Large Amplitude Free Vibration Analysis of Tapered Timoshenko Hinged-Hinged Beam Using Coupled Displacement Field Method

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Abstract

Tapered beams are more efficient compared to uniform beams as they provide a better distribution of mass and strength and also meet special functional requirements in many engineering applications like architecture, aeronautical, civil, mechanical, automobile, nuclear and robotics. The authors proposed a new method called Coupled Displacement Field (CDF) method in which the displacement field such as total rotation is assumed such that the assumed displacement must satisfy the kinematic and force boundary condition of the beam. The lateral transverse displacement is derived from the coupling equation which is derived from the static equilibrium conditions of the beam. By the application of principle of minimum total potential energy for different beam boundary conditions, the fundamental frequency parameter value is calculated in terms of taper ratio and slenderness ratio for various maximum amplitude ratios of the tapered Timoshenko shear flexible hinged-hinged beam boundary condition.

Keywords: large amplitude free vibrations, Coupled Displacement Field method, tapered Timoshenko beams, slenderness ratio; taper ratio

1. Introduction

Many authors developed different methods to find the free vibration behaviour of shear flexible beams for a long period of time and are mentioned below. The free vibration of nonuniform beams with general shape and arbitrary boundary conditions was analyzed [1]. Free vibrations of tapered beams with general boundary condition is ealuated by using the ordinary differential governing equation of beams which can be solved by numerical methods and the natural frequencies are calculated by combining the Runge Kutta method and the determinant search method [2]. The dynamic behaviour of beams with linearly varying cross-section was studied by the equation of motion in terms of Bessel functions, and the boundary conditions lead to the frequency equation which is a function of four flexibility coefficients [3]. Natural vibration frequencies of tapered beams by using Euler-Bernoulli beam theory in the presence of an arbitrary number of rotationally, axially and elastically flexible constraints were studied by the dynamic analysis, performed by means of the so-called cell discretization method (CDM), according to which the beam is reduced to a set of rigid bars, linked together by elastic sections, where the bending stiffness and the distributed mass of the bars is concentrated [4]. Wentzel, Kramers, Brillouin (WKB) approximation was used to study the transverse free vibration of a class of variable-cross-section beams in which the governing equation of motion of the Euler–Bernoulli beam including axial force distribution is utilized to obtain a singular differential equation in terms of the natural frequency of vibration and a WKB expansion series is applied to find the solution [5]. Green's function method was used for the free vibration problem of non uniform Bernoulli-Euler beams, to find the Green's function of the fourth order differential operator, occurring at the beam's equation of motion, the power series method is proposed [6]. The differential transformation method (DTM) was used for free vibration analysis of beams with uniform and non-uniform cross sections [7].

The Coupled Displacement Field method applied to free vibration analysis of uniform Timoshenko beams for different beam boundary conditions [8]. The vibrations of an isotropic beam with a variable cross-section is studied by uysing the governing equation by reducing it to an ordinary differential equation in spatial coordinate for a family of cross-section geometries with exponentially varying width [9]. Non-linear vibration analysis was premeditated by establishing equations of motion for taper Timoshenko beams [10]. A mathematical model for vibrations of non-uniform flexural beams was presented for free vibrations of non-uniform viscoelastic flexural beams by getting an analytical solution for the fourth order differential equation of beam vibration under appropriate boundary conditions by factorization and calculated mode shapes and damped natural frequencies of the beam for wide range of beam characteristics [11]. The concept of coupled displacement method was sucessfully applied for large amplitude free vibrations of shear flexible beams and the approach leads to only one undetermined coefficient, in the case of single-term admissible functions, which can easily be used in the principle of conservation of total energy, neglecting damping, to solve the problem [12]. The natural frequencies and dynamic behaviour vibration of linearly tapered beams subjected to different combinations of edge supports by finite element algorithmic procedures are evaluated [13]. The Green's function method is used in frequency analysis of a beam with varying cross section for the beam carrying an arbitrary number of attached discrete systems.The exact solution of the problem concerns a beam with quadratically varying cross-section area [14]. The vibrational characteristics of tapered beams with continuously varying rectangular cross-section of depth and breadth proportional to x_s and x_t respectively, where both s and t are arbitrary real numbers for a truncated beam and arbitrary positive numbers for a sharp ended beam and x is the axial co-ordinate measured from the sharp end of the beam and obtained the eigen frequency equation by the Rayleigh-Ritz method [15].

