Dynamic Analysis of Optimized Two-Phase Auxetic Structure

Eligiusz IDCZAK
Institute of Applied Mechanics, Poznan University of Technology
ul. Jana Pawla II 24, 60-965 Poznan, Poland
eligiusz.j.idczak@doctorate.put.poznan.pl

Tomasz STRĘK
Institute of Applied Mechanics, Poznan University of Technology
ul. Jana Pawla II 24, 60-965 Poznan, Poland
tomasz.strek@put.poznan.pl

Abstract

This paper presents a dynamic analysis of earlier optimized auxetic structure. This optimization based on the distribution of two materials in such way to obtain a minimal value of Poisson's ratio (PR), which indicates the auxetic properties. The initial optimized shape was so-called star structure, which if is made from one material has the PR close to 0.188. After optimization with the goal function of PR-minimization, the obtained value was equal to -9.5043. Then the eigenfrequencies for the optimized structure were investigated. The calculations were carried out by means of Finite Element Method (FEM). For optimization of the value of Poisson's ratio was used algorithm MMA (Method of Moving Asymptotes). The computing of single material properties (PR, Young's modulus, density) for the whole shape was made by means of SIMP method (Solid Isotropic Method with Penalization).

Keywords: negative PR, auxetics, eigenfrequency, topology optimization, dynamic analysis

1. Introduction

Dynamic analysis is one of the most important parts of investigations of structures and constructions. It allows evaluating their real behaviours under influence of real forces. The carrying out of dynamic simulations is mainly important during the first step of the design process to find out how the newly designed structure or shape will behave under dynamic exerting forces.

The first step for dynamic analysis is computing of eigenfrequencies. Natural frequencies are the frequencies at which a system tends to oscillate in the absence of any driving or damping force. The calculating of eigenfrequencies allows checking how the structure will behave after load and how will cooperate with other elements of the system. It is the initial stage of the checking the usefulness of the shape for the industrial applications.

The most common auxetic metamaterials are cell structures which are consisted of many repeated single cells. These structures are constantly developed because of their advantageous properties like low density, beneficial damping behaviour, energy absorption and many others. The examples of auxetic cellular metamaterials are: re-entrant honeycombs (Fig. 1), rotating units (Fig. 2.), star-shaped structures (Fig. 3.), "missing rib" (Fig. 4.) "double arrowhead" (Fig. 5.), chiral and anti-chiral structures (Fig. 6.).

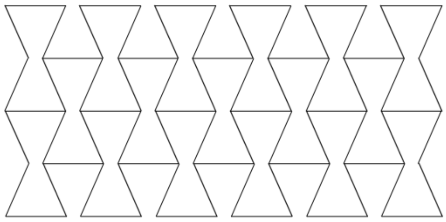

Figure 1. Re-entrant honeycomb υ = -0.39 [1]

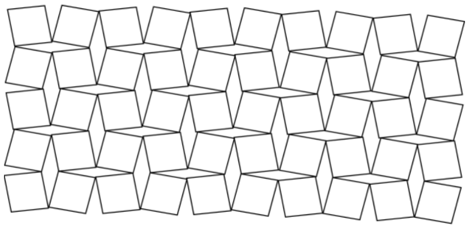

Figure 2. Rotating squares υ = -0.963 [2]

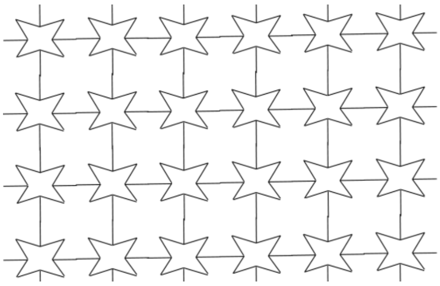

Figure 3. Star structure υ = -0.25 ÷ -0.35 [3]

