A Semi-Active Control of Lateral Vibrations of the Overhung Rotor Using Dampers with the Magneto-Rheological Fluid

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Abstract

In the paper there is proposed an algorithm of an efficient semi-active control of steady-state periodic lateral vibrations of the overhung rotor-shaft system. This algorithm has been developed using fundamentals of the Optimal Control Theory. In the considered system the control is realized by means of the linear dampers with the magneto-rheological fluid built in the bearing housing. The computational example demonstrates possibilities of the applied approach resulting in an additional reduction of out-of-resonance and near-resonance harmonic oscillation amplitudes in comparison with an analogous passive control.

Keywords: overhung rotor-shaft, lateral vibrations, semi-active control, Optimal Control Theory

1. Introduction

Heavy rotors suspended in bearings in an overhung way constitute a wide class of rotating machinery. Typical examples of this group are pumps, compressors, blowers, gas turbines, crushers, beater mills, drums of washing machines and many others. As it follows e.g. from [1,2], at high rotational speeds they are sensitive to gyroscopic effects associated by their lateral vibrations excited mainly by residual unbalances as well as by assembly misalignments, rubbing effects in bearings, sealings or blade rims and by other sources. Such oscillations are usually very detrimental and a suppression of their amplitudes is an important challenge in order to assure precise motions of such rotor-shaft systems, possibly small bearing reactions, minimized danger of material fatigue and low level of generated noise. This target can be effectively achieved by means of a semi-active control of lateral vibrations affecting the rotor-shaft systems with overhung rotors. For this purpose, similarly as e.g. in [3], actuators with the magneto-rheological fluid (MRF) are going to be applied. Such an approach seems to be very convenient for rotor machines like vacuum pumps, turbo-chargers, washing machines, precise spindles and others rotating with high speeds in steady-steady state operating conditions under harmonic external excitations due to residual unbalances and the mentioned above dynamic effects. It is to emphasize that, contrary to a control of transient or resonant vibrations, for which many algorithms turned out to be effective, a suppression of forced, steady-state oscillations with frequencies far away from resonance zones is an extremely difficult task. Here, in cases of the abovementioned rotor machines even a few-percent minimization of fluctuation amplitudes can be very fruitful from the viewpoint of material fatigue, precision of motion, dynamic interaction with an environment, detrimental noise generation and many other factors. Thus, in order to achieve this target, in the paper for the actuators with the MRF a control strategy based on the

Optimal Control Theory (OCT) will be applied for the high-speed overhung rotor-shaft under steady-state harmonic lateral vibrations. The obtained results of simulations are going to be compared with the analogous ones determined for additional passive damping applied into the considered system as well as using the numerical optimization control algorithm.

2. Modelling of the rotor-shaft and mathematical formulation of the problem

In many cases the high-speed rotating machines are characterized by heavy, lumped overhung rotors attached on short, dumpy shafts suspended on relatively flexible bearing supports. Thus, deformations of such rotor-shafts can be neglected and then only rotor-shaft inertial parameters and bearing support visco-elastic properties play a predominant role in lateral vibrations of these objects. According to [1], if a maximal static deflection of such rotor-shaft is of the same order as the bearing clearances, its dynamic behaviour can be investigated by means of a rigid body model of four degrees of freedom. Then, the generalized coordinates corresponding to them describe two translational displacements of the rigid body mass center in the two mutually perpendicular directions with respect of the rotor-shaft rotation axis as well as two angular displacements with respect of mutually perpendicular axes passing the mass center of this rigid body. In order to take into consideration a rotor-shaft support in a possibly general way, the anisotropic and non-symmetrical visco-elastic properties of bearings have been assumed in the form of stiffness and damping coefficients containing also the proper cross-coupling terms. The proposed rigid body model of the overhung rotor shaft supported on two bearings is presented in Fig. 1.

