Application of the Continuous Dynamic Absorbers in Local and Global Vibration Reduction Problems in Beams

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Abstract

The paper deals with the application of the continuous dynamic absorbers in vibration reduction problems in beams. The Euler-Bernoulli beam of variable cross-section is subjected to the concentrated and distributed harmonic excitation forces. The beam is equipped with a system of the continuous vibration absorbers. The problem of the forced vibration is solved employing the Galerkin's method and Lagrange's equations of the second kind. Performing time-Laplace transformation the amplitudes of displacement may be written in the frequency domain, similarly the time-averaged kinetic energy of any part of the beam. The results of some local and global vibration control optimization problems concerning the placement and parameters of the continuous vibration absorbers are presented.

Keywords: tuned mass damper, dynamic vibration absorber, continuous absorber, beam vibration, vibration control

1. Introduction

The main aim of dynamic vibration absorbers (DVA) and tuned mass dampers (TMD), properly located and tuned to the excitation force frequency, is the reduction of structure vibrations in the point of attachment [1,2]. The problem of vibration analysis and the proper selection of absorbers parameters was investigated in several theoretical studies [3-10].

Certain general rules concerning the proper location of dynamic absorbers can be given [7,18]. In continuous systems, such as beams, in case of its loading by a concentrated force the best place for the dynamic absorber attachment is usually the point of the applied load. The discrete absorbers efficiency depends significantly on the accuracy of their placement since even a slight deviation from the optimal position significantly decreases their effectiveness. Finding the optimal positions of absorbers for a distributed force applied is more complicated, especially for global problems of vibration reductions. Systems of dynamic absorbers tuned – in dependence of the excitation force bandwidth – into one [11-14] or into a few frequencies [3-5] are applied in several cases.

Continuous absorbers, in comparison with discrete absorbers, are efficient for various locations of excitation forces and at the appropriate tuning can be efficient within a wide frequency range. Continuous absorbers are especially suitable for damping the running structural waves in long one-dimensional continuous systems, such as beams [15]. This type dynamic absorbers are applied also for reducing vibrations of plates and shells at low frequencies [19] as well as for problems related to sound emission [20].

The computational algorithm, allowing to determine the amplitude-frequency characteristics of displacement and energy for the Euler-Bernoulli beam of a variable cross-section subjected to harmonic excitations of concentrated and distributed forces, with the system of continuous dynamic absorbers attached, is presented in the hereby paper. The presented examples of numerical calculations concern the application of the continuous absorbers in global vibration reduction problems in beams.

2. Theoretical model

The system considered in the paper is shown in Fig. 1. The beam of length l and with any given boundary conditions is given, its physical and geometrical parameters are functions of the position: mass density $\rho(x)$, cross-section area $A(x)$, geometrical moment of inertia $I(x)$, Young modulus $E(x)$, viscous damping coefficient $\alpha(x)$ (the Voigt-Kelvin rheological model was assumed). The beam subjected to harmonic excitations (both concentrated and distributed) is equipped with the system of continuous dynamic vibration absorbers.

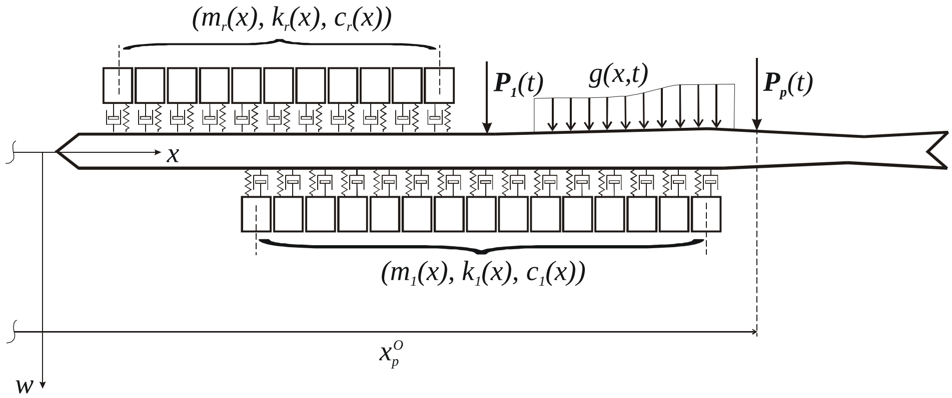

Figure 1. Beam with the system of ϱ continuous dynamic vibration absorbers

When the Euler-Bernoulli beam model is taken into account, the expressions for the kinetic and potential energy and for the dissipation potential take the following forms:

