Nonlinear Dynamic Phenomena in Fixed and Rotating Mechanical Structures

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Abstract

A review of selected nonlinear phenomena which may occur in fixed or rotating structures has been presented in the paper. At first, a self, parametrically and externally exited oscillator with added time delay control has been studied. It has been shown that the interaction between different vibration types may produce an untypical resonance curve, with five solutions occurring, observed by an internal resonance loop. The existence of the loop may be controlled by adding a time delay input signal. A proper selection of the time delay may reduce the loop or eliminate it totally. In the second problem a rotating hub-beam structure has been studied. The blade, apart from passive layers, has been composed of two active PZT layers which enabled active vibration control. A nonlinear coupling of the structure (plant) and the controller resulted in the so called saturation phenomenon which has been effectively used for the vibration reduction.

Keywords: nonlinear vibrations, rotating structures, resonance, saturation control, composite beams

1. Introduction

Nonlinear dynamic effects may occur in civil, mechanical or aerospace structures and sometimes, even *small nonlinearities* may result in large qualitative and quantitative changes in structural dynamics [1, 2]. The literature on nonlinear mechanics is very reach but some of special phenomena have been selected and presented in book [3]. As a classical example we may mention *pendulum-like systems*. In practice, a pendulum can be used as a dynamic absorber mounted in high buildings, bridges or chimneys. However, applying a specific semi-active damper and an additional levitating magnet inside the pendulum [3, 4] it is possible to supress vibrations and to harvest energy at the same time, taking advantage from nonlinear effects. Nonlinear mechanics must be used to explain undesired response in slender footbridges, towers or sagged cables. The famous London Millennium Bridge event [3] may serve as an example or sagged cables and towers [1, 2] which under specific fluid flow conditions may perform large oscillations of complex nature, characterized by bifurcation scenarios leading to a transition to very complex regular or chaotic oscillations.

The nonsmooth problems belongs to another and specific area of nonlinear dynamics. The methodology for solving them and mechanical engineering examples have been studied in [3] for vibro-impact systems in the form of a moling device, the opening and closing of a fatigue crack on the host system dynamics, and nonlinear interactions between a rotor and snubber ring system.

The main challenges of nonlinear dynamics are a proper reduced-order modelling, obtaining asymptotic solutions and dedicated experimental investigations which allow understanding of the observed nonlinear phenomena.

Smart active or semi-active elements, like for example: magnetorheological dampers, piezoelectric patches or shape memory alloys actuators embedded inside the structure, together with robust control algorithms, may eliminate regions of dangerous behaviour [3, 5]. We may also take advantage of the nonlinear phenomena to design an active structure to work more effectively. Nonlinear effects can be introduced artificially in control strategy to get specific behaviour, for example to enhance response of the system to harvest energy [6] or, to supress oscillations by supplying small amount of energy [7].

In the present paper we present a brief *review of specific dynamics* of (i) a self, parametrically and externally exited system with added time delay control and (ii) dynamics of a rotor with embedded active PZT layers and applied a nonlinear saturation control strategy.

2. Dynamics and Control of Nonlinear Structures

Let us consider a nonlinear oscillator of Duffing type which includes self, parametric and external excitations and time delay signals of displacement and velocity. The delayed ordinary differential equation of motion have the form [8]

$$\ddot{x} + \left(-\alpha + \beta x^2\right)\dot{x} + \left(1 - \mu\cos 2\Omega t\right)x + \gamma x^3 = f\cos\Omega t + g_1 x\left(t-\tau\right) + g_2\dot{x}\left(t-\tau\right) \quad (1)$$

Parameters α and β represent nonlinear van der Pol damping, μ and Ω are amplitude and frequency of parametric excitation, f amplitude of external excitation, g_1, g_2 gains of the delayed signals and τ is time delay.

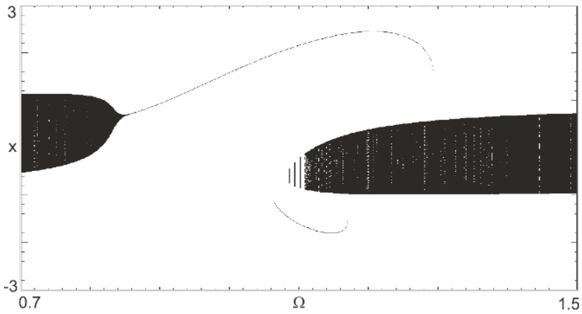

Figure 1. Bifurcation diagram for a nonlinear system without control $g_1 = g_2 = 0$;
$\alpha = 0.01$, $\beta = 0.05$, $\gamma = 0.1$, $\mu = 0.2$, $f = 0.15$

The dynamics of the structure can be studied directly solving numerically Eq.(1) or analytically by approximate methods. The presented equation has been solved analytically by the multiple time scale method, taking into account a second order approximation [8]. On the basis of the analytical solutions, which for the sake of brevity are not presented here, it is possible to study bifurcation scenario and influence of selected parameters on the system response.