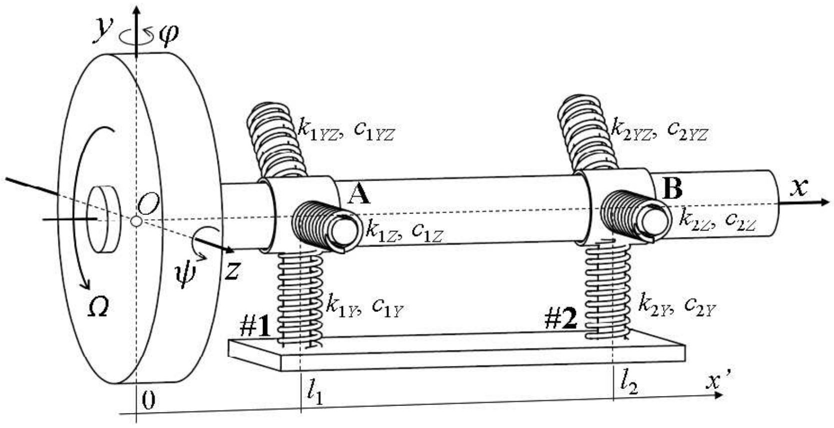


Figure 1. The rigid-body model of the double-bearing overhung rotor

Motion of the rotor-shaft has been described in the inertial orthogonal coordinate system *Oxyz* with the origin placed in the rigid body model center of gravity *O*. Axis *Ox* coincides with the bearing axis and axes *Oy*, *Oz* respectively determine the vertical and horizontal direction. The plains of bearing interaction cross *Ox* axis in points A and B distant of l_1 in the case of bearing #1 and of l_2 in the case of bearing #2, as shown in Fig.

1. The motion equation of the assumed rotor-shaft rigid body model have the following form:

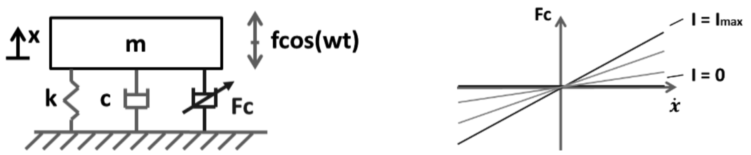
$$\mathbf{M}\cdot\ddot{\mathbf{r}}(t)+\left(\mathbf{C}+\Omega\cdot\mathbf{G}\right)\cdot\dot{\mathbf{r}}(t)+\mathbf{K}\cdot\mathbf{r}(t)=\mathbf{F}\left(t,\Omega^{2}\right) \tag{1}$$

where $\mathbf{r}(t)$= col [$y(t)$, $z(t)$, $\psi(t)$, $\varphi(t)$] is the generalized coordinate vector with components corresponding respectively to the translational displacements along *Oy* and *Oz* axes and to the angular displacements around *Oz* and *Oy* axes. Symbol **M** denotes the diagonal inertial matrix, **C** and **K** are respectively the symmetrical bearing damping and stiffness matrices and **G** is the skew-symmetrical matrix of gyroscopic effects. The external excitation vector **F** has the following components:

$$\mathbf{F}\left(t,\Omega^{2}\right)=\begin{bmatrix} Mg+M\varepsilon\cdot\Omega^{2}\sin(\Omega t)+U(t) \\ M\varepsilon\cdot\Omega^{2}\cos(\Omega t)+V(t) \\ 0 \\ 0 \end{bmatrix} \tag{2}$$

where ε is the eccentricity of the rotor-shaft residual static unbalance, M denotes the entire mass of the rigid rotor and $U(t)$, $V(t)$ are the control forces acting in the vertical and horizontal direction, respectively. Such equations are very convenient here for a demonstration of relatively easy implementation of the proposed algorithm of semi-active control of the steady state forced lateral vibrations of the considered object.

The rotating machines usually operate in steady-state conditions at constant rotational speeds, more or less far away from the critical ones associated with the corresponding lateral eigenvibration modes. Thus, the goal of this paper is to propose a computationally effective numerical method for determination of the optimal control function applied here for the mechanical system under periodical vibrations due to the residual unbalance. In order to distinguish such successive mutually uncoupled eigenmodes of the considered gyroscopic, nonconservative rotor-shaft system, it is necessary to perform a complex modal analysis of Eqs. (1) according e.g. to the approach presented in [2,4]. Then, the investigations reduce to control of steady-state harmonic oscillations of simple single degree-of-freedom oscillators shown in Fig. 2 (a).



a) b)

Figure 2. Single DOF dynamic oscillator (a), controllable damper force function (b)

An equation of motion of such oscillator has the following form:

$$m\cdot\ddot{x}(t)+c\cdot\dot{x}(t)+k\cdot x(t)=\kappa\cdot\left(f\cdot\cos(\Omega t-\varphi)-u(\dot{x}(t))\right) \tag{3}$$

where according to [4], the modal damping coefficient $c = 2\delta m$, the modal stiffness $k = \left(\delta^2 + \omega^2\right)m$, $\kappa = \left(\delta r - \omega s\right)$, m is the modal mass, δ, ω denote respectively the real and imaginary part of the complex eigenvalue corresponding to the considered eigenmode, r, s are respectively the real and imaginary part of the complex left eigenvector component, $x(t)$ denotes the modal displacement of the controlled eigenmode and φ is phase shift angle.