$$T = \frac{1}{2}\int_0^l \varrho(x)A(x)\left(\frac{\partial w}{\partial t}\right)^2 dx \tag{1}$$

$$V = \frac{1}{2}\int_0^l E(x)I(x)\left(\frac{\partial^2 w}{\partial x^2}\right)^2 dx \tag{2}$$

$$R = \frac{1}{2}\int_0^l E(x)I(x)\alpha(x)\left(\frac{\partial^3 w}{\partial t \partial x^2}\right)^2 dx \tag{3}$$

The beam deflection is described by the functional series

$$w(x,t) = \sum_{i=1}^{n} q_i(t)\varphi_i(x) \tag{4}$$

in which eigenfunctions of the beam of a constant cross-section (for boundary conditions of the given problem), without the attached dynamic absorbers, are assumed as basic functions $\varphi_i(x)$. Time functions $q_i(t)$ tare generalized coordinates which should be determined.

After the substitution of series (4) into equations (1)-(3) the expressions for the kinetic and potential energy and for the dissipation potential take the following forms:

$$T = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} m_{ij}\dot{q}_i\dot{q}_j \tag{5}$$

$$V = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} k_{ij}q_i q_j \tag{6}$$

$$R = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij}\dot{q}_i\dot{q}_j \tag{7}$$

Numerical factors m_{ij}, k_{ij}, b_{ij} occurring in the above shown expressions, are defined as follows:

$$m_{ij} = \int_0^l \rho(x)A(x)\varphi_i(x)\varphi_j(x)dx \tag{8}$$

$$k_{ij} = \int_0^l E(x)I(x)\varphi_i''(x)\varphi_j''(x)dx \tag{9}$$

$$b_{ij} = \int_0^l E(x)I(x)\alpha(x)\varphi_i''(x)\varphi_j''(x)dx \tag{10}$$

For the arbitrary beam load applied $H(x, t)$ the generalized force for the i-th generalized coordinate equals to:

$$H_i(t) = \int_0^l H(x,t)\,\varphi_i(x)dx \tag{11}$$

Using the Lagrange's equations of second kind the differential equations system for the generalized coordinates $q_i(t)$ is obtained:

$$\sum_{j=1}^{n} m_{ij}\ddot{q}_j + \sum_{j=1}^{n} b_{ij}\dot{q}_j + \sum_{j=1}^{n} k_{ij}q_j = H_i(t), \quad i = 1 \ldots n \tag{12}$$

Applying the time Laplace transform (with initial conditions being zero) to system (12) the linear system of algebraic equations is obtained, from which it is possible to obtain transforms $Q_i(s)$ of functions $q_i(t)$:

$$\sum_{j=1}^{n} m_{ij}s^2 Q_j(s) + \sum_{j=1}^{n} b_{ij}sQ_j(s) + \sum_{j=1}^{n} k_{ij}Q_j(s) = H_i(s), \quad i = 1 \ldots n \tag{13}$$

The transform of the beam deflection line is given by the series

$$W(x,s) = \sum_{i=1}^{n} Q_i(s)\varphi_i(x) \tag{14}$$

The load of the considered beam (Fig. 1) consists of p concentrated forces $P_k(t)$ applied in points of coordinates x_k^0, of a distributed load $g(x, t)$ and of r distributed loads $f_k(x, t)$ originated from continuous dynamic absorbers:

$$H(x,t) = \sum_{k=1}^{p} P_k(t)\,\delta(x - x_k^0) + g(x,t) + \sum_{k=1}^{r} f_k(x,t) \tag{15}$$

Thus, the generalized force $H_i(t)$ for the generalized coordinate $q_i(t)$ equals to:

$$H_i(t) = \sum_{k=1}^{p} P_k(t)\,\varphi_i(x_k^0) + \int_0^l g(x,t)\,\varphi_i(x)dx + \sum_{k=1}^{r}\int_0^l f_k(x,t)\,\varphi_i(x)dx, \quad i = 1 \ldots n \tag{16}$$

The Laplace transform of the generalized force is given by the expression:

$$H_i(s) = \sum_{k=1}^{p} P_k(s)\,\varphi_i(x_k^0) + \int_0^l g(x,s)\,\varphi_i(x)dx + \sum_{k=1}^{r}\int_0^l f_k(x,s)\,\varphi_i(x)dx, \quad i = 1 \ldots n \tag{17}$$

where $P_k(s)$, $g(x, s)$, and $f_k(x, s)$ are the Laplace transforms of functions: $P_k(t)$, $g(x, t)$, $f_k(x, t)$.