The solution for the large amplitude free vibration problems using energy method involves assuming suitable admissible functions for lateral displacement and the total rotation which leads to two coupled nonlinear differential equations in terms of lateral displacement and the total rotation. This can be overcome with less computational efforts by Coupled Displacement Field method in which lateral displacement and total rotation are coupled through the static equilibrium equation of the shear flexible beam.

2. Coupled Displacement Field (CDF) method

The concept of coupled displacement field method is explained in detail. In the Coupled Displacement Field Method (CDF) with the single term admissible function for total rotation θ, the function for transverse displacement w is derived using the coupling equation. The coupling equation has been derived from the kinematic and static boundary conditions of beam.

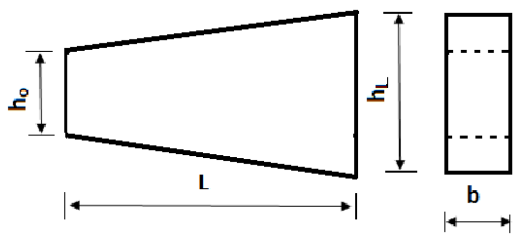

Figure 1. Tapered Timoshenko beam with linearly varying height (depth)

2.1. *Coupling equation*

From the kinematics of a shear flexible beam theory

$$\bar{u}\,(x, z) = z\theta \tag{1}$$

$$\bar{w}\,(x, z) = w\,(x, z) \tag{2}$$

where $\bar{u}$ is the axial displacement and $\bar{w}$ is the transverse displacements at an any point of the beam, z is the distance of the any point from the neutral axis and θ is the total rotation anywhere on the beam axis and x, z are the independent spatial variables. The axial and shear strains are given by

$$\varepsilon_x = z\,\frac{\partial\theta}{\partial x} \tag{3}$$

$$\gamma_{xz} = \frac{\partial w}{\partial x} + \theta \tag{4}$$

Now, the expressions for the strain energy 'U' and the work done 'W' by the externally applied loads are given by

$$U = \frac{EI}{2}\int_0^L \left(\frac{d\theta}{dx}\right)^2 dx + \frac{kGA}{2}\int_0^L \left(\frac{dw}{dx} + \theta\right)^2 dx \tag{5}$$

$$W = \int_0^L p(x)w(x)dx \tag{6}$$

where EI is the flexural rigidity, GA is the shear rigidity, k is the shear correction factor (taken as 5/6 in the present study), $p(x)$ is the static lateral load per unit length acting on the beam, E is the Young's modulus, G is the shear modulus, x is the axial coordinate and L is the length of the beam. Applying the principle of minimization of total potential energy, as

$$\delta(U - W) = 0 \tag{7}$$

The following equilibrium equations can be obtained

$$kGA\left(\frac{d^2w}{dx^2} + \frac{d\theta}{dx}\right) + p = 0 \tag{8}$$

$$EI\frac{d^2\theta}{dx^2} - kGA\left(\frac{dw}{dx} + \theta\right) = 0 \tag{9}$$

where θ is total rotation, w is transverse displacement. Equations (8) and (9) are coupled equations and can be solved for obtaining the solution for the static analysis of the shear deformable beams. A close observation of equation (8) shows that it is dependent on the load term p and equation (9) is independent of the load term p. Hence, equation (9) is used to couple the total rotation θ and the transverse displacement w, so that the two undetermined coefficients problem (for single term admissible function) becomes a single undetermined coefficient problem and the resulting linear free vibration problem becomes much simpler to solve.

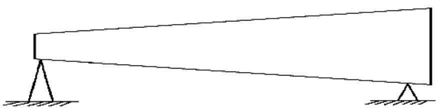

Figure 2. Tapered Timoshenko hinged-hinged beam (depth taper) with axially immovable ends

An admissible function for tapered Timoshenko hinged-hinged beam θ which satisfies all the applicable boundary conditions and the symmetric condition is assumed in the beam domain as

$$\theta = a\frac{\pi}{L}\cos\left(\frac{\pi}{L}x\right) \tag{10}$$

$$\frac{d\theta}{dx} = -a\frac{\pi^2}{L^2}\sin\left(\frac{\pi}{L}x\right) \tag{11}$$

$$\frac{d^2\theta}{dx^2} = -a\cos\left(\frac{\pi}{L}x\right)\frac{\pi^3}{L^3} \tag{12}$$

where a is the central lateral displacement of the beam which is also the maximum lateral displacement. Rewriting equation (9)

$$\frac{dw}{dx} = -\theta + \frac{EI}{kGA}\frac{d^2\theta}{dx^2} \tag{13}$$

By integrating the above equation, lateral displacement can be obtained as

$$w = -a\left[1 + \frac{\pi^2}{L^2}\frac{EI}{kGA}\right]\sin\left(\frac{\pi}{L}x\right) \tag{14}$$

