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Thermoelastic Wheel-Rail Contact Problem for a Multi-Layer Structure

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Abstract

Wheel-rail thermoelastic contact problem is analysed and numerically solved in the paper. The surface of the rail is assumed to consist from layers having distinct constant material parameters and a functionally graded material layer between. Thermal and mechanical properties of the graded layer are dependent on its depth rather than constant as it is considered in the literature. Numerous laboratory experiments indicate that graded materials layers or coatings covering the conventional steel body can reduce the magnitude of contact and/or thermal stresses as well as the noise and the rolling contact fatigue. The contact phenomenon includes friction as well as frictional heat generation and wear. Quasistatic numerical approach is used to solve numerically this contact problem. Numerical results are provided and discussed.

Keywords: thermoelastic rolling contact problem, functionally graded materials, quasistatic method

1. Introduction

Two-dimensional rolling contact problems including friction, frictional heat generation and wear are solved numerically in this paper. The unilateral contact of a rigid wheel with an elastic rail lying on a rigid foundation is considered. The friction between the bodies is described by Coulomb law [1,2,3]. The coefficient of friction is assumed constant. Due to the heat conduction, the frictional heat flow is directed into the coated medium [4]. We employ Archard's law of wear [5]. In the model the wear is identified as an increase in the gap between bodies.

The thermoelastic contact or rolling contact problems were considered by many authors (see references in [1,3,6,7,8,9,10,11,12]). Numerous laboratory experiments indicate [2,8] that the use of a coating material attached to the conventional steel body reduce the magnitude of residual or thermal stresses. It leads to the reduction of the rolling contact fatigue and noise. However in a conventional coating structure homogeneous materials are used. The abrupt change in the mechanical properties of the materials at the surface coating-substrate interface results in stress concentration or degraded bonding strength [9]. Thermoelastic rolling contact problem with two layer surface model with the material properties governed by the power law are considered in [12].

In this paper, following [13], we assume that between the homogeneous coating layer and the homogeneous substrate there exists the graded interlayer which properties depend on its depth according to the exponential law. We consider also thermoelastic contact phenomenon with the frictional heat flow rather than elastic contact model as in [13].

In the paper we take special features of this rolling contact problem and use so-called quasistatic approach [14] to solve it numerically. In this approach the inertial terms in elastic and heat equations are replaced by the stationary terms reflecting the dynamics of the body and heat transfer rather than completely neglect them as in the classical quasistatic formulation. Therefore, after brief introduction of the thermoelastic model of the rolling contact problem with friction and wear in the framework of two-dimensional linear elasticity theory the general coupled time dependent system describing this physical phenomenon is formulated. This system is transformed into equivalent stationary system in so-called quasistatic formulation To solve numerically this stationary system we will decouple it into mechanical and thermal parts. Finite element method is used as a discretization method. The numerical results including the distribution of temperature field in the contact zone are provided and discussed.

2. Problem formulation

Consider deformations of an elastic strip lying on a rigid foundation (see Figure 1). The strip has constant height h and occupies domain $\Omega \subset R^2$ with the boundary Γ.