The Laplace transform of the continuous beam load originated from the k-th continuous dynamic absorber (with zero initial conditions) equals to:

$$f_k(x,s) = -\frac{\left(c_k(x)s + k_k(x)\right)m_k(x)s^2}{m_k(x)s^2 + c_k(x)s + k_k(x)}\sum_{j=1}^{n} Q_j(s)\varphi_j(x) \tag{18}$$

where by: $m_k(x)$, $k_k(x)$, $c_k(x)$ the linear densities of the mass, stiffness and damping coefficients (describing the continuous dynamic absorber) are marked, respectively. When expression (18) is inserted into (17), after rearrangements the system of linear algebraic equations is obtained from system (13). The transforms $Q_i(s)$ can be determined from the system:

$$\sum_{j=1}^{n}\left(m_{ij}s^2+b_{ij}s+k_{ij}+\sum_{k=1}^{r}F_{ij}^{k}(s)\right)Q_j(s)=\sum_{k=1}^{p}P_k(s)\varphi_i(x_k^0)+G_i(s) \qquad i=1\ldots n \tag{19}$$

where the following notations are introduced:

$$F_{ij}^{k}(s)=\int_0^l\frac{\left(c_k(x)s+k_k(x)\right)m_k(x)s^2}{m_k(x)s^2+c_k(x)s+k_k(x)}\varphi_i(x)\varphi_j(x)dx \qquad G_i(s)=\int_0^l g(x,s)\varphi_i(x)dx \tag{20}$$

The solution of equations system (19) provides in s-domain – after using equation (14) – the transform of the beam deflection line for arbitrary boundary conditions. When considering the steady state, substituting s = $j\omega$ ($j=\sqrt{-1}$)allows to determine the amplitude of the beam deflection line as the function of frequency. Analogous amplitude-frequency characteristics can be obtained for the bending moment, shear force and the time-averaged kinetic energy of the beam.

The developed computational algorithm allows to determine the mentioned above amplitude-frequency characteristics for the beam described by arbitrary functions (within the geometrical model applicability): $\rho(x)$, $A(x)$, $I(x)$, $E(x)$, $\alpha(x)$.

3. Numerical calculations – tunable continuous vibration absorber

A cantilever steel beam, with rectangular cross-section, excited by uniform distributed harmonic force: $g(x, t) = g_0 sin\omega t$ distributed along the segment $\langle 0.3l, 0.6l\rangle$ is considered, with the continuous absorber attached (Fig. 2). The parameters describing the system are collected in Table 1 (the internal damping in the beam is neglected).

Table 1. Parameters of the beam and absorber

Quantity	Symbol	Unit	Value
Mass density	ρ	kg/m^3	7800
Length	l	m	1.0
Young's modulus	E	N/m^2	2.1e11
Cross-section width	b	m	0.05
Cross section height	h	m	0.005
Total mass of the absorber	-	kg	0.098

The first four natural frequencies of the beam without the absorber attached are equal to: f_1 = 4.19 Hz, f_2 = 26.26 Hz, f_3 = 73.54 Hz, f_4 = 144.11 Hz.

It is assumed that the linear densities of the absorber mass, stiffness and damping coefficients are constant along its segment: $m(x)$ = const, $k(x)$ = const, $c(x)$ = const, the total mass of the continuous absorber is equal to 5% of the total beam mass, which means 0.098 kg.

Depending on whether the local optimization problem is considered (e.g. minimization of the vibration amplitude of the selected point of the beam) or the global one (e.g. minimization of the time-averaged kinetic energy of the selected part of the beam), the optimal solutions (i.e. width, location and physical absorber parameters) may be completely different. The solutions also depend on whether the problem of tuning the absorber around a selected frequency is considered (passive method) or the problem of tuning in real-time to the excitation frequency in a wider frequency band (semi-active method).