As shown in Fig. 2b, for the assumed linear relationship between the shaft/bearing vibratory velocity and the control force F_c generated by the MRF damper built in the bearing housing, one can express in (3): $F_c = u(\dot{x}(t))$, where u denotes the control variable. The slope of the damping force curve depends on the instant value of the control current I. Control current cannot exceed the boundary limits $I \in \langle 0, I_{max} \rangle$. Also, it is assumed that the control current can change its value instantly. Because the controllable damper characteristic is linear, it may be assumed that:

$$\begin{cases} u = I \\ u_{min} = I_{min} = 0 \\ u_{max} = I_{min} \end{cases} \tag{4}$$

For the simplification of further considerations it is convenient to transform Equation (3) into the state-space representation:

$$\begin{cases} \dot{q}_1 = q_2 \\ \dot{q}_2 = -\dfrac{k}{m} q_1 - \dfrac{c}{m} q_2 - \dfrac{\kappa}{m}(f\,cos(\Omega t) - u q_2) \end{cases} \tag{5}$$

where state variables are defined in the following form:

$$[\boldsymbol{q}] = \begin{bmatrix} x \\ \dot{x} \end{bmatrix} = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} \tag{6}$$

In order to define the optimal control problem it is necessary to introduce a performance index which will represent a measure of vibration level. One of possible choices is to select the performance index as a single scalar value that will represent the average motion mean energy of the considered system:

$$J = \int_0^{t_f} \left[\frac{1}{2}\left(kq_1^2 + mq_2^2\right) + ru^2 \right] dt = \int_0^{t_f} E dt \tag{7}$$

In the above equation, apart from the motion energy component $1/2(kq_1^2 + mq_2^2)$, the other component has been added, namely ru^2. This expression refers to the amount of energy consumed by the controlled damping element. This component has been added into Eq. (7) in order to simplify further transformations. The term ru^2 should be treated as negligible, since a minimization of the control energy has not been considered as a primary goal for mechanical systems under periodical excitation. Therefore, it is assumed that scalar r nearly equals zero. Variable E denotes the integrand function.

Using the Optimal Control Theory (OCT) it is possible to derive the set of equations specifying the optimal control function profile u^*, providing a minimization of functional *J*. For this purpose, it is necessary to apply the common OCT control function derivation procedure given in [5,6]. It starts with a definition of the Hamiltonian function:

$$H = E + \boldsymbol{\lambda}'\dot{\boldsymbol{q}} \tag{8}$$

Next, using the necessary condition for minimization of functional *J*, namely: $\delta J = 0$, the following set of equations can be derived:

$$\begin{cases} \dot{\boldsymbol{q}} = \dfrac{\partial H}{\partial \lambda} \\ \dot{\boldsymbol{\lambda}} = -\dfrac{\partial H}{\partial \boldsymbol{q}} \\ H(\boldsymbol{q}^*, \boldsymbol{\lambda}^*, u^*) \le H(\boldsymbol{q}^*, \boldsymbol{\lambda}^*, u) \\ \boldsymbol{u} \in \langle 0, u_{max} \rangle \end{cases} \tag{9}$$

where $\boldsymbol{\lambda}$ denotes the costate vector. Upon an expansion of the third inequality standing in (9) and an application of the Pontryagin principle, finally the following set of equations defining the optimal control can be derived:

$$\begin{cases} \begin{cases} \dot{q}_1 = \dfrac{\partial H}{\partial \lambda_1} = q_2 \\ \dot{q}_2 = \dfrac{\partial H}{\partial \lambda_2} = -\dfrac{k}{m} q_1 - \dfrac{c}{m} q_2 + \dfrac{\kappa}{m}(f\,cos(\Omega t) - u q_2) \end{cases} \\ \begin{cases} \dot{\lambda}_1 = -\dfrac{\partial H}{\partial q_1} = -k q_1 + \dfrac{k}{m}\lambda_2 \\ \dot{\lambda}_2 = -\dfrac{\partial H}{\partial q_2} = -m q_2 - \lambda_1 + \dfrac{c}{m}\lambda_2 + \lambda_2 u \\ u^* = sat\left(sign\left(\lambda^*{}_2 q^*{}_2\right)\right) \end{cases} \end{cases} \tag{10}$$

In order to find exact function values, all equations of the above system have to be solved simultaneously. It requires a specification of boundary values of the state and costate vectors. For the considered vibrating system one can assume that under optimal control function this system will eventually fall into steady-state vibrations, starting from an arbitrary initial state condition. Different initial state conditions will only affect a duration time of the transient phase of motion up to the instant, when the steady-state vibration phase shall be established. Concluding, the initial condition for the state vector can be arbitrarily chosen as: $\boldsymbol{q}(0) = 0$.

The second condition follows directly from the fundamentals of the OCT. Provided that the considered system of Eqs. (10) has to be integrated in the finite time range $t \in \langle 0, T_f \rangle$, the optimal problem in the OCT nomenclature can be classified as *free-end, fixed-time* problem, [5]. The phrase "*free-end*" refers to a lack of constraints specified for the state vector at the end of the simulation time window. The phrase "*fixed-time*" refers to the finite value of the simulation time range T_f. For such kind of the optimal control problem the OCT provides the additional boundary condition, i.e.: $\boldsymbol{\lambda}(T_f)=0$.

Concluding, because the known boundary conditions are specified partially at the beginning and partially at the end of the simulation time window, this problem can be classified as the Two-Point Boundary Value Problem (TPBVP). The TPBVPs are generally considered as difficult numerical problems. In order to solve the TPBVP for the considered system, the following algorithm has been developed:

1. initialize the $\boldsymbol{\lambda}(0)$ vector with random values,
2. integrate the coupled state-costate equations on the time interval $\langle 0, T_f \rangle$ assuming $\boldsymbol{q}(0)=0$ and taking (0) from point 1,
3. after an integration check, whether terminal condition has been satisfied $\boldsymbol{\lambda}(T_f)=0$,
4. conditional step:
   a. if the terminal condition from step 3 has been satisfied, terminate the algorithm,
   b. if the terminal condition from step 3 has not been satisfied, find the new estimation of the (0) condition by means of the external, numerical optimization algorithm; then, repeat the steps 1-4 as long as terminal condition is not being satisfied.

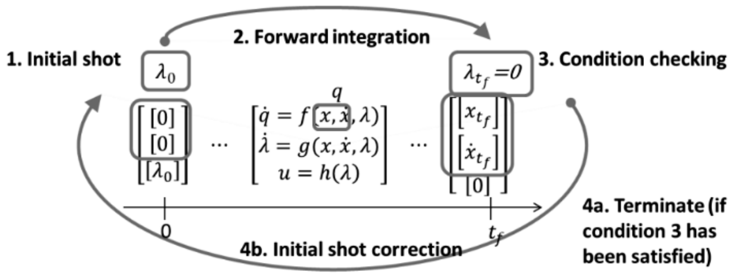


Figure 3. Optimal control problem computational algorithm

The algorithm described above can be illustrated by means of the following diagram presented in Fig. 3. It is important to choose the sufficiently large T_f value, so the steady-state phase of motion could be significantly longer than either transient phase at the beginning or at the end of the simulation time window.

3. Computational example

In the computational example the rigid overhung rotor-shaft of the industrial blower supported on two identical rolling bearings is used as an object of considerations. This rotor-shaft of a total weight ca. 60.13 kg and of the bearing span 0.275 m is characterized by a relatively heavy impeller and light shaft, as shown in Fig. 1. Its total polar and diametral mass moments of inertia are respectively equal to 7.02 and 12.75 kgm^2. It is assumed that bushings of the isotropic and radially stiff rolling bearings are embedded in the bearing housings by means of layers made of relatively soft and viscous vulcanized rubber. The bearing suspension stiffness coefficients are assumed constant within the entire shaft rotational speed range 0-7200 rpm.

In Fig. 4a there are presented the imaginary parts and in Fig. 4b the real parts of four eigenvalues of the considered rotor-shaft, where the grey lines correspond to the original system and the black ones to the system equipped with the MRF damper built in the bearing support #1 and operating passively. From the obtained plots it follows that