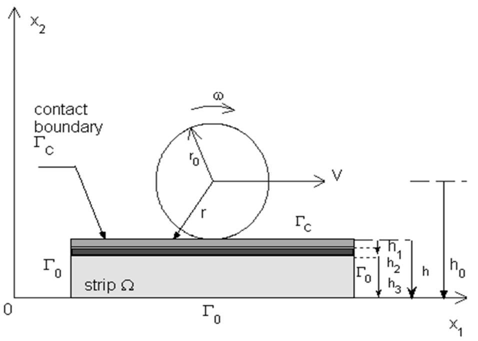

Figure 1. Wheel rolling over the strip

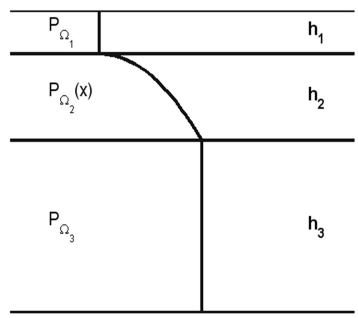

Figure 2. Three-layers model

A wheel rolls along the upper surface Γ_C of the strip. The wheel has radius r_0, rotating speed ω and linear velocity V. The axis of the wheel is moving along a straight line at a constant altitude h_0 where $h_0 < h + r_0$, i.e., the wheel is pressed in the elastic strip. It is assumed, that the head and tail ends of the strip are clamped, i.e., we assume that the length of the strip is much bigger than the radius of the wheel. Moreover it is assumed, that there is no mass forces in the strip. The body is clamped along a portion Γ_0 of the boundary Γ of the domain Ω. The contact conditions are prescribed on a portion Γ_C of the boundary Γ. Moreover, $\overline{\Gamma_0} \cap \overline{\Gamma_C} = \emptyset$. $\overline{\Gamma} = \overline{\Gamma_0} \cup \overline{\Gamma_C}$.

We denote by $u=(u_1,u_2)$, $u = u(x, t)$, depending on the spatial variables $x=(x_1,x_2) \in \Omega$, and time variable $t \in [0,T]$, $T>0$, a displacement of the strip and by θ the absolute temperature of the strip. Assume $\Omega=\Omega_1 \cup \Omega_2 \cup \Omega_3$ where Ω_1, Ω_2 and Ω_3 denote the homogeneous coating layer, graded interlayer, and substrate layer, respectively. The heights of these layers are h_1, h_2, h_3, respectively. In the middle layer Ω_2 material parameters depend on the height of the layer according to the exponential law. The displacement u of the strip satisfies the evolution equation [9] in the cylinder $\Omega \times (0,T)$:

$$\rho \frac{\partial^2 u}{\partial t^2} = A^* D A u - \alpha(3\lambda + 2\gamma)\nabla\theta , \quad (1)$$

The temperature θ of the strip satisfies the parabolic equation in the cylinder $\Omega \times (0,T)$:

$$\rho c_p \frac{\partial \theta}{\partial t} = \bar{\kappa}\Delta\theta \quad (2)$$

The following initial and boundary conditions are imposed:

$$u(0)= u_{0i} , \quad u'(0)= u_{1i} , \quad i=1,2, \quad \theta(0) = \theta_0 \text{ in } \Omega, \quad (3)$$

$$u = 0 \text{ on } \Gamma_0 \times (0,T) \text{ and } B^*D\,Au = F \quad \text{on } \Gamma_C \times (0,T), \quad (4)$$

$$\frac{\partial \theta}{\partial n} = q(t) \text{ on } \Gamma \quad (5)$$

where $u(0)=u(x,0)$, $u' = du/dt$, u_{0i} and u_{1i} θ_0 $q(t)$ are given functions, ρ is a mass density of the strip material, α is a coefficient of thermal expansion, $\bar{\kappa}$ is a thermal conductivity coefficient, c_p is a heat capacity coefficient, $\Gamma_0 = \Gamma \setminus \Gamma_C$. The operators A, B and D are defined as follows [10]

$$A=\begin{pmatrix} \frac{\partial}{\partial x_1} & 0 & \frac{\partial}{\partial x_2} \\ 0 & \frac{\partial}{\partial x_2} & \frac{\partial}{\partial x_1} \end{pmatrix}^*, B=\begin{pmatrix} n_1 & 0 & n_2 \\ 0 & n_2 & n_1 \end{pmatrix}^*, D=\begin{pmatrix} \lambda+2\gamma & \lambda & 0 \\ \lambda & \lambda+2\gamma & 0 \\ 0 & 0 & 2\gamma \end{pmatrix}, \quad (6)$$

