical conclusions, related to the analysis of the random measurement sample. The type B uncertainty is determined on the basis of the expert knowledge related to available information on possible systematic errors, e.g. errors resulting from Certificates of Standardisation of the equipment applied in the experimental process.

Both information on the possible errors of the type A and B should be treated equally, when estimating the uncertainty. This fact generates several methodological problems related to the compilation of the type A random error with the type B error being the determined variable.

The basis assumption of the GUM convention - in the type A uncertainty estimation process - is building the model of the measurement result, as the random variable Y described by the density probability function $g(y)$, for which two basic parameters i.e.: expected value $\mu(y)$ and standard deviation $\sigma(y)$ are determined. According to the idea contained in these guidelines, the uncertainty is understood as the numerical measure of the measurement inaccuracy, described in probabilistic categories and interpreted in the interval way. This interval is formed around the average value considered equitable with the expected value with the discussed parameter. It is given by the following relation:

$$P = P\{y \in (y - U; y + U)\} \tag{1}$$

determining with the required probability, equal the confidence level α, the fact that inside this interval the unknown, but real, measured value is present. This interval – determining the error of the measurement result U – is called the expanded uncertainty. It is estimated at assuming the known distribution $g(y)$ of the discussed parameter of the analysed vibroacoustic effect, using the condition:

$$\int_{-U}^{+U} g(y)dy = \propto = np. \cong 95\ \% \tag{2}$$

The expanded uncertainty: $U = k\ u$, is the product of standard uncertainty u and the expansion factor k, which is the quantile of the probability distribution of the measurement error, for the required confidence level α.

The standard way of working out the results given by the measuring series y_i; $i=1, 2, ..., n$, is based on calculating: average

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i \tag{3}$$

from control results and related to them standard deviation $s(y_i)$ during observations:

$$s(y_i) = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y}^2} \tag{4}$$

and the standard deviation of the average distribution observation y_i:

$$s(\bar{y}) = \frac{s(y_i)}{\sqrt{n}} = u_A(\bar{y}) \tag{5}$$

equated with the average uncertainty called the standard type A uncertainty $u_A(\bar{y})$.

In the common application of this approach it is assumed that the distribution of the measured value is - in approximation - the normal distribution, which at assuming the confidence level being 95% leads to the expansion factor 2. In this case the measurement result is hedged with the uncertainty interval:

$$y = \bar{y} \mp 2\, u_A(\bar{y}) \tag{6}$$

The value of the discussed parameter Y can be inaccessible directly in measurements. This value can be a function of several measured values X_i, being random variables described by affiliated to them density probability functions $g_i(x_i)$, with expected values $\mu_i(x_i)$ and standard deviations $\sigma_i(x_i)$. It is usually assumed, in such investigation procedure, that the function of this parameter $y=f(x_1, x_2, .. \quad .., x_n)$ is selected in such way as to have input quantities not correlated and random values X_i independent.

At such assumption concerning the discussed value Y, its expected value $\mu(y)$ and standard deviation $\sigma_i\,(x_i)$ are expressed as follows :

$$\mu\,(y) = \sum_{i=1}^{N} c_i\ \mu_i\,(x_i); \qquad \sigma(y) = \sqrt{\sum_{i=1}^{N} c_i^2 \sigma_i^2(x_i)} \tag{7}$$

where: $c_i = \frac{\partial f}{\partial\, x_i}$, and between functions of density probability distribution the convolution occurs:

$$\begin{aligned} g(y) &= g_1(x_1) * g_2(x_1) \ldots \qquad \ldots g_N(x_1) = g_i(x_i) * g_{i+1}(x_{i+1}) = \\ &\int_{-\infty}^{\infty} g_i(x_i) * g_{i+1}(x - x_{i+1})\, dx_i = \int_{-\infty}^{\infty} g_i(x - x_i) * g_{i+1}(x_{i+1})\, dx_i \end{aligned} \tag{8}$$

The problem of selecting the probability distribution for the estimated measurement results is the most difficult for the uncertainty estimation in such case. The determination of the convolution of the density probability of measured variables requires performing complex calculations not providing solution in a closed form. Generally the distribution form is either determined by numerical operations or approximated by the „*Monte Carlo*" method.