It may be noted here that because of the coupled displacement field concept, the transverse displacement w distribution contains the same undetermined coefficient a as the θ distribution and satisfies all the applicable essential boundary and symmetric conditions.

$$w(0) = w(L) = \left.\frac{dw}{dx}\right|_{x=L/2} = 0 \tag{15}$$

2.2. *Linear free vibrations*

Linear free vibrations can be studied, once the coupled displacement field for the lateral displacement w, for an assumed θ distribution is evaluated using the principle of conservation of total energy at any instant of time, neglecting damping, which states that $U + T$ = constant. The expression for U and T are given by

$$U = \frac{E}{2}\int_0^L I\left(\frac{d\theta}{dx}\right)^2 dx + \frac{kG}{2}\int_0^L A\left(\frac{dw}{dx} + \theta\right)^2 dx \tag{16}$$

$$T = \frac{\rho\omega_L^2}{2}\int_0^L Aw^2 dx + \frac{\rho\omega_L^2}{2}\int_0^L I\theta^2 dx \tag{17}$$

$$A = A_0\left(1 + \frac{\alpha}{L}x\right), I = I_0\left(1 + \frac{\alpha}{L}x\right)^3, \alpha = \frac{(h_L - h_0)}{h_L} \tag{18}$$

where T is the kinetic energy, h_L, h_0, are the height of the beam at left end $x = 0$ and the right end $x = L$ respectively, A_0 and I_0 are cross sectional area and area moment of inertia at right side, A is the area at any cross section, α is the taper ratio. Substituting equations (11), (13) and (18) in equations (16) and after simplification

$$U=\frac{a^2EI_0\pi^2}{2L}\left[\left(0.5+0.0871\alpha^3+0.4241\alpha^2+0.75\alpha\right)+3.12\frac{\pi^2}{\beta^2}\left(0.5+0.25\alpha\right)\right] \tag{19}$$

Substituting equations (10), (14) and (18) in equations (17) and after simplification

$$T=\frac{a^2\rho A_0\omega_L^2L^3}{2\pi^2} \cdot\left[\left(1+3.12\frac{\pi^2}{\beta^2}\right)^2\left(0.5+0.25\alpha\right)+\frac{\pi^2}{\beta^2}\left(0.5+0.1629\alpha^3+0.5759\alpha^2+0.75\alpha\right)\right] \tag{20}$$

By the application of principle of minimum total potential energy principle $\left(\frac{\partial(U-T)}{\partial a}=0\right)$ with respect to undetermined coefficient *a*, the fundamental frequency parameter is obtained and is given as below

$$\lambda=\frac{\rho A_0\omega_L^2L^4}{EI_0} \tag{21}$$

$$=\pi^4\frac{\left[\left(0.5+0.0871\alpha^3+0.4241\alpha^2+0.75\alpha\right)+3.12\frac{\pi^2}{\beta^2}\left(0.5+0.25\alpha\right)\right]}{\left[\left(1+3.12\frac{\pi^2}{\beta^2}\right)^2\left(0.5+0.25\alpha\right)+\frac{\pi^2}{\beta^2}\left(0.5+0.1629\alpha^3+0.5759\alpha^2+0.75\alpha\right)\right]} \tag{22}$$

where λ is the non dimensional fundamental frequency parameter, $\beta = L/r$ (slenderness ratio) and r is radius of gyration for the cross-section of the beam.