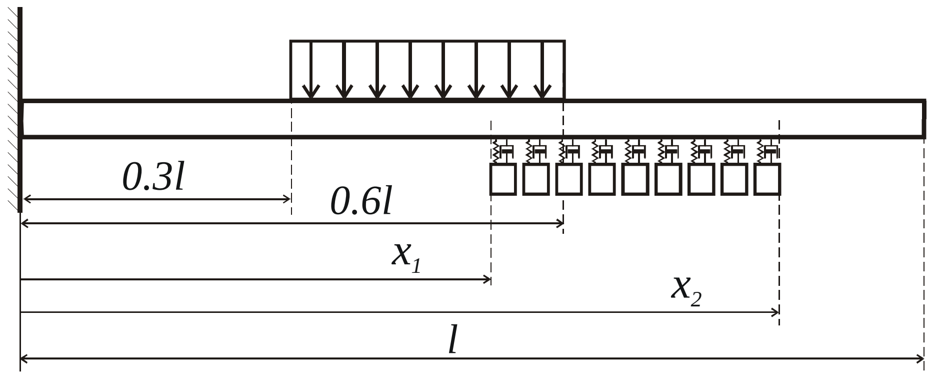

Figure 2. Beam with the attached dynamic continuous vibration absorber

For example, for the problem of passive minimization of the vibration amplitude of the free end of the beam shown in Fig. 2, in the bandwidth around the first natural frequency f_1 = 4.19 Hz, the best result is obtained for the discrete damper placed at the end of the beam. The calculated for this case the optimal stiffness and damping coefficients of the damper are: K_{OPT} = 47.70 N/m , C_{OPT} = 1.08 Ns/m.

Due to the first mode shape the problem of passive minimization of the time-averaged kinetic energy of the beam around the first natural frequency has a similar solution: the discrete damper placed at the end of the beam with the optimal parameters almost the same like given earlier.

The width and location of the optimal absorber in vibration reduction problems considered in a wider frequency band can be different depending on the criterion taken (local or global). In the case of the absorber tuned in real-time to the excitation frequency (semi-active method), the best solution for the problem of minimization the vibration amplitude of the beam free end may also occur the discrete absorber placed at the end of the beam. In this case, however, a new resonant frequency [18, 23-24] may appear with a node at the beam end, so such location of a discrete damper may be inadequate in the energy minimization problems. The vibration suppression efficiency may be improved by using several discrete translational and rotational absorbers [22-24].

In further calculations the continuous absorber (Fig. 2) is assumed to be tuned so that it is resonant at each frequency, without energy dissipating appliances ($c(x) = 0$).

The aim of calculations is to find the optimal width and placement of the continuous vibration absorber in a given frequency range, as a measure of vibration is used the time -averaged kinetic energy of the whole beam.

The results of the numerical calculations are presented in Fig. 3 and Fig. 4.

For comparison it is first shown in Fig. 3 the calculated time-averaged kinetic energy for the case with the single discrete absorber placed in different positions on the beam. The numbers in the figure represent the distance from the support (in meters).

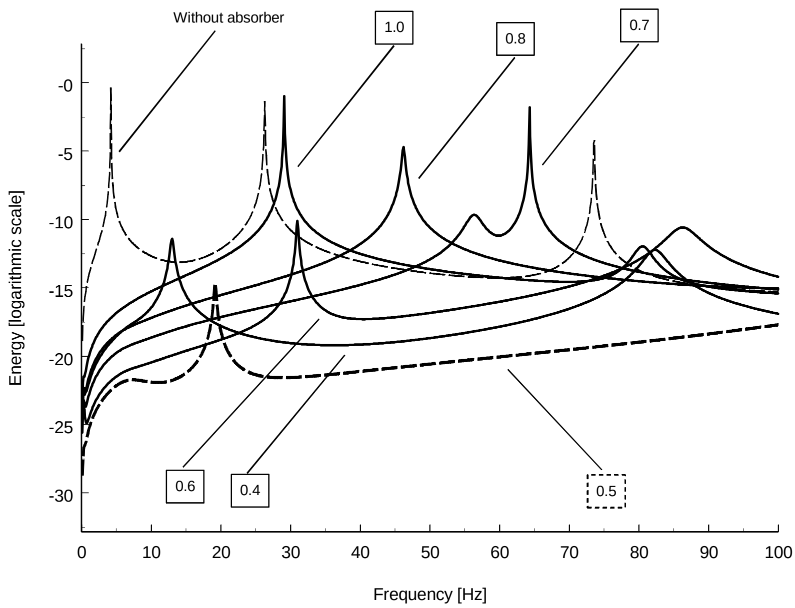

Figure 3. Time-averaged kinetic energy of the beam with the one discrete absorber attached in different positions – the absorber is tuned to be resonant at each frequency