2. Methodological problems in applications of classic solutions of the uncertainty estimation determined by the gum convention

The problem of the uncertainty type A estimation in acoustic investigations is related to the determined statistical drawing up process. On the basis of the random measurement test the acoustic parameters of the considered effects are determined and equated with these effects. The correctness of such conclusion drawing is relevant to the correctness of applying the proper statistical methods. Their improper application leads to significant errors, which was broadly described in papers [4,5].

The lack of the probabilistic properties of the identified effect in relation to assumptions of the applied method of the statistical conclusions drawing should be recognised as the most often made methodological errors. Especially the attention should be directed to: randomness of the measurement test, verification of the assumptions correctness of the applied method of the statistical conclusions drawing, selection of the appropriately numerous measurement tests, and also to the selection of the proper estimators for the assessment of position statistics of the investigated acoustic effects, which determine uncertainties of considered identifications.

Thus, the correct application of the recommended in the '*Guide to the uncertainty'* [1] procedures of uncertainty estimations, requires fulfilling the determined class of assumptions, which acceptability should be analysed in-depth and which is often, unfortunately a marginalised activity. Especially, the uncertainty assessment of the result of the environment acoustic hazards control is related to the assumption, which takes the normal distribution form as the representative of the mathematical observation model for the sound level measurement results L_{Ai} ; $i=1, 2, .. \; .., n$. *T*he condition of the lack of the correlation of results in measurement series is essential. It seems obvious, that in case of inaccuracy of these assumptions, the average sound level value (representing the control assessment) or another noise indicator from the test measurement and their standard deviation (also from the test), cannot be the best estimation of the measurement result and thereby the best assessment of its standard uncertainty type A.

The majority of scientists intuitively assume the normality of the sound measurement results distribution of the tested population (*from which the random test for the estimation of the controlled noise indicators is taken)*. They are relating it to the results of the central limited theorem of Lindeberg-Levy, determining the convergent form of the random events distribution with the normal distribution. They do not consider the mechanism, which generates the sound level measurements results.

Taking into account this mechanism leads to distributions significantly differing from the normal distribution, as it was shown in papers [22,23]. This fact was also confirmed in works [8-11], in which the likelihood of this assumption was verified.

The condition that the results of sound level measurements in processes of controlling acoustic hazards must not be correlated can also be not fulfilled. A high level of noise disturbances can essentially influence successive calculations of the equivalent sound level constituting the investigated random test. Also measured tests of the equivalent sound level, necessary for calculating the controlled noise indicators, estimated in not distant time intervals can be correlated by external factors generating noises. This is documented by the results presented in paper [31], in which the effects related to this fact are also shown. They cause essential increasing of the standard uncertainty of the controlled noise indicator results.

Doubts related to a small likelihood of two out of three basic assumptions of the applied methodology of noise indicators estimation - according to the convention GUM presented in the 'Guide to the Uncertainty' [2]; generate the need of the development of new model formalisms for the realisation of such tasks. Their realisation ideas will be presented in the next item.

3. New concepts and their model formalisms dedicated to the uncertainty assessments in environment acoustic investigations

The described above limitations of generally applied estimation methods of the uncertainty of the acoustic investigations results with restricted assumptions, generated approaches of looking for the new model. In works of Department of Mechanics and Vibroacoustics AGH an attention was directed towards model formalisms allowing analysing statistic measurement data in which departures from classic assumptions, mainly from the assumption of normality of distribution of the controlled noise indicator results and of not correlated random sample results, are possible. The attention was directed towards the currently developed methodology of statistical investigations of effects, which have the same specificity of conditions, colloquially called ‘not classic statistical methods’ [15,20].