3. Large Amplitude free vibrations

For an assumed θ distribution, the coupled displacement field for the lateral displacement w is evaluated, after the lateral displacement w is calculated, the large amplitude free vibrations can be studied using the principle of conservation of total energy at any instant of time neglecting damping.

$$U + T + W = \text{const.} \tag{23}$$

Work done due to large amplitudes

$$W = \frac{T_a}{2}\int_0^L \frac{1}{2}\left(\frac{dw}{dx}\right)^2 dx \tag{24}$$

where w is transverse displacement obtained from coupling equation. From Woinowsky-Krieger equation

$$T_a = \frac{E}{2Lr^2}\int_0^L I_0 \left(\frac{dw}{dx}\right)^2 dx \tag{25}$$

where

$$w = a\sin\frac{\pi x}{L}, \frac{dw}{dx} = \frac{a\pi}{L}\cos\left(\frac{\pi x}{L}\right) \tag{26}$$

T_a is the tension developed in the beam because of large deformations. W is the work done by the tension developed because of large amplitudes, ρ is the mass density. T_a is evaluated in terms of the amplitude ratio (a/r). Substituting the values of w (obtained from coupled displacement field),equation (25) in equation (24) and solving the work done due to large amplitudes becomes

$$W = \frac{EI_0\pi^2 a^4}{16r^2L}\frac{(2-\alpha)(\pi^2\alpha^2 + 3\alpha^2 - 2\pi^2\alpha + 2\pi^2)}{8\pi^2}\left(1 + \frac{\pi^2}{L^2}3.12r^2\right) \tag{27}$$

Substituting equations (19), (20) and (27) in equation (23) and simplifying,
the following form is obtained

$$\dot{a}^2 + a^2\alpha_1 + a^4\alpha_2 = \text{const.} \tag{28}$$

where

$$\alpha_1 = \frac{EI_0 2\pi^4}{\rho L^4 A_0}\frac{\left[(0.5 + 0.0871\alpha^3 + 0.4241\alpha^2 + 0.75\alpha) + \frac{3.12\pi^2}{\beta^2}(0.5 + 0.25\alpha)\right]}{\left[\left(1 + \frac{3.12\pi^2}{\beta^2}\right)^2(0.5 + 0.25\alpha) + \frac{\pi^2}{\beta^2}(0.5 + 0.1629\alpha^3 + 0.5759\alpha^2 + 0.75\alpha)\right]} \tag{29}$$

$$\alpha_2 = \frac{EI_0\pi^2}{35\rho L^4 A_0 r^2}\frac{\left[(2-\alpha)(\pi^2\alpha^2 + 3\alpha^2 - 2\pi^2\alpha + 2\pi^2) + \left(1 + \frac{3.12\pi^2}{\beta^2}\right)^2\right]}{\left[\left(1 + \frac{3.12\pi^2}{\beta^2}\right)^2(0.5 + 0.25\alpha) + \frac{\pi^2}{\beta^2}(0.5 + 0.1629\alpha^3 + 0.5759\alpha^2 + 0.75\alpha)\right]} \tag{30}$$

The ratio of non linear and linear frequency is expressed as

$$\left(\frac{\omega_{NL}}{\omega_L}\right)^2 = 1+\frac{3}{2}\left(\frac{\alpha_2}{\alpha_1}\right)\left(\frac{a_m}{r}\right)^2 \tag{31}$$

$$\left(\frac{\omega_{NL}}{\omega_L}\right)^2 = 1+\frac{3}{70\pi^2}\frac{\left[(2-\alpha)\left(\pi^2\alpha^2+3\alpha^2-2\pi^2\alpha+2\pi^2\right)\left(1+\frac{3.12\pi^2}{\beta^2}\right)^2\right]}{\left[\left(0.5+0.871\alpha^3+0.4241\alpha^2+0.75\alpha\right)+\frac{3.12\pi^2}{\beta^2}\left(0.5+0.25\alpha\right)\right]}\left(\frac{a_m}{r}\right)^2 \tag{32}$$