The interactions between self, parametric and external excitations can be observed by setting gains $g_1 = g_2 = 0$. In this case the mentioned above, three different types of vibrations interact.

The bifurcation diagram in Fig. 1 has been obtained by direct numerical simulations of Eq.(1). The black zones correspond to quasi-periodic oscillations while a dotted line represents harmonic solution with frequency equal to excitation frequency Ω. This phenomenon corresponds to so called quenching of self-excited vibrations [9]. As we may see inside the resonance zone an additional line which represents additional harmonic solutions occurs. This phenomenon is a result of interactions of self and parametric vibrations and external harmonic force. If external force is equal to zero ($f = 0$) then this phenomenon does not take place. We note that unstable solutions are not visible in Fig. 1. More information on the system behaviour can be drawn on the basis of analytical solution presented in [8].

a) b)

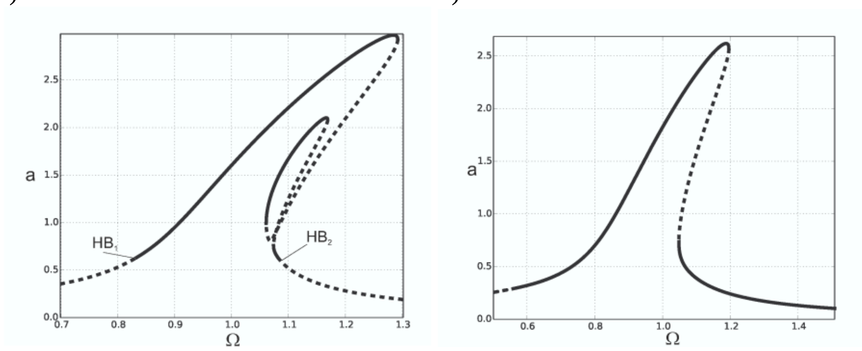

Figure 2. Resonance curves for a nonlinear system (a) without control ($g_1 = g_2 = 0$) and with control ($g_1 = 0.1$, $\tau = 0.5$, $g_2 = 0$) (b)

The resonance curve obtained analytically is presented in Fig. 2(a) with stable and unstable solutions represented by solid and dashed lines respectively. Indeed, inside the resonance zone the untypical loop is obtained. But the only upper part of the loop is stable. The stable solutions correspond to those presented in bifurcation diagram in Fig. 1. Outside the resonance zone there are two Hopf bifurcation points HB_1 and HB_2 (Fig. 2a). In these points the periodic solution becomes unstable and quasi-periodic oscillations arise, those represented by dark zones in Fig. 1. The occurrence of the loop as well as Hopf bifurcation can be fully controlled by the time delay feedback. On the basis of analytical solutions obtained for the delayed differential equation we can control interaction between different vibration types. By adding time delay input $g_1 = 0.1$, and time delay $\tau = 0.5$ we change the system response and the additional solutions, as well as quasi-periodic motions are eliminated in Fig. 2(b). Hopf bifurcations HB_1, HB_2 may be controlled and the curve changes its course into typical Duffing type.

Specific nonlinear phenomena may be used to enhance control strategy. One of the method which takes advantage of nonlinear dynamics to control the system response with small amount of input energy has been proposed for fixed structures in [5, 7]. Due to quadratic coupling of the plant and the especially tuned controller, the saturation control strategy has been successfully applied in theory [7] and implemented in the laboratory setup [5]. This strategy has also been applied for self and externally excited systems in [10] and then for rotating structures with PZT layers in [11].

Let us take into account a rotating structure presented in Fig. 3. The structure is composed of a rigid hub with an attached composite thin-walled beam with a rectangular box cross-section. The reinforcing fibres of the composite beam are placed creating so called Circumferentially Asymmetric Stiffness (CAS) which gives strong bending-twisting coupling of the blade. Furthermore, there are two PZT layers embedded into the composite structure as presented in Fig. 3. Considering possible large oscillations nonlinear constitutive equations for the active piezo-layers are taken into account. The model of the composite beam has been presented in [12] and then developed for composite with active layers in [13].