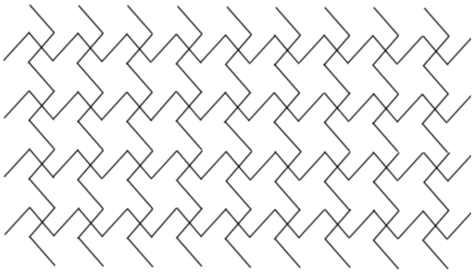

Figure 4. Missing rib structure υ = -0.6 [4]

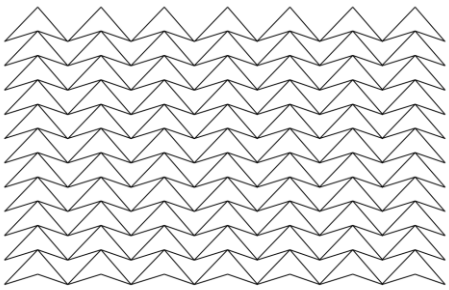

Figure 5. Double arrow head topology

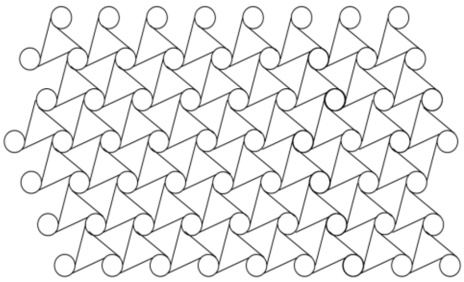

Figure 6. Chiral honeycomb υ = -0.98 [5]

As presented above the values of negative PR for homogenous cellular structures is not less than -1. But through material distribution by means of topology optimization, the two-phase auxetic cell can have the value of PR on the level -9.5.

In the literature, there are many examples of dynamic analysis of auxetics. First works about deformation mechanism of structures with negative PR were presented by Gibson [6] and the inventor of name "auxetic" - R. Lakes in his papers [7-9]. Scarpa [10] shows various dynamic characteristics of open cell compliant polyurethane foam with auxetic behavior. In the works [11, 12] was shown the dynamic analysis of periodic auxetic chiral structures. The authors numerically and experimentally proved that the chiral structures deform when were excited at one of its eigenfrequencies. This is particularly important because resonance can be used to minimize the power required for the occurrence of localized deformations.

The most common analysis is connected with simulations and experiments which confirm whether the structures with negative PR have favourable properties for damping the vibrations or acoustic isolations. It is also defined the specific ratios for checking the damping's properties like VTL (Vibration Transmission Loss) or STL (Sound Transmission Loss) [13-15]. The natural frequencies for the earlier optimized structure with the criterion of minimal internal energy were described in the papers [16, 24].

The auxetics are also checked to find out which deformation will take place under influence of harmonic loading force and to compute the value of crushing strength of the structure. Dynamic comparing analysis of re-entrant honeycomb (negative PR) and hexagonal honeycomb (positive PR) proves the relation between negative PR and crushing strength [17].

The dynamic behaviour of the composite with auxetic core is presented in work [18]. Authors presented effective properties and dynamic response of a sandwich panel made of two face sheets and core.

Other optimized shapes with the goal function of minimization of the Poisson's ratio were shown in the papers [22, 23].

2. Properties and methods

To compute the effective values of parameters (Young's modulus, Poisson's ratio, density) of the two-phase shape cell is used effective interpolation method. Here it's used the solid isotropic material with a penalization (SIMP) scheme. The most important part of SIMP is the insertion of an interpolation function as a function of the continuous variable. This interpolation function defines the mechanical properties. The particular parameters in SIMP method for the minimization of PR fulfill the equations:

$$E(r) = E_1 + (E_2 - E_1) r^p \tag{1}$$

$$\nu(r) = \nu_1 + (\nu_2 - \nu_1) r^p \tag{2}$$

where: $r = r(x)$ – control variable for computing the effective values of properties, p – penalization parameter, which in optimizations is equal to 3 (previous works about SIMP recognize penalization parameter equal to 3 as the most effective), E_1, E_2, υ_1, υ_2 – Young's moduli and Poisson's ratios of first and second material. Density of material represents continuous variable $0 < r < 1$. In this method, the continuous variables are described as material densities. SIMP also can evaluate bulk and shear moduli and other physical properties.