4. Numerical results and discussion

The concept of coupled displacement field and harmonic balance method are used to determine the ratios of non linear radian frequency ω_{NL} to the linear radian frequency ω_L of tapered Timoshenko beams with the two most practically used hinged-hinged beam boundary condition. Suitable single term trigonometric admissible functions are used to represent the total rotatio θ in the coupled displacement field method. The corresponding coupled lateral displacement w is derived using the coupling equation. The numerical results are obtained in terms of ω_{NL}/ω_L for various maximum amplitude, taper parameter and slenderness ratios. To assess the accuracy of the results, the present results obtained from the coupled displacement filed method are compared with the existing literature. Table.1 shows the variation of linear non dimensional Fundamental frequency parameter with slenderness ratio and taper ratio for hinged-hinged beam boundary condition. For the sake of comparison and validation of the coupled displacement filed method, the same results obtained by the other researchers are also included in Table 1. It is observed from Table 1 that the non dimensional linear fundamental frequency parameter value increases with increase in taper ratio for a given slenderness ratio. It is also observed from Table 1, the non dimensional linear fundamental frequency parameter value increases with increase in slenderness ratio for a given taper parameter. Table 2 Table 3 and Table 4 show the variation of frequency ratio ω_{NL}/ω_L with maximum amplitude and taper parameter for different slenderness ratios such as 20, 50 and 100 are given respectively for hinged-hinged beam boundary condition. It is found from Table 2, Table 3 and Table 4 that frequency ratio is function of three parameters such as maximum amplitude ratio, taper parameter and slenderness ratio. It is In general found from Table 2, Table 3 and Table 4 that frequency ratio increases with increase of maximum amplitude ratio for a given taper parameter and slenderness ratio. It is also observed from Table 2, Table 3 and Table 4 that frequency ratio decreases with increase of taper parameter for a given slenderness ratio and amplitude ratio. This is mainly because of as taper ratio increases mass of the beam also increases.

Table 1. $\lambda^{1/2}$ values for a tapered Timoshenko hinged-hinged beam (depth taper)

α	β							
	10		20		40	80	100	
	CDF Method	Ref.[13]	CDF Method	Ref.[13]	CDF Method	CDF Method	CDF Method	Ref.[13]
0	8.3912	8.388	9.4107	9.411	9.7470	9.8384	9.8496	9.850
0.1	8.6916	8.683	9.8415	9.829	10.2267	10.3317	10.3446	-
0.15	8.8435	-	10.0595	-	10.4695	10.5816	10.5953	-
0.2	8.9962	8.955	10.2789	10.228	10.7141	10.8333	10.8480	
0.25	9.1496	-	10.4996	-	10.9604	11.0869	11.1024	-
0.3	9.3036	9.205	10.7214	10.610	11.2082	11.3420	11.3585	-
0.35	9.4580	-	10.9443	-	11.4574	11.5987	11.6161	-
0.4	9.6127	-	11.1681	-	11.7079	11.8569	11.8752	-
0.45	9.7676	-	11.3926	-	11.9596	12.1163	12.1356	-
0.5	9.9225	-	11.6178	-	12.2124	12.3770	12.3973	-
0.55	10.0774	-	11.8435	-	12.4661	12.6389	12.6601	-
0.6	10.2321	-	12.0697	-	12.7208	12.9018	12.9241	-
0.65	10.3866	-	12.2962	-	12.9764	13.1658	13.1891	-
0.7	10.5407	-	12.5230	-	13.2327	13.4307	13.4551	-
0.75	10.6943	-	12.7500	-	13.4897	13.6965	13.7219	-
0.8	10.8475	-	12.9771	-	13.7474	13.9631	13.9897	-
0.85	11.0000	-	13.2042	-	14.0057	14.2304	14.2582	-
0.9	11.1519	-	13.4313	-	14.2644	14.4986	14.5274	-
0.95	11.3030	-	13.6583	-	14.5237	14.7673	14.7974	-
1	11.4533	-	13.8852	-	14.7834	15.0368	15.0681	-

Table 2. ω_{NL}/ω_L values for a tapered Timoshenko hinged-hinged beam for β = 20

a_m/r	α = 0.25		α = 0.5		α = 0.75		α = 1	
	CDF Method	Ref.[10]	CDF Method	Ref.[10]	CDF Method	Ref.[10]	CDF Method	Ref.[10]
0.10	1.0009	1.0009	1.0005	1.0007	1.0003	1.0006	1.0002	1.0005
0.20	1.0036	1.0037	1.0019	1.0030	1.0011	1.0025	1.0007	1.0021
0.30	1.0081	-	1.0042	-	1.0025	-	1.0016	-
0.40	1.0144	1.0146	1.0075	1.0119	1.0044	1.0100	1.0028	1.0085
0.50	1.0224	-	1.0118	-	1.0069	-	1.0044	-
0.60	1.0321	1.0325	1.0169	1.0266	1.0099	1.0224	1.0064	1.0190
0.70	1.0434	-	1.0230	-	1.0134	-	1.0087	-
0.80	1.0564	1.0570	1.0299	1.0467	1.0175	1.0394	1.0113	1.0336
0.90	1.0709	-	1.0377	-	1.0221	-	1.0143	-
1.00	1.0868	1.0878	1.0464	1.0721	1.0272	1.0608	1.0177	1.0519
1.10	1.1042	-	1.0559	-	1.0328	-	1.0213	-
1.20	1.1230	1.1239	1.0662	1.1022	1.0389	1.0864	1.0253	1.0740
1.30	1.1430	-	1.0773	-	1.0455	-	1.0297	-
1.40	1.1642	-	1.0891	-	1.0526	-	1.0343	-
1.50	1.1865	1.1878	1.1017	1.1552	1.0602	1.1315	1.0393	1.1131