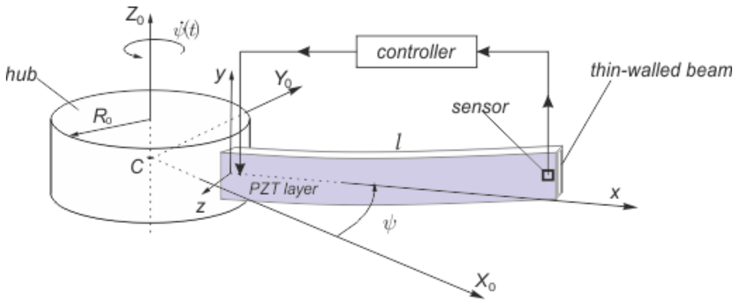

Figure 3. Model of a rotating structure with active elements

The final model of a rotating structure has the form

$$
\begin{aligned}
&(1+J_h+\alpha_{h12}q_1^2)\ddot{\psi}+\zeta_h\dot{\psi}+\alpha_{h11}\ddot{q}_1+\alpha_{h13}q_1\dot{q}_1\dot{\psi}=\mu \\
&\ddot{q}_1+\zeta_1\dot{q}_1+\alpha_{12}\ddot{\psi}+(\alpha_{11}+\alpha_{13}\dot{\psi}^2)q_1+\alpha_{14}q_1\dot{q}_1\dot{\psi}+\alpha_{15}sgn(q_1)q_1^2+\alpha_{16}q_1^3=g_1q_c^2 \\
&\ddot{q}_c+\zeta_c\dot{q}_c+\omega_{0c}^2q_c=g_2q_cq_1
\end{aligned}
\tag{2}
$$

where q_1 is a generalised coordinate of the beam representing complex bending-twisting motion of the piezo-composite blade, ψ represents rotation of the structure, α_{ij} are the coefficients coming from modal projection. Damping of the system is assumed to be viscous and represented by ζ_1 and ζ_h damping coefficients for the beam and for the hub, respectively. The system is excited by torque $\mu = \mu_0 + \rho\sin\omega t$ composed of constant μ_0 and harmonic component with amplitude ρ and frequency ω.

Input signal q_c, given in square and multiplied by gain g_1 is added to the right hand side of Eq.(2) and it comes from additional oscillator written in the third equation of set (2). This oscillator plays a role of a controller with coordinate q_c. This controller is coupled with beam oscillations by a product of signals q_1 and q_c and gain g_2. A very important requirement for the proposed control strategy is a proper tuning of the controller natural frequency to the rotating beam frequency, keeping them in ratio: $\omega_{0c} = \omega_1 / 2$. Frequency ω_1 is crucial in the proposed control strategy and it is defined as:

$$\omega_1 = \sqrt{\frac{\alpha_{11} + \alpha_{13}\dot{\psi}^2}{1 - \frac{\alpha_{12}\alpha_{h1}}{1 + J_h}}}. \tag{3}$$

In a case of rotating structure this frequency has to take into account angular velocity of the hub $\dot{\psi}$ and mass moment of hub inertia J_h.

Analysis of Eq.(2) is performed numerically for data [11,13]:

$$\alpha_{11} = 10.86,\ \alpha_{12} = 1.77,\ \alpha_{13} = 0.35,\ \alpha_{14} = -1.55,\ \alpha_{15} = -2.33,\ \alpha_{16} = 0,$$
$$J_h = 1.0,\ \alpha_{h11} = -0.53,\ \alpha_{h12} = -0.40,\ \alpha_{h13} = -0.81,$$
$$\zeta_h = 0.1,\ \zeta_1 = 0.033,\ \zeta_c = 0.001,\ g_1 = 0.01,\ g_2 = 1.$$

The exciting torque supplied to the hub is defined by parameters $\mu_0 = 0,\ \rho = 0.005$. This means that the hub is excited periodically. At first we consider only hub-blade dynamics without any control $q_1 = g_2 = 0$. Due to a nonlinear electric field the blade resonance curve demonstrates softening phenomenon. In Fig. 4 (a) we present resonance curves for selected amplitudes of excitation ρ. For large oscillations of the blade the nonlinear softening effect with unstable solutions is clearly observed.

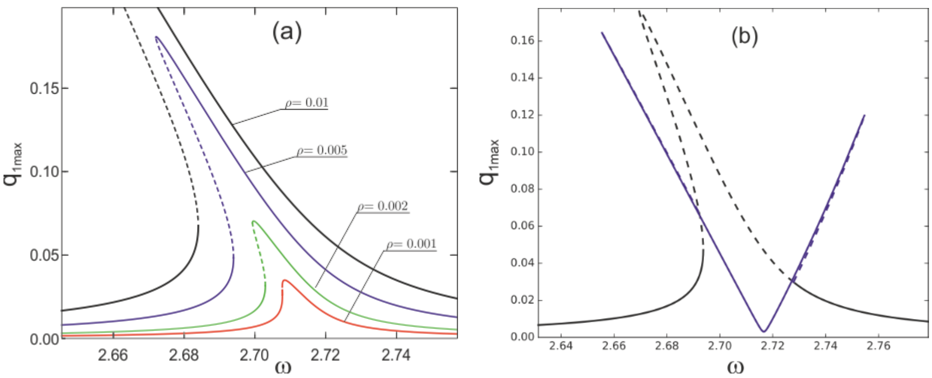

Figure 4. Resonance curves of a rotating active structure without control for selected amplitudes of excitation ρ (a) and saturation control effect around the resonance zