Free Vibrations of Non-Prismatic Slender System Subjected to the Follower Force Directed Towards the Positive Pole

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Abstract

The paper contains the results of theoretical and numerical studies within the scope of kinetic criterion of stability loss of slender non-prismatic column subjected to the follower force directed towards the positive pole (the case of specific load). Shape of the system approximation by a linear function and polynomial of degree 2 was considered. On the basis of the Bernoulli – Euler's theory, the mechanical energy was defined. The differential equations of motion and natural boundary conditions were determined according to the Hamilton's principle. The issue of free vibrations was solved using the small parameter method. Within the range of numerical calculations, the changes in the eigenvalues were presented as a function of external load with variable geometrical parameters of the system, including parameters resulting from the shape approximation and parameters of loading structure.

Keywords: slender systems, non-prismatic systems, free vibrations, specific load

1. Introduction

Non-prismatic systems are commonly used in mechanics and mechanical constructions. Due to increasing technical requirements for the designers, an optimal shapes of structures, that will ensure an increase in transferred load or mass reduction are looked for. The issue of dynamics of slender non-prismatic systems is the subject of many scientific publications.

The dynamic analysis of Bernoulli – Euler's beam with stepped variable flexural stiffness with discrete elements was presented in work [1]. The problem was solved on the basis of the mode summation method. The results regarding to the issue of stability and free vibrations of non-prismatic column under Euler's load were shown in publication [2]. The solution of vibration problem of beam with stepped variable cross-section was presented in [3].

In scientific papers, the shape optimization was based on different methods, such as the Lagrange multiplier formalism [4], modified simulated annealing algorithm [5], finite element method [6], cellular automata method [7] or using Green's function properties [8].

2. The physical model of the system

A slender non-prismatic column of rectangular cross-section subjected to the chosen case of specific load is considered in this paper. The physical model of analysed system is presented in Figure 1. To model cross-section variable along the axis, the structure was divided into n segments of constant length l and thickness h and variable width b. It is assumed that total volume of each segments V_{obj}, total length of the column $L = l \cdot n = const.$ and the values of material density ρ as well as Young's modulus E of each parts are constant. In addition, the value of width b of segments must satisfy the condition that $b \geq h$. The column's shape was described by linear function $b(x) = 2a(Z) \cdot x + d$ and by polynomial of degree 2 $b(x) = 2\left[a(p,q) \cdot [x - p]^2 + q\right]$, where $0 \leq x \leq L$.

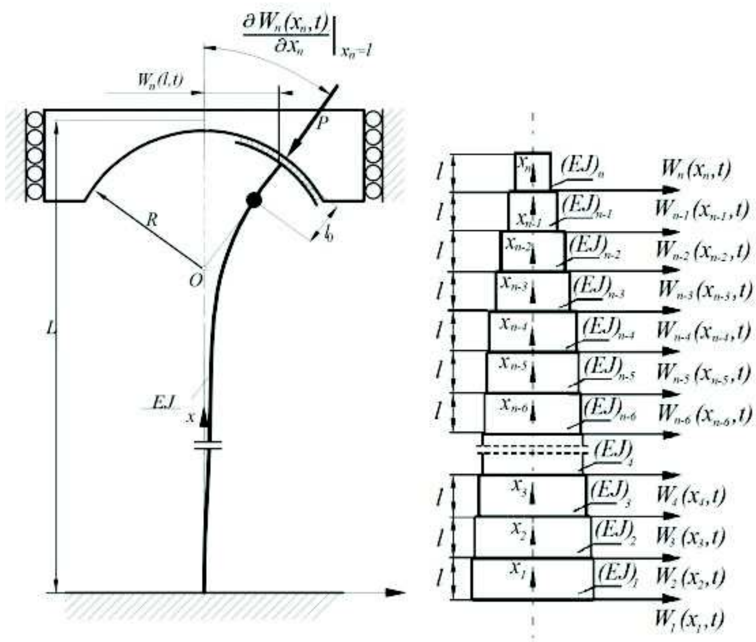

Figure 1. The scheme of physical model of considered column

The load by follower force directed towards the positive pole (the case of specific load, see [9]) is achieved by loading and receiving heads of circular outlines. The direction of the force P is tangential to the line of deflection of end of system ($x = L$) and additionally passes through stationary point O located on the non-deformed axis of the column at the distance of R from its free end (positive pole). The system is connected with receiving head through infinitely rigid element l_0, which consideration is necessary for reasons relating to the construction.