To the solutions of statistical conclusions drawing, related to these methods, enabling the estimation of the expected values of the investigated populations and assessment of their variance (i.e. allowing to create confidence intervals for the realised estimations) belong the methods based on the model formalism of:

- time series [16,26],
- kernel estimators [16],
- bootstrap analysis [13,19,20],
- Bayes’ analysis [21],
- propagation of distributions [22-25].

The analysis of the application possibility of these solutions and their adaptation for the needs of the estimation of long-term noise indicators and building of assigned to them confidence intervals was the subject of numerous scientific works in the Department of Mechanics and Vibroacoustics and related to them Ph.D. Thesis, either finished or in the realisation stage . Their results were presented in several publications [15,-17,19-21, 22-25].

It is assumed, in the process of assigning to results of control measurements $\{x_1, x_2,..,x_n\}$ the representative in the form of **time series** [26]; (*being a sequence of random values of variable X describing the state of the analyzed acoustic effect*); that its probabilistic structure can be shaped by the mechanism:

$$X_t = \mu_t + \varphi_t + \xi_t ; \quad t = 1,2,. \ldots, n \tag{9}$$

It has the following components: trend μ_t related to a constant tendency forcing the level of changed values of the analyzed acoustic parameters, cyclic component φ_t - representing cyclic changes related to recurrent characteristic excitations, and the residual component representing random disturbances (or inaccuracy of the model description) ξ_t of the normal distribution $N(0, \sigma_\xi^2)$.

In contrast to the classic model of the random control test (which assumes that random observations are of a normal distribution), the proposed approach assumes the presence of a certain mechanism forcing changes of control results values, which can be represented with the accuracy characteristic to Gaussian disturbances of the expected value being zero and variance σ_{ξ}^{2} .

The estimation problem, of the expected value and variance of the analyzed acoustic parameters in such approach, is reduced to the identification of the time series structure. It requires determining the proper approximation $\widehat{\mu_t}$,$\widehat{\varphi_t}$ for components μ_t and φ_t, which should provide the correct variability description of successively observed results of control tests (with a random Gaussian error ε_t, of the expected value being zero and variance σ_{ε}^{2}).

The correct selection of the approximation for the observed series of changes of the acoustic parameters values being controlled, requires the identification of its variability properties. Helpful in this process are generally applied solutions allowing to resolve problems concerning:

- stationary of the analysed time series;
- presence of the cyclic component (in this time series);
- homogeneity of the observation set and properties, including the random component variance.

The results of such identifications are helpful in selecting the correct modelling of time series formed from the control results values.

The realised examples of such analyses, in relation to the estimation of noise indicators determining the acoustic climate and assessments of their uncertainties, are given in the Ph.D. thesis of R. Bal [16]. They were referred to the results of the continuous noise monitoring, recorded at one of the main arteries in Krakow. They provided recommendations for the proper model selection for the estimation of the long-term sound indicators, describing the acoustic climate in the analysed areas and for the assessment of their uncertainty.

The application of kernel estimators allowing the likelihood estimation of the density probability distribution function of the analysed acoustic parameters and related to them uncertainty assessments can become the helpful solution [18]. The unknown function of the density probability $f(x)$ of n-dimensional random variable X, is – according to this procedure – calculated on the basis of experimentally determined values of m-element test: $x_1, x_2, \ldots, x_m$ of the analysed random variable from the following dependence:

$$\hat{f}(x) = \frac{1}{mh^n}\sum_{i=1}^{m} K\left(\frac{x-x_i}{h}\right) \tag{10}$$

in which the function $K(x)$ meets the condition:

$$\int_{-\infty}^{\infty} K(x)dx = 1 \tag{11}$$

It is called the nucleus, while the positive index h is called the smoothing parameter. It is possible to apply various nuclear functions, presented in paper [18], in the estimation process. The selection is related to the condition of the proper adapting of estimator $\hat{f}(x)$ to the real density function $f(x)$, characterised by the effectiveness factor, defined as:

$$Ef\ (K) = \frac{3}{5\sqrt{5}}\left[\int_{-\infty}^{\infty} t^2\ K(\,t)dt\right]^{\frac{1}{2}}\left[\int_{-\infty}^{\infty} K(t)dt^2\right]^{-1} \tag{12}$$