It is visible that the vibration suppression efficiency of the discrete absorber (tuned to the excitation force frequency) depends largely upon the absorber position. Due to the appearing a new resonant frequency of the structure composed of the beam with absorber, there is no position of the absorber appropriate in the whole frequency band considered. Additionally the discrete absorber is very sensitive to inaccurate location and tuning.

In Fig. 4 is shown the time-averaged kinetic energy for the case with the single continuous absorber of different width and placed in different positions on the beam. The numbers in the figure represent the values of x_1 and x_2 (Fig. 2).

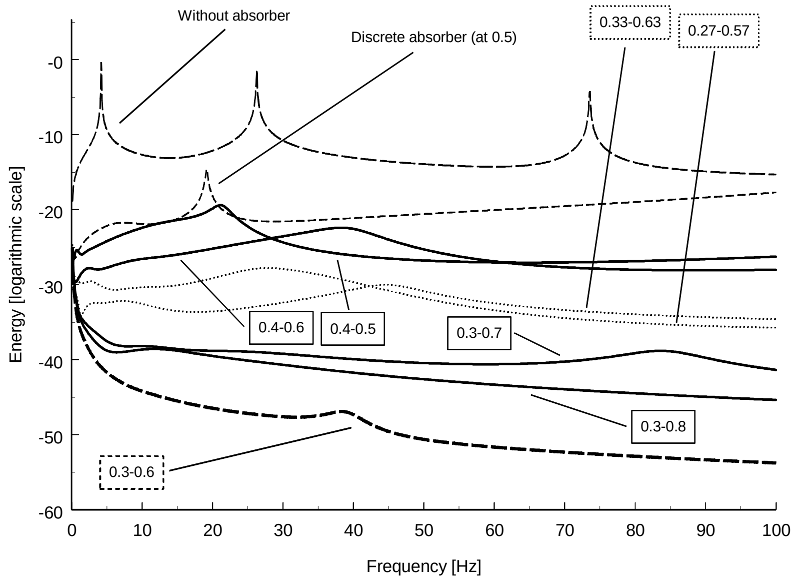

Figure 4. Time-averaged kinetic energy of the beam with the one continuous absorber attached in different positions and with different segment widths – the absorber is tuned to be resonant at each frequency

It results from the diagrams in Fig. 4 that the continuous absorbers may have the suppression efficiency many orders of magnitude higher than the discrete absorber.

It is possible to find the width and position of the continuous absorber segment which are considered optimal in the entire given bandwidth, because there doesn't appear any new resonant frequency in the system.

The continuous absorber is also sensitive to inaccurate location and tuning, but even when placed not exactly at the optimal location it can posses the vibration suppression efficiency much more higher than the discrete absorber.

For another type of loading optimization can give different results, as for the other frequency bands. A further improvement of the vibration reduction would be achievable, when the real time change not only of stiffness but also of damping was possible. Detuning the absorbers, both discrete and continuous, can also be beneficial [18].

4. Conclusions

Continuous dynamic absorbers can be efficient in cases when the points of loading attachment is not accurately determined as well as in cases of distributed loads. They can be applied in places where placements of one or a few absorbers of significant masses is technically impossible. By the appropriate tuning they can be efficient within a broad frequency band.

The computational model presented in the hereby paper can be used in local and global problems of the optimization the continuous dynamic absorbers locations and parameters in beams. The numerical algorithm created for calculation of the continuous absorbers may also be applied to calculation of the discrete absorbers. It can be obtained by taking the very narrow segment over which the continuous absorber is distributed or by describing the densities of the mass, stiffness and damping coefficients using the δ-Dirac distribution. The advantage of this approach is that the number of unknowns in the solved systems of equations does not depend on the number of discrete dampers used.

The computational model of the continuous dynamic absorber, presented in this study, can be adjusted to vibration reduction problems in more complex one-dimensional systems such as frames or curvilinear beams. It can be also expanded to problems of vibration reductions in plates or shells.

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