The Poisson's ratio for the homogenous, isotropic elastic solid material is the negative ratio of transverse to longitudinal strain at every point in a body under longitudinal loading. In a material with two phases where the ratios may be changed classic definition Poisson's ratio isn't possible to use. The new definition of Poisson's ratio was described by many authors and introduces small change to an equation and it is calculated as a negative ratio of the average transverse to longitudinal strains [19]:

$$\nu_{eff} = -\frac{\overline{\varepsilon_t}}{\overline{\varepsilon_l}} \tag{3}$$

where: $\overline{\varepsilon_t}$ – average transverse strain, $\overline{\varepsilon_l}$ – average longitudinal strain. When the force is applied along the *y*-axis, the average transverse strain is defined as:

$$\overline{\varepsilon_t} = \frac{\int_{\Gamma_1} u_1 d\Gamma}{L_x \int_{\Gamma_1} d\Gamma} \tag{4}$$

where Γ_1 is the boundary parallel ($x = L_x = 0.4$) to the boundary with prescribed displacement applied. The average longitudinal strain is defined as:

$$\overline{\varepsilon_l} = \frac{\int_{\Gamma_2} u_2 d\Gamma}{L_y \int_{\Gamma_2} d\Gamma} \tag{5}$$

where Γ_2 is the boundary ($y = L_y = 0.4$) where a load is applied.
Because of using SIMP scheme effective Poisson's ratio must be dependent on control variable r, like in the equation (6):

$$\nu_{eff}(r) = -\frac{\overline{\varepsilon_t(r)}}{\overline{\varepsilon_l(r)}} \tag{6}$$

SIMP scheme for control variable function must have two constraints - pointwise inequality (7) and integral inequality (8), which are given below:

$$0 \le r(x) \le 1 \text{ for } x \in S \tag{7}$$

$$0 \le \int_S r(x) dS \le A_f \cdot S \tag{8}$$

where: x - defined coordinate, A_f – fraction of the second material of the domain S. The order of optimization order is as follows: FEM – discretization, the redefinition of minimization function with applied constraints and at the end the value of control variable is calculated at every mesh node as:

$$r(x) = \sum_{i=1}^{N} r_i \cdot \Phi_i(x) \tag{9}$$

where: $\Phi_i(x)$ are the shape functions, i – is the number of an element node, N – is the amount of all nodes.

3. Equation of motion of the solid

The Navier's equation of motion of solid has the form [20]:

$$\rho\frac{\partial^2\mathbf{u}}{\partial t^2}-\nabla\cdot\boldsymbol{\sigma}=0 \tag{10}$$

where: **u** – the vector of displacements, ρ - the density, **σ** - the stress tensor and is defined as [21]:

$$\boldsymbol{\sigma}=\mathbf{D}\boldsymbol{\varepsilon}=\lambda(\nabla\cdot\mathbf{u})\mathbf{I}+2\mu\boldsymbol{\varepsilon} \tag{11}$$

where: **D** – constitutive matrix, **I** – identity matrix, **ε** – the strain tensor, defined as:

$$\boldsymbol{\varepsilon}=\frac{1}{2}(\nabla\mathbf{u}+(\nabla\mathbf{u})^T) \tag{12}$$

and μ, λ – Lame constants, which fulfills the equations:

$$\lambda=\frac{E\cdot\nu}{(1-2\nu)(1+\nu)},\qquad \mu=G=\frac{E}{2(1+\nu)} \tag{13}$$

where: E is Young's modulus, G is shear modulus, υ is Poisson's ratio.

The Navier's equation of motion with linear constitutive relation between stresses and deformations [20] is:

$$\rho\frac{\partial^2\mathbf{u}}{\partial t^2}-(\mu\nabla^2\mathbf{u}+(\lambda+\mu)\nabla\nabla\cdot\mathbf{u})=0 \tag{14}$$

A harmonic displacement is defined by equation as below:

$$\frac{\partial^2\mathbf{u}}{\partial t^2}=-\omega^2\mathbf{u} \tag{15}$$

where: ω – frequency. The displacement vector has the complex form and is defined as:

$$\mathbf{u}(x)=\mathbf{u_1}(x)+i\mathbf{u_2}(x) \tag{16}$$

where the harmonic displacement is a real part of complex form:

$$\mathbf{u}(\mathbf{x},t)=\mathrm{Re}[\mathbf{u}(\mathbf{x})e^{-i\omega t}] \tag{17}$$

According to aforementioned equations the harmonic equation of motion fulfills the formula:

$$-\rho\omega^2\mathbf{u}-(\mu\nabla^2\mathbf{u}+(\lambda+\mu)\nabla\nabla\cdot\mathbf{u})=0\,. \tag{18}$$

4. Numerical results

Optimized star-shape structure with the criterion of minimal PR by means of distribution of two materials is shown in Figure 7 (material 1 – green colour and material 2 – blue colour). Deformation of the compressed structure is presented in Figure 8. The parameters of materials 1 and 2 are presented in Table 2. The minimal effective PR after optimization has the value: -9.5043. The optimization Method of Moving Asymptotes (MMA) and Solid Isotropic Method with Penalization (SIMP) were used to received two-phase structures with lower Poisson's ratio [22].

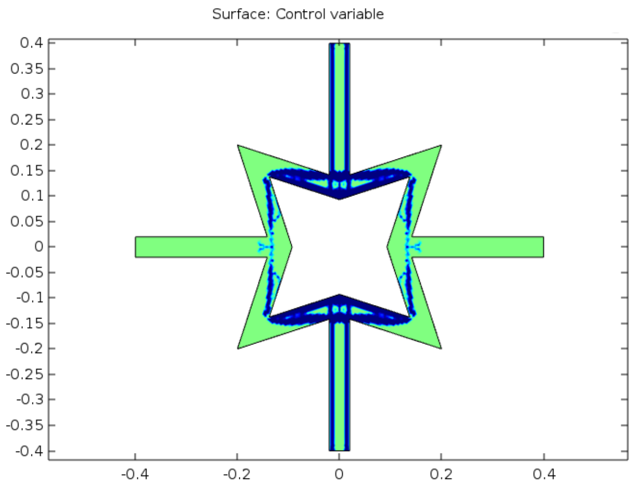

Figure 7. Distribution of two materials in star-shape structure with the minimal PR

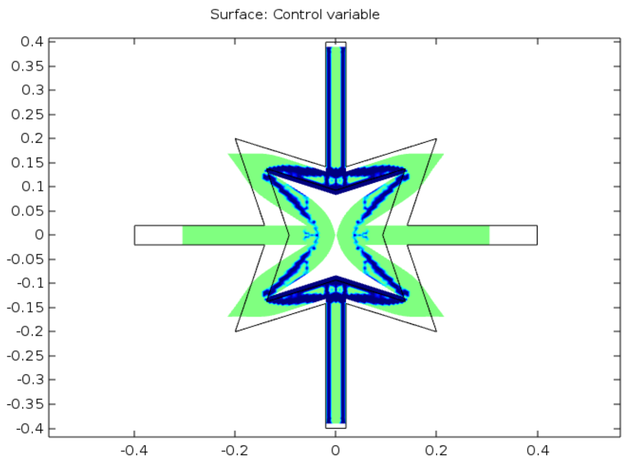

Figure 8. Deformation of structure with minimal PR

Table 1. Material properties of two-phase structure

No.	Material 1	Material 2
Poisson's ratio	0.1	0.33
Young's modulus [MPa]	1e7	1e11

Dynamic analysis of optimized shape consists in the investigation of the deformations after the harmonic load. The seven eigenfrequencies of the structure were determined – in table 2 are the values of each eigenfrequency. To compare the values for two-phase structure presented in table 2, the results for homogenous one-phase structure are also presented. The mode shapes for each natural frequency are shown in the Figures 9-15 (color figures available only online).