The adaptation of this solution for the needs of the acoustic environment control, together with its effectiveness assessment was the subject of the Ph.D. Thesis of B. Stępień [20] and analyses given in papers [19,20]. They Illustrate conditions of the effective estimation of the long-term noise indicators expected values and assessments of their uncertainties.

Similar analyses were performed in relation to the **bootstrap method** [13] recommended for the uncertainty assessment in acoustic investigations. Its solution leads to the distribution function determination for the expected value and variance of the analyzed acoustic parameters, on the grounds of the results of the individual random sample $\{x_1,\ x_2,.\quad .,x_n\}$. It does not require the determined assumptions concerning the measuring test probability distribution. It provides the way of creating research statistics. Bootstrap copies constitute its data base. Their data are generated in such way that from the set of measurement results $\{\ x_i\ \}$ the test of the determined size is drawn $\{\ x_i^*\ \}$ and the drawn numbers are not removed from the test. In such way not one but several copies, allowing to calculate the statistical characteristic of the analyzed acoustic parameters, are formed. Functional properties of the bootstrap method (*based on copied data*) analyzed in the estimation process of controlled noise indicators and assessments of their uncertainty, were published in several papers [19,20]. The proposed solution occurred to be the efficient tool in the estimation of the expected value and variances of the controlled noise indicators.

The successive estimation method of the controlled noise indicators [ ] and assessments of their uncertainties (*recommended for using in environment control*), currently being developed in KMiWA AGH, is the solution based on the **Bayes' method.** The estimated parameter, is a random variable, for which the *a priori* distribution is assumed on the grounds of logical premises and analyses as well as on other information originated from outside of the control test. From the formal point of view the proposed Bayes' mathematical formalism is reduced to treating the estimated acoustic parameters and assessments of their uncertainties (*being random variables, not known a priori*) in relation with the classic reasoning, based on the probability mathematics. Especially two probabilistic principles are applied: *determinations* versus the value of the observed measurement test (i.e. statistic control data) and *the determination of boundary distributions* '*a posteriori'*, for the variable being under investigations (i.e. possible future values of the controlled acoustic parameter).

Bayes' reasoning answers directly (intuitively) the question concerning the probability of the hypothesis, in relation to the obtained measurements results. Thus, it should be expected that this solution can have *a good, wider application perspective* in practical control of acoustic investigations. In favour of this approach application will act more and more general access to computational tools (e.g. *allowing multidimensional numerical integration*), inseparably related to the Bayes' analysis. The studies performed within this scope in the Department of Mechanics and Vibroacoustics [21] are reminding its realisation grounds and showing its practical potential from the perspective of the already realized long-term noise indicators and assessments of their uncertainties.

An important direction of works, concerning uncertainty assessments in the estimation of the environment acoustic state, is looking for the density probability distribution $p[x(\sum_{i=1}^{n} q_i)]$, being the summary result of all measured variables q_i, contributing to the final assessment of the controlled acoustic parameter.

To this aim it is possible to look for the solution by the **method of propagation of partial distributions** related to transformations of measuring results q_i, in the analyzed control assessment. As the example of such task can serve the problem of the logarithmic mean estimation $L_{śr}$ = 10 log($1/n \sum_{i=1}^{n} 10^{0.1L_i}$), being the sum of independent sound level random results L_i, $i = 1, 2, . . . , n$. The analytical solution of this problem is difficult, due to the necessity of performing complex transformations leading to the determination of the looked for probability distribution:

$$p[x(\textstyle\sum_{i=1}^{n} q_i)] = p[x(q_1)] * p[x(q_2)] * . \quad .* \ p[x(q_n)] \tag{13}$$

which is the convolution of the partial variables distributions. On its grounds, it is possible to estimate the expected value of the controlled noise indicator and the expansion factor $k(\propto)$, allowing to determine confidence intervals for the controlled variable.