3. The mathematical model

On the basis of the physical model of non-prismatic column (comp. Figure 1), the total mechanical energy of the system was defined. The potential energy V consists of:

- energy of bending elasticity:

$$V_1 = \sum_{i=1}^{n} \frac{(EJ)_i}{2} \int_0^l \left(\frac{\partial^2 W_i(x_i,t)}{\partial x_i^2} \right)^2 dx_i \tag{1}$$

- potential energy V_2 resulting from the external load:

$$V_2 = -\frac{P}{2} \int_0^l \left(\frac{\partial W_i(x_i,t)}{\partial x_i} \right)^2 dx_i + PU_n(l,t) + \frac{1}{2} P (R - l_0) \left(\left. \frac{\partial W_n(x_n,t)}{\partial x_n} \right|_{x_n=l} \right)^2 \tag{2}$$

$$V = V_1 + V_2 \tag{3}$$

The kinetic energy T of the system is formulated in the following form:

$$T = \sum_{i=1}^{n} \frac{(\rho A)_i}{2} \int_0^l \left(\frac{\partial W_i(x_i,t)}{\partial t} \right)^2 dx_i + \frac{1}{2} m \left(\frac{\partial W_n(l,t)}{\partial t} \right)^2 \tag{4}$$

The solution of the problem of free vibrations of column was obtained on the basis of Hamilton's principle (see [2,9]), using the properties of the calculus of variation:

$$\delta \int_{t_1}^{t_2} (T - V) dt = 0 \tag{5}$$

where t_1, t_2 – coordinates of time, δ – variation operator.
Known a priori geometrical boundary conditions and continuity conditions were written as follows:

$$W_1(0,t) = \left. \frac{\partial W_1(x_1,t)}{\partial x_1} \right|_{x_1=0} = 0, \tag{6-7}$$

$$W_i(l,t) = W_{i+1}(0,t) \tag{8}$$

$$\left. \left(\frac{\partial W_i(x_i,t)}{\partial x_i} \right) \right|_{x_i=l} = \left. \left(\frac{\partial W_{i+1}(x_{i+1},t)}{\partial x_{i+1}} \right) \right|_{x_i+1=0}, \tag{9}$$

$$W_n(l,t) = (R - l_0) \left. \frac{\partial W_n(x_n,t)}{\partial x_n} \right|_{x_n=l}, \tag{10}$$

where the condition (10) results from the geometry of loading head.

Taking into account the variation of mechanical energy (1-4) and conditions (6-10) in the equation (5), the following relations were obtained:
– differential equations of motion:

$$(EJ)_i \frac{\partial^4 W_i(x_i,t)}{\partial x_i^4} + P \frac{\partial^2 W_i(x_i,t)}{\partial x_i^2} + (\rho A)_i \frac{\partial^2 W_i(x_i,t)}{\partial t^2} = 0 \tag{11}$$

– missing natural boundary condition and continuity conditions:

$$\left.\frac{\partial^3 W_n(x_n,t)}{\partial x_n^{\,3}}\right|_{x_n=l} - \frac{1}{(R-l_0)}\left.\frac{\partial^2 W_n(x_n,t)}{\partial x_n^{\,2}}\right|_{x_n=l} - \frac{m}{(EJ)_n}\left.\frac{\partial^2 W_n(x_n,t)}{\partial t^2}\right|_{x_n=l} = 0 \tag{12}$$

$$(EJ)_i\left.\left(\frac{\partial^2 W_i(x_i,t)}{\partial x_i^{\,2}}\right)\right|_{x_i=l} = (EJ)_{i+1}\left.\left(\frac{\partial^2 W_{i+1}(x_{i+1},t)}{\partial x_{i+1}^{\,2}}\right)\right|_{x_i+1=0} \tag{13}$$

$$(EJ)_i\left.\left(\frac{\partial^3 W_i(x_i,t)}{\partial x_i^{\,3}}\right)\right|_{x_i=l} = (EJ)_{i+1}\left.\left(\frac{\partial^3 W_{i+1}(x_{i+1},t)}{\partial x_{i+1}^{\,3}}\right)\right|_{x_i+1=0} \tag{14}$$

The solution of the differential equations of motion was obtained on the basis of small parameter method, which consists of expanding of nonlinear members of differential equations into the power series with respect to the amplitude parameter ε ($\varepsilon \ll 1$).