Table 2. Values of eigenfrequecies for structures

No. of eigenfrequency		1	2	3	4	5	6	7
Two-phase	Value [Hz]	20.472	55.239	71.912	117.17	151.83	158.53	186
One-phase	Value [Hz]	96.63	351.64	443.02	620.82	800.53	944.99	1088.7

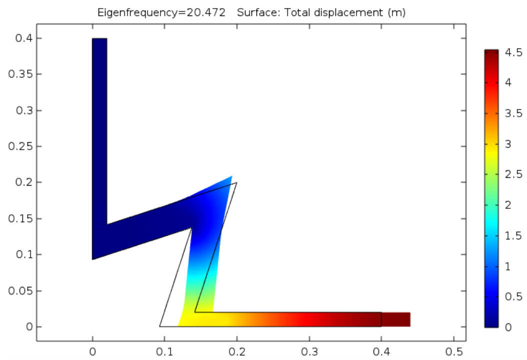

Figure 9. Mode shape for the quarter of optimized structure (eigenfrequency 20.472 Hz)

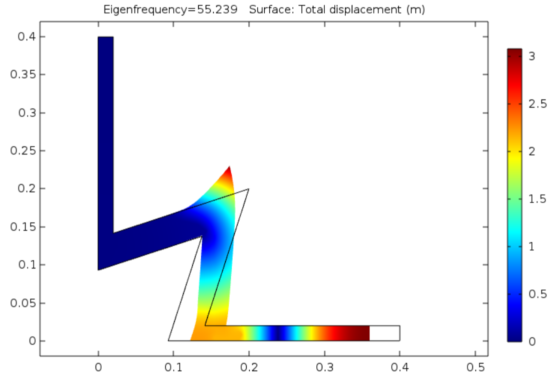

Figure 10. Mode shape for the quarter of optimized structure (eigenfrequency 55.239 Hz)

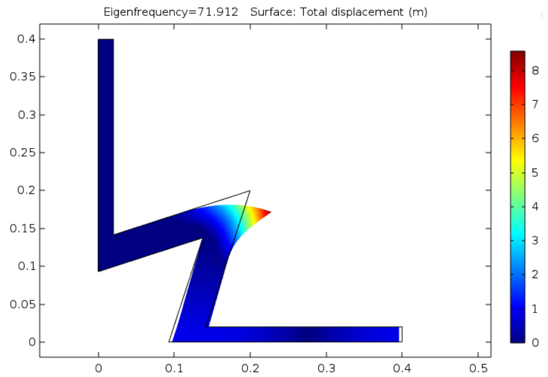

Figure 11. Mode shape for the quarter of optimized structure (eigenfrequency 71.912 Hz)

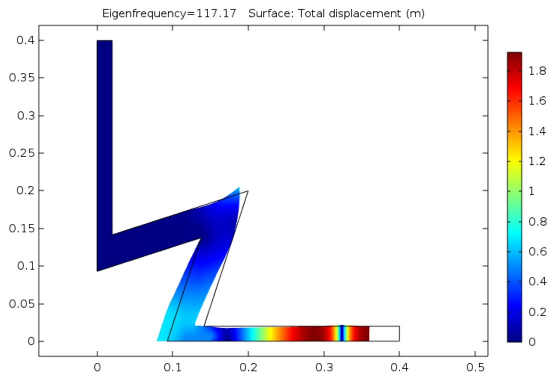

Figure 12. Mode shape for the quarter of optimized structure (eigenfrequency 117.71 Hz)