This problem was applied for the task of the estimation – of the mentioned above – average sound level $L_{śr}$ = 10 log($1/n \sum_{i=1}^{n} 10^{0.1L_i}$), determined by the sum of independent random results L_i, $i = 1, 2, . . . , n$ of sound level measurement [6, 7]. The possibility of obtaining – by this method – the recurrent algorithm for the expected value and variance (of this variable) estimation at the assumed form of the probability distribution of controlled results, was indicated.

The new document of the Joint Committee for Guides in Metrology (JCGM 102:2008) [28] Guide to Uncertainty [2] (edited by the International Standardisation Organisation) corresponds with the studies in KMiWA, the estimation approach to controlled variables, based on the distributions propagation method. The ISO document propagates new standard in the scope of uncertainty calculations of the control result by distributions propagation method. The probability distribution of the controlled variable – according to the recommendations contained in this document – should be calculated by means of the Monte Carlo simulation by the mathematical model of input values, contrary to the analytical approached being developed in Department of Mechanics and Vibroacoustics AGH.

4. Research challenges

The analysis of the realisation basis of uncertainty assessments in identifications of acoustic investigations indicates that its correctness is fully attributed to the correctness of the statistic inferences with respect to the performed acoustic measurements. Analyses of assumptions related to inferences are indispensable as well as looking for the proper interpretation for the mechanism of generating measurements random test results. Fulfilling these conditions is necessary for the correct statistic inferences, which are aimed at the determination of the confidence interval containing – with the determined probability – the hypothetic, true value of the acoustic variable being under control.

Applications of estimated uncertainty solutions, based on not fully random techniques of obtaining control data and also not having properties of random test, required for the correct uncertainty assessment, is quite common in investigation practice of the identification of the determined acoustic effects parameters (*including numerous control procedures of the environment acoustic hazards*).

It especially concerns the problems listed below:

- The lack of investigation specifications related to the analyzed acoustic effect and – connected with it – randomness requirement and representativeness of the random test (taking the measurement test for inferences from not properly defined, or not defined at all, investigation population).
- None reflections on the correctness of assumptions of the estimated uncertainty solutions, including: the normality of the measurement results distribution and the lack of the correlation of the random test results.
- Not taking into consideration the requirement of the proper testing sample size versus the realised assessments of the measurement results uncertainty.
- Not proper verification of the hypotheses - being assumptions of the assumed estimative solutions of uncertainties.
- Using the same data base in processes of formulating and of testing of the assumed investigation hypothesis - in relation to the identified acoustic effects.

Inferences concerning acoustic investigations uncertainties – according to the GUM convention – i.e. on the bases of the measurements results distribution creating random measurement series, are burdened by numerous faults and limitations in acoustic tests. The impossibility to assure properly numerous measurements series (in the majority of realised acoustic investigations) belongs to these limitations. This is usually related to a large size of the investigation task, its costs and labour-consumption. Assumptions contained in them are difficult to be accepted (*as can be noticed in several references*), and their correctness is questioned by several environmental noise tests.

There is also a serious mathematical problem in selecting the model formalism, which would allow to join the type A uncertainty with type B, it means the uncertainty estimated by statistic methods with information on possible error ranges given by the a priori expertise.

In case when information – on the possible error of the acoustic measurement – are not sufficient or of a small reliability the characteristics of the identified effect could be restricted only to the statistic description and not to the statistic inferring in relation to the uncertainty assessment of the obtained results. The observed correctness in the test should be treated as the test representative, however without attributing to it the error size with the determined probability of its correctness. The statistic inferring application, recommended by the GUM convention in uncertainty assessments, seems unjustified.