4. The Results of Numerical Calculations

To compare the results, the following dimensionless parameters were determined:

– external load parameter

$$\lambda = \frac{PL^2}{(EJ)_{pr}} \tag{15}$$

– parameter of frequency of natural vibrations

$$\Omega = \frac{\omega^2 (\rho A)_{pr} L^4}{(EJ)_{pr}} \tag{16}$$

– parameters describing cross-section variable along the axis of the column

$$Z^* = \frac{b_1 - b_n}{L}\cdot 100\%, \quad p^* = \frac{p}{L}, \quad q^* = \frac{q}{L}, \tag{17-20}$$

– radius of loading head parameter

$$R^* = \frac{R - l_0}{L}\,, \tag{21}$$

where the subscript „*pr*" refers to the geometrical parameters of prismatic column (a comparative system).

The results of numerical computations in the scope of kinetic criterion of stability loss were shown in Figures 2 and 3.The considerations are limited to presentation of changes in two first frequencies of natural vibrations of column (Ω_1, Ω_2) as a function of the parameter of external load. In Figure 2., the changes in first frequency of natural vibration of non-prismatic system for different values of taper parameter Z (shape approximation by linear function) was illustrated. The results regarding to the approximation by quadratic function were presented in Figure 3., taking into account variable location of a vertex of parabola (p^*, q^* parameters).

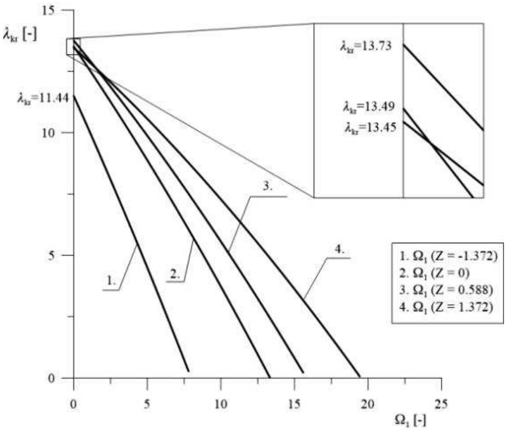

Figure 2.The first frequency of vibration of non-prismatic column approximated by linear function (R^*=1.3) for selected values of taper parameter Z

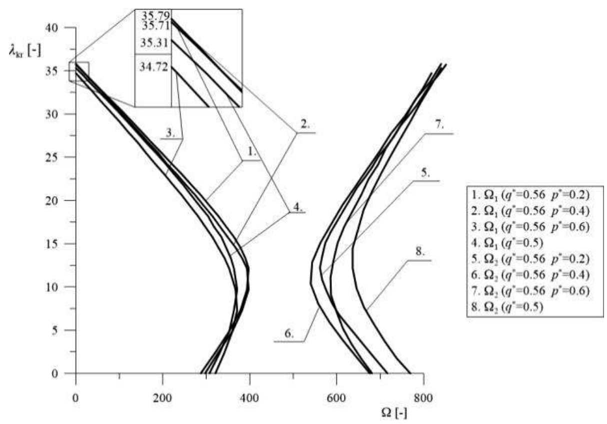

Figure 3. The characteristic curves of column approximated by quadratic function (R^*=0.3) for chosen values of parameters p^* and q^*

The value of critical load for presented curves on the plane dimension less parameter of external load – dimensionless parameter of frequency of free vibrations is determined for $\Omega = 0$. The results regarding to the values of critical load parameter, obtained on the basis of the kinetic criterion of stability loss, show compliance with the results from the energetic method (the static criterion of loss of stability).Presented courses of changes in eigenvalues have the positive, zero or negative slope, depending on the value of external load and radius of loading head. Therefore, considered structures may be classified as a divergent or divergent pseudo fluttering type of system.

5. Conclusions

The analysis of free vibrations of non-prismatic column subjected to the follower force directed towards the positive pole was presented in this paper. On the basis of conducted numerical calculations, the following conclusions were formulated:
– shape of system approximation effects the value of frequency of vibration. The value of critical load of the system depends on the parameters describing shape of the column and geometrical parameters of loading structure,
– depending on the value of radius of the loading head parameter, the system under consideration may be classified as the divergent or divergent pseudo fluttering type of system,
– approximation of the shape of the considered column is restricted by the condition which states that the value of width *b* of each segments of system must be greater than or equal to the thickness *h* of segments.

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