Table 3. ω_{NL}/ω_L values for slenderness ratio β (L/r) = 50 for higed-hinged tapered Timoshenko beam

a_m/r	α			
	α = 0.25	α = 0.5	α = 0.75	α = 1
	CDF Method	CDF Method	CDF Method	CDF Method
0.10	1.0005	1.0002	1.0001	1.0000
0.20	1.0018	1.0010	1.0005	1.0004
0.30	1.0041	1.0021	1.0012	1.0008
0.40	1.0073	1.0038	1.0022	1.0014
0.50	1.0114	1.0060	1.0034	1.0022
0.60	1.0164	1.0086	1.0049	1.0032
0.70	1.0222	1.0116	1.0067	1.0043
0.80	1.0290	1.0152	1.0088	1.0056
0.90	1.0366	1.0192	1.0111	1.0071
1.00	1.0450	1.0236	1.0137	1.0088
1.10	1.0542	1.0285	1.0165	1.0106
1.20	1.0642	1.0338	1.0196	1.0127
1.30	1.0750	1.0396	1.0230	1.0148
1.40	1.0865	1.0458	1.0266	1.0172
1.50	1.0987	1.0524	1.0305	1.0197
2	1.1697	1.0914	1.0536	1.0348
3	1.3521	1.1959	1.1170	1.0767
4	1.5724	1.3284	1.2001	1.1327
5	1.8168	1.4815	1.2991	1.2009

Table 4. ω_{NL}/ω_L values for a tapered Timoshenko hinged-hinged beam for β = 100

	α							
	0.25		0.5		0.75		1	
a_m/r	CDF Method	Ref.[10]	CDF Method	Ref.[10]	CDF Method	Ref.[10]	CDF Method	Ref.[10]
0.10	1.0010	1.0010	1.0004	1.0008	1.0003	1.0007	1.0002	1.0006
0.20	1.0033	1.0040	1.0017	1.0033	1.0010	1.0028	1.0006	1.0025
0.30	1.0075	-	1.0039	-	1.0022	-	1.0014	-
0.40	1.0132	1.0158	1.0069	1.0132	1.0040	1.0113	1.0025	1.0098
0.50	1.0206	-	1.0107	-	1.0062	-	1.0040	-
0.60	1.0295	1.0353	1.0154	1.0294	1.0089	1.0252	1.0057	1.0219
0.70	1.0400	-	1.0209	-	1.0121	-	1.0078	-
0.80	1.0519	1.0619	1.0272	1.0516	1.0158	1.0444	1.0101	1.0387
0.90	1.0653	-	1.0344	-	1.0199	-	1.0128	-
1.00	1.0800	1.0950	1.0423	1.0795	1.0245	1.0685	1.0158	1.0597
1.10	1.0961	-	1.0509	-	1.0296	-	1.0191	-
1.20	1.1134	1.1344	1.0603	1.1127	1.0351	1.0972	1.0227	1.0849
1.30	1.1319	-	1.0704	-	1.0411	-	1.0266	-
1.40	1.1516	-	1.0813	-	1.0475	-	1.0308	-
1.50	1.1724	1.2033	1.0928	1.1712	1.0543	1.1479	1.0352	1.1296

5. Conclusions

The concept of the Coupled Displacement Field (CDF) method applicable to beams presented in this paper is successfully applied to study the large amplitude free vibration behaviour of tapered Timoshenko beams with axially immovable ends. Elegant and accurate closed form expression for $\left(\frac{\omega_{NL}}{\omega_L}\right)^2$ for the hinged-hinged beam boundary condition is obtained in terms of maximum amplitude ratio, taper ratio and slenderness ratio for the assumed single term admissible function for the total rotation θ.

Acknowledgements

The authors would like to thank the authorities of University College of Engineering, Jawaharlal Nehru Technological University Kakinada (JNTUK), for sponsoring and presenting the research paper under TEQIP-II.

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