where $n=(n_1,n_2)$ is the outward normal versor to the boundary Γ of the domain Ω, λ and γ are Lame coefficients, A* denotes a transpose of A. In Ω_2 operator D is assumed to depend on the depth of the graded interlayer according to the exponential law. By $\sigma=(\sigma_{11}, \sigma_{22}, \sigma_{12})$ and F we denote the stress tensor in domain Ω and surface traction vector on the boundary Γ, respectively. The surface traction vector $F=(F_1,F_2)$ on the boundary Γ_C is a priori unknown and is given by conditions of contact and friction. Under the assumptions that the strip displacement is small the contact conditions on the boundary $\Gamma_C \times (0,T)$ take a form:

$$u_2+g_r + w \le 0, \quad F_2 \le 0, \quad (u_2+g_r + w)F_2 = 0, \quad g_r=h-h_0+\sqrt{r_0^2-(u_1+x_1)^2} , \quad (7)$$

$$| F_1 | \leq \mu | F_2 |, \; F_1 \frac{du_1}{dt} \leq 0, \; (| F_1 | - \mu | F_2 |) \frac{du_1}{dt} = 0, \tag{8}$$

where μ is a friction coefficient. Conditions (5)-(6) describe the non-penetration condition as well as Coulomb law of friction, respectively [1,6]. Assuming that the dimensional wear coefficient k>0 is given the wear w=w(x,t) is governed by the equation [4]:

$$\frac{dw}{dt} = k\, V\, F_2. \tag{9}$$

Remark, in (7) the wear w increases the gap between the contacting surfaces.

2.1 Material properties of functionally graded materials

In subdomains Ω_1 and Ω_3 the operator D characterizing the properties of the material occupying strip Ω or the conductivity coefficient take different constant values, respectively (see Figure 2). In the subdomain Ω_2 the operator D or the conductivity coefficient are assumed to depend on the depth of the layer. This dependence is governed by the exponential law [8,9]:

$$P(x_2) = P_{\Omega 1} \exp\left(n \frac{x_2 + h_1}{h_2}\right), \quad x_2 \in [-h_2-h_1, -h_1], \tag{10}$$

where $n=\log(P_{\Omega 1}/P_{\Omega 3})$, h_1, h_2 are given parameters, x_2 denotes the spatial variable and $P(x_2)$, $P_{\Omega 1}$, $P_{\Omega 3}$ denote the height dependent material property (material density, conductivity coefficient or Young modulus) of layer Ω_2 as well as the material properties of layers Ω_1 and Ω_3, respectively. The continuity of the displacements, temperatures and the stresses along the interfaces $\partial\Omega_1 \cap \partial\Omega_2$ and $\partial\Omega_2 \cap \partial\Omega_3$ are assumed.

3. Quasistatic formulation

Taking into account the special features of the contact problem (1)-(9) one can reformulate it in the framework of the quasistatic approach. This approach is based on the assumption that for the observer moving with a wheel its displacement does not depend on time [14].

Consider an observer moving with the wheel with the constant linear velocity V. We introduce the new Cartesian coordinate system $O'x_1'x_2'$ hooked in the middle of the wheel. The systems $O'x_1'x_2'$ and Ox_1x_2 are related by: $x_1' = x_1 - V\,t$ and $x_2' = x_2$. Therefore the displacement $u(x_1',x_2')$ does not depend on time [14] and we obtain:

$$\frac{du}{dt} = (x_1', x_2') = \frac{du}{dt}(x_1 - Vt, x_2) = 0\,. \tag{11}$$

It implies:

$$\frac{du}{dt} = -V \frac{du}{dx_1} \text{ and } \frac{d^2u}{dt^2} = V^2 \frac{d^2u}{dx_1{}^2}\,. \tag{12}$$

Using the same arguments for the temperature field we obtain

$$\frac{\partial \theta}{\partial t} = -V \frac{\partial \theta}{\partial x_1} \quad \text{and} \quad \frac{\partial w}{\partial t} = -V \frac{\partial w}{\partial x_1} . \tag{13}$$