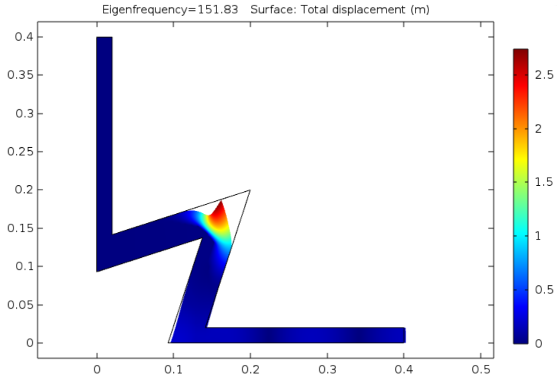

Figure 13. Mode shape for the quarter of optimized structure (eigenfrequency 151.83 Hz)

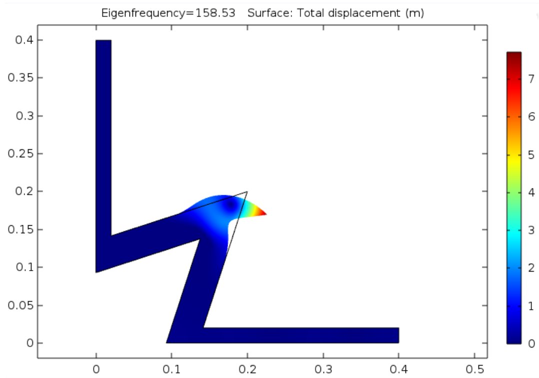

Figure 14. Mode shape for the quarter of optimized structure (eigenfrequency 158.53 Hz)

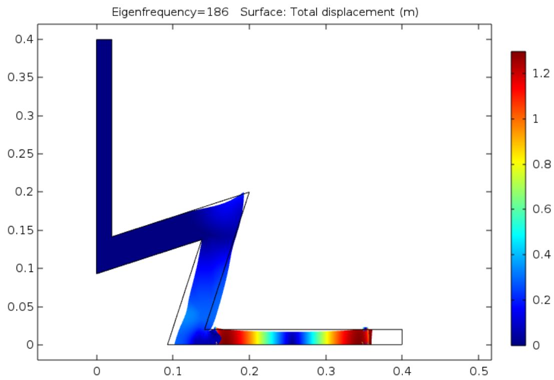

Figure 15. Mode shape for the quarter of optimized structure (eigenfrequency 186 Hz)

5. Conclusions

Dynamic properties of the auxetic structure composed of two materials were investigated. The eigenfrequencies of minimized shape and amplitude of forced vibration were determined.

The knowledge about these dynamic properties can be useful for deciding how serviceable will be the whole structure. In the future may be possible to apply analyzed or similar structure to the industrial applications.

Acknowledgments

This work was supported by grants of the Ministry of Science and Higher Education in Poland: 02/21/DS PB/3493/2017 and 02/21/DSMK/3498/2017. The simulations have been carried out at the Institute of Applied Mechanics, Poznan University of Technology.