The proposed in papers [30,31] formalisation based on interval algebra seems to be the worthy recommendation method of solving the problem. In this case the metrological interpretation for both ways of defining the measurement result uncertainty, i.e. described in papers [30,31] and guidelines of the GUM, can be quite similar. However, within the range of the mathematical formalisation of both ways essential differences can be seen. In case, when the uncertainty is defined as the interval of possible values of measurement variables, successive steps towards obtaining the interval result of the control measurement can be consistently realised from the measuring process description. It does not require meeting – difficult for the measuring practice – assumptions of the GUM convention.

This behaviour differs from the solutions recommended in the Guide, which applications can be really limited to laboratories, since only in laboratories the numerous, homogeneous and not correlated measurement series can be achieved. Such requirement is difficult to be accepted in the control of the acoustic environment state. Essential variancies of measuring conditions, in which the results reproducibility is disturbed, occur in acoustic environment measurements. This type of limitations create uncomfortable situations in which legally recognised way of uncertainty determining has a limited application and which in effect causes – very often – ignoring the needs of the verification of assumptions applied in the uncertainty assessment method, by persons performing measurements.

The fact that for the solutions determined by the GUM convention it is difficult to find the mathematical justification in physical interpretations of the measurements of analysed acoustic effects, to assumed statistical models used for assessments of uncertainties of their identification, is the essential argument for the possible marginalising of these solutions.

The approach based on the Bayes' method can be interesting for the application [20,21]. Admittedly, in such case a priori knowledge of probabilistic error characteristics is needed, but the modern technique provides several useful tools, and thus this will not constitute an essential limitation.

Summarising, it can be stated that current analyses of the uncertainty in acoustic investigations based on the GUM convention (*in basic assumptions and formal ways of their model solutions*) are weakly justified. There is a noticeable gap between their assumptions and constrains supplied so far by investigation experiences from the environment acoustic monitoring.

Limitations in the currently binding assessments of the uncertainty of acoustic investigations results, sketched in the hereby paper, can become a source of inspiration for searching and development of better formal bases for the calculation procedures of their uncertainty.

Thus, broader investigations concerning the model formalisms, indicated in the paper, based on '*not classic statistics methods'* (free from limitations of the current methods of the uncertainty assessment) should be undertaken. This would allow to verify the divergence level in uncertainty assessments of the controlled acoustic effects.

References

1. EA-04/16 rev00 December 2003 *EA guidelines on the expression of uncertainty in quantitative testing*.
2. *GUM - Guide to the Expression of Uncertainty in Measurements*. BIPM, IEC, IFCC, ISO, IUPAC, IUPAP, OIML. International Organization for Standardization, Switzerland 1993, 1995.