Using (12)-(13) the inertial terms in equations (1)-(2) are replaced by the stationary terms depending on the wheel velocity and spatial derivatives of displacement or temperature fields and reflecting the dynamics of the moving body rather than completely neglected it as in the classical quasistatic formulation [1]. Taking into account (12)-(13), quasistatic approximation of the contact problem (1)-(10) takes the form: find displacement u and temperature θ satisfying:

$$A^*D(x)Au - \rho V^2 u_{1,1} - \alpha(3\lambda + 2\gamma)\ \nabla\theta = 0 \ \text{ in } \Omega, \tag{14}$$

$$-V \frac{\partial \theta}{\partial x_1} = \bar{\kappa} \frac{\partial^2 \theta}{\partial x_2^2} , \tag{15}$$

as well as the boundary conditions

$$u = 0 \quad \text{on } \Gamma_0, \qquad B^*D(x)\ Au = F \quad \text{on } \Gamma_C, \tag{16}$$

$$u_2 + g_r + w \leq 0, \quad F_2 \leq 0, \quad (u_2 + g_r + w)F_2 = 0, \qquad \text{on } \Gamma_C , \tag{17}$$

$$F_1 \mid \leq \ \mu \mid F_2 \mid, \quad F_1 u_{1,1} \leq 0, \ \ (\mid F_1 \mid - \mu \mid F_2 \mid)\ u_{1,1} = 0, \quad \text{on } \Gamma_C , \tag{18}$$

$$-\kappa \frac{\partial \theta}{\partial x_2} = \bar{\alpha}[\frac{\theta}{r} F_2 + (1 - \frac{k\rho c\theta}{\mu})\mu V F_2] \qquad \text{on } \Gamma_C, \tag{19}$$

$$\frac{dw}{dx_1} = -k\ F_2, \qquad \text{on } \Gamma_C , \tag{20}$$

where $u_{i,\,j} = \frac{\partial u_i}{\partial x_j}$, $u_{i,jk} = \frac{\partial^2 u_i}{\partial x_i \partial x_k}$, i,j,k=1,2 and r denotes thermal resistant constant. Moreover, $u_{0i} = u_{1i} = 0$ is set in (2).

3.1 Friction Regularization

In order to ensure the existence of solutions to the problem (14)-(20) we have to regularize it, i.e., we will consider it as the problem with the prescribed friction. Let ε > 0 be a regularization parameter. We use the following formula relating tangential and normal tractions on the contact boundary Γ_C [14]:

$$F_1 = F_1(\varepsilon, F_2, u_1\ ) = -\ \mu \mid F_2 \mid \text{arc tan}\ \frac{V u_{1,1}}{\varepsilon} . \tag{21}$$

4. Numerical methods and results

Finite element method is used to approximate thermoelastic contact problem (14)-(21) as the approximation method. Problem (14)-(21) is a coupled thermoelastic problem. Remark, the contact traction depends on the thermal distortion of the bodies and wear process. On the other hand, the amount of heat generated due to friction depends on the contact traction. The main solution strategies for coupled problems are global solution algorithms where the differential systems for the different variables are solved together or operator splitting methods. In this paper we employ operator split algorithm. The numerical algorithm consists first in calculating for a given temperature field and wear the corresponding displacement and stress fields, i.e., in solving the mechanical subproblem. Next for the calculated displacement and stress fields we solve the thermal part of the system and calculate wear. The algorithm is terminated when the calculated temperature becomes steady, i.e., the temperature changes from iteration to iteration are less than the prescribed tolerance. The convergence of the operator split algorithm is shown using Fixed Point Theorem (see references in [12]). For details of the method see [14].

The obtained distributions of normal and tangential temperature distributions in the contact zone for different values of parameter n=0.28, 0, -0.28 are displayed in Figures 3 and 4, respectively. These distributions are strongly dependent on parameter η. The temperature is rapidly decreasing inside the strip and in front of the wheel. Behind the wheel the decrease of temperature is mild.