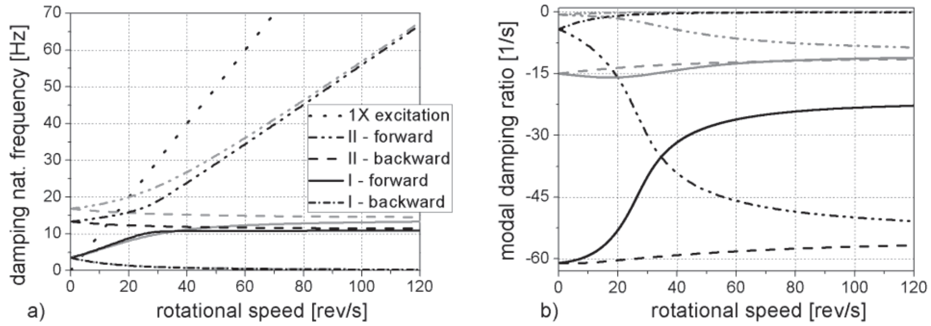


Figure 4. Imaginary (a) and real (b) parts of the rotor-shaft eigenvalues

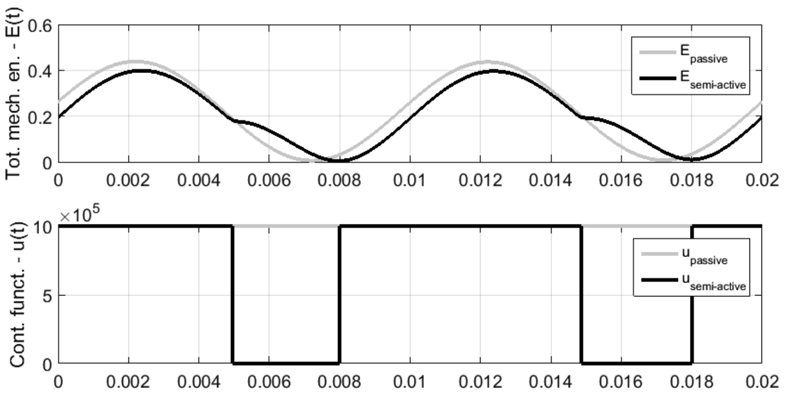


Figure 5. Entire vibratory mechanical energy profiles for the passive and semi-actively damped system for the 1st eigenmode backward precession of 6.1 Hz at 3000 rpm

the optimal passive control effectively stabilizes the backward and forward branches of the second eigenmode and the forward branch of the first eigenmode. But it has almost no influence on a stabilization of its backward branch characterized by the close to zero natural frequency and modal damping coefficient at greater rotational speeds, Fig. 4. However, the semi-active control realized using the MRF damper and the proposed control algorithm can result in an effective stabilization of this almost no damped backward precession of the 1st eigenmode excited here e.g. by means of periodic retarding frictional loads in the bearings. As shown in Fig. 5, the semi-active control minimizes fluctuation amplitudes of this backward mode by ca. 8%. Moreover, the semi-active control suppresses lateral vibration amplitudes even by 10% for the first eigenmode forward precession induced by unbalances at the overcritical rotational speed 110 rev/s, i.e. 6600 rev/min, as it follows from the time-history plots depicted in Fig. 6.

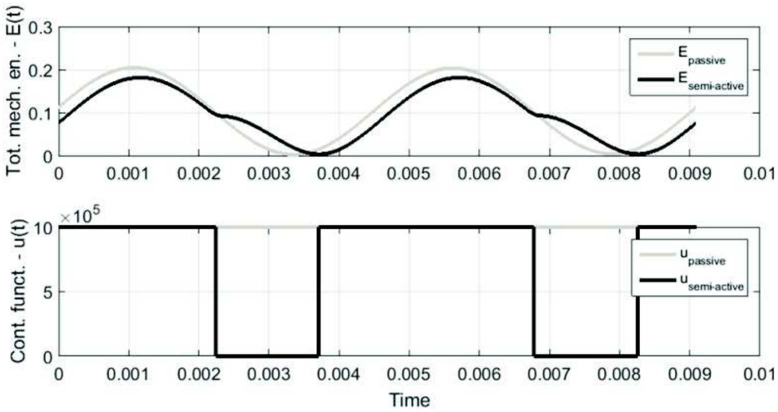


Figure 6. Entire vibratory mechanical energy profiles for the passive and semi-actively damped system for the 1st eigenmode forward precession of 13.67 Hz at 6600 rpm.

4. Conclusions

In the paper there were considered passively and semi-actively controlled periodic lateral vibrations of the rigid overhung rotor suspended on flexible bearings equipped with the MRF dampers. From the results of an eigenvalue analysis it follows that additional passive damping introduced into this system can effectively suppress its oscillation amplitudes and increase stability regions only for sufficiently stable eigenmodes. But it is not the case for unstable or almost stable eigenmodes, e.g. due to gyroscopic effects or skew-symmetrical bearing properties. Here, the semi-active control realized according to the proposed algorithm based on the Optimal Control Theory seems to be a very advantageous and universal tool for engineering applications tool for stabilization of vibrating mechanical systems and for an attenuation of their oscillation amplitudes.

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