3. EN ISO 4871: *Acoustics – Declaration and verification of noise emission values of machinery and equipment*.
4. P.I. Good, J.W. Hardin, *Common Errors in Statistics* (*and How to Avoid Them*), John Wiley & Sons, 2003.
5. D. Huffa, *How To Lie With Statistics*, W.W. Norton & Company, 1954, 1982, 1993.
6. DIN 45 641 *Mittelung von Schallpegeln*, Juni 1990.
7. VDI 3723: *Anwendung Statistische Methoden bei der Kennzeichnung schwankender Gerauschimmisionen* (*Application of statistical methods for the description of variating ambient nosie levels*), VDI-Handbuch Larmminderung, 1993.
8. T. Wszołek, M. Kłaczyński, *Effect of traffic noise statistical distribution on $L_{Aeq,T}$ measurement uncertainty*, Archives of Acoustics, **31** (2006) 311–318 (supplement).
9. M. Gałuszka, *Statistical distribution levels and energy road noise*, Proc. I International Conference Environment Monitoring, AGH, Kraków 2010.
10. W. Batko, B. Stępień, *Analysis of the Probability Distribution of traffic Noises*, Conference Proc. XXXV–th Winter School of Vibroacoustic Hazards Control 5- 16, Gliwice-Ustroń 2007 (in Polish).
11. W. Batko, R. Bal, *Verification of the calculation assumptions applied to solutions of the acoustic measurements uncertainty*, Proc. 16th International Conference on Noise Control'13, 26-29 May, Ryn-Poland, 2013.
12. C. Domański C., K. Pruska, *Nieklasyczne metody statystyczne*, PWE, Warszawa 2000.
13. B. Efron, R. Tibshirani, *An Introduction to the Bootstrap*, Chapman & Hall, New York 1993.
14. S. Martinez, *Geraeuschmessungen und Immissionsprognosen: Ermittlung von Mess- und Prognoseunsicherheiten, Anwendung der Bootstrap- und der Gaußschen Methode*, Laermbekaempfung BD. **3**, 1 (2008) 16–25.
15. W. Batko, R. Bal, *Analysis of usefulness of R.G. Brown model for estimation process of long term noise ratio*, Archives of Acoustics, **33** (2008).
16. R. Bal, *Analysis of estimation conditions of the long–term noise indicators,* Ph.D. Thesis, AGH, 2010 (in Polish).
17. W. Batko, R. Bal, *Modeling of Estimation Tasks of the Year Long Noise Indicators*, Mechanics and Mechanical Engineering, Technical University of Lodz, **15** (2011) 11–20.
18. P. Kulczycki, *Estymatory jądrowe w analizie systemowej*, WNT, Warszawa 2005.
19. W. Batko, B. Stępień, *Application of the bootstrap estimator for uncertainty analysis of the long-term noise indicators*, Acta Physica Polonica A, **118** (2010) 11–16.
20. B. Stępień, *Estimation of uncertainties of the environment noise hazard indicators*, Ph. D. Thesis, AGH, 2011 (in Polish).

21. W. Batko, B. Stępień, *Application of the Bayesian inference for estimation of the long-term noise indicators and their uncertainty*, Acta Physica Polonica A, **119** (2011) 916–920.
22. W. Batko, B. Przysucha, *Determination of the Probability Distribution of the Mean Sound Level*, Archives of Acoustics, **35** (2010) 543–550.
23. W. Batko, B. Przysucha, *Random Distribution of Long - Term Indicators of Variable Emission Conditions*, Acta Physica Polonica A, **119-6-A** (2011) 1086–1090.
24. W. Batko, B. Przysucha, *Determination of the distribution form for sound level measurements results* (sent for publication in Archives od Acoustics).
25. W. Batko, B. Przysucha, *Cascade method of the determination of the sound level probability distribution function* (sent for publication in Archives od Acoustics).
26. G.E.P Box, G.M. Jenkins, *Analysis of time series: forecasting and control*, PWN, 1983 (in Polish).
27. L. Robert, R.L. Launer, G.N. Wilkinson, *Robustness in Statistics*, Academic Press, New York 1979.
28. *Evaluation of measurement data – Supplement 1 to the Guide to the expression of uncertainty in measurement – Propagation of distributions using a Monte Carlo method* (GUM), BIPM, JCGM 101:2008.
29. W. Batko, P. Pawlik, *Uncertainty Evaluation in Modelling of Acoustic Phenomena with Uncertain Parameters Using Interval Arithmetic*, Acta Physica Polonica A, 121 (2012) A-152-155.
30. W. Batko, P. Pawlik, *New approach to uncertainty assessment of acoustic effects in the environment*, Archives of Acoustics, **37** (2012) 57–61.
31. W. Batko, R. Bal, *Analiza skorelowania wyników pomiarowych w ocenach stanu zagrożeń hałasowych środowiska*, Rozdz. w monografii „Systemy wspomagania w inżynierii produkcji: środowisko i bezpieczeństwo w inżynierii produkcji. Red. W. Biały, A. Kuboszek, pp. 67–74, Wydawnictwo PA NOVA S.A., Gliwice 2013.