References

1. E. A. Friis, R. S. Lakes, J. B. Park, *Negative Poisson's ratio polymeric and metallic foams.*, Science, **23**(12) (1988) 4406 – 4414.
2. M. N. Ali, J. J. C. Busfield, I. U. Rehman, *Auxetic oesophageal stents: structure and mechanical properties,* Journal of Materials Science: Materials in Medicine, **25**(2) (2014) 527 – 553.
3. M. Rad, A. Zaini, A. Amran, *Computational Approach in Formulating Mechanical Characteristics of 3D Star Honeycomb Auxetic Structure*, Advances in Materials Science and Engineering, (2015) 1 – 11.
4. C. W. Smith, J. N. Grima, K. E. *Evans A novel mechanism for generating auxetic behaviour in reticulated foams: missing rib foam model*, Acta Materialia, **48**(17) (2000) 4349 – 4356.
5. A. Alderson, K. L. Alderson, D. Attard, K. E. Evans, K. E. R. Gatt, J. N. Grima, W. Miller, N. Ravirala, C. W. Smith, C. W. K. Zied, *Elastic constants of 3-, 4- and 6-connected chiral and anti-chiral honeycombs subject to uniaxial in-plane loading*, Composites Science and Technology, **70**(7) (2010) 1042 – 1048.
6. L. J. Gibson, M. F. Ashby, *The mechanics of three-dimensional cellular materials*. Proc. R. Soc. A Math. Phys. Eng. Sci., **382** (1982) 43 – 59.
7. R. S. Lakes, *Foam Structures with a Negative Poisson's Ratio*. Science (80-.), **235** (1987) 1038 – 1040.
8. R. S. Lakes, *Deformation Mechanisms in Negative Poisson's Ratio Materials: Structural Aspects*, J. Mat. Sci., **26** (1991) 2287 – 2292.
9. D. Prall, R.S. Lakes, *Properties of a Chiral Honeycomb with Poisson's Ratio of -1*, Int. J. of Mechanical Sciences, **39** (1996) 305–314.
10. F. Scarpa, L. G. Ciffo, J. R. Yates, Dynamic properties of high structural integrity auxetic open cell foam. Smart Mater. Struct., **13 (**2004) 49 – 56.
11. A. Spadoni, M. Ruzzene, F. Scarpa, *Dynamic response of chiral truss-core assemblies*, Journal of intelligent material systems and structures, **17**(11) (2006) 941 – 952.

12. A. Spadoni, M. Ruzzene, *Structural and Acoustic Behavior of Chiral Trusscore Beams*, Proceedings of the ASME (IMECE 2004) Noise Control and Acoustic Division 2004.
13. H. R. Joshi, *Finite Element Analysis of effective mechanical properties, vibration and acoustic performance of auxetic chiral core sandwich structures*, All Theses., Clemson University, Clemson, South Carolina 2013.
14. R. R Galgalikar, *Design automation and optimization of honeycomb structures for maximum sound transmission loss*, Clemson University, Clemson, South Carolina 2012.
15. E. Idczak, T. Stręk, *Computational Modelling of Vibrations Transmission Loss of Auxetic Lattice Structure*, Vibrations in Physical Systems, **27** (2016) 124 – 128.
16. M. Nienartowicz, T. Stręk, *Finite Element Analysis of Dynamic Properties of Thermally Optimal Two-Phase Composite Structure*, Vibrations in Physical Systems **26** (2014) 203 – 210.
17. W. Liu, M. Wang, T. Luo, Z. Lin, *In-plane dynamic crushing of re-entrant auxetic cellular structure*, Materials & Design, **100** (2016) 84 – 91.
18. T. Strek, H. Jopek, M. Nienartowicz, *Dynamic response of sandwich panels with auxetic cores*, Phys. Status Solidi B, **252**(7) (2015) 1540 – 1550.
19. T. Strek, H. Jopek, K. W. Wojciechowski *The influence of large deformations on mechanical properties of sinusoidal ligament structures*, Smart Materials and Structures, **25**(5) (2016) 05402.
20. B. Lautrup, *Physics of Continuous Matter, Exotic and Everyday Phenomena in the Macroscopic World*, IOP 2005.
21. O. C. Zienkiewicz, R. L. Taylor, *The Finite Element Method, Volume 2, Solid Mechanics*, Butterworth-Heinmann 2000.
22. E. Idczak, T. Strek, *Minimization of Poisson's ratio in anti-tetra-chiral two-phase structure*, IOP Conf. Ser.: Mater. Sci. Eng., **248** (2017) 012006.
23. T. Stręk, H. Jopek, E. Idczak, K. W. Wojciechowski, *Computational Modelling of Structures with Non-Intuitive Behaviour*, Materials, **10** (2017) 1386.
24. M. Nienartowicz, T. Stręk, *Modeling and FEM analysis of dynamic properties of thermally optimal composite materials*, 11th World Congress on Computational Mechanics, WCCM 2014, 5th European Conference on Computational Mechanics, ECCM 2014 and 6th European Conference on Computational Fluid Dynamics, ECFD 2014.