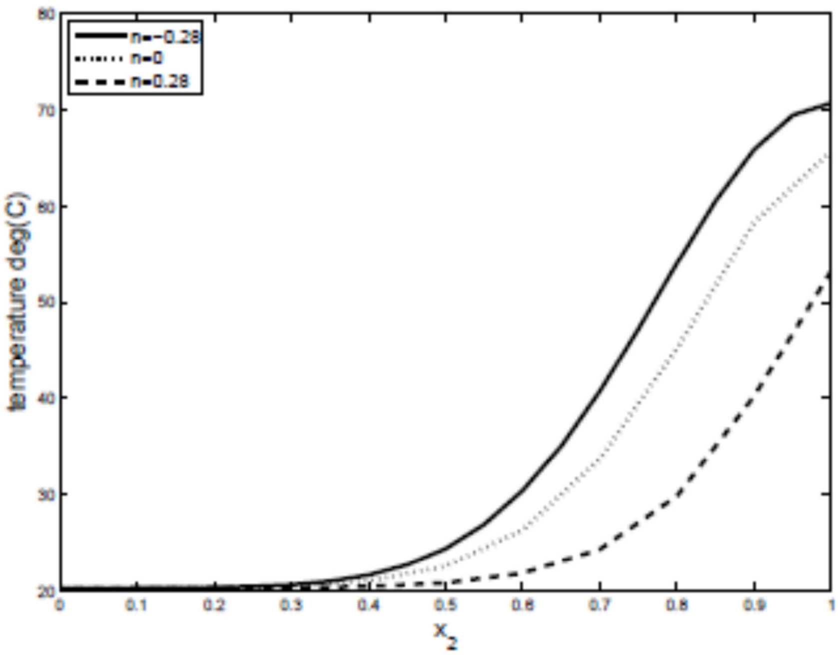

Figure 3. Rail temperature distribution along x_2 direction

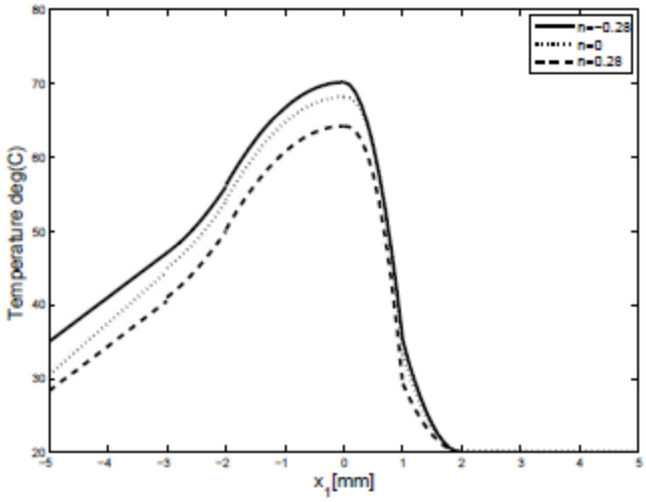

Figure 4. Rail temperature distribution along x_1 direction

5. Conclusions

The thermoelastic rolling contact problem where the properties of the elastic layer between the homogeneous surface coating and the substrate of the rail are dependent on its depth is solved numerically using the quasistatic approach. The material properties of the graded layer are assumed to be governed by the exponential law. The applied exponential model of the graded material allows to control the normal contact pressure, temperature and the size of the contact area comparing to the pure homogeneous case. The dependence of the obtained stress distributions on the parameter n is stronger than on the nonhomogenity index in power law (see [12]). The decrease in the nonhomogeneity index n reduces the maximum normal contact pressure and temperature at a cost of the widening of the contact area. The relationship between the applied normal load and the size of the contact zone is nonlinear. Remark also, that using the quasistatic approach we can observe dynamic phenomena of the rolling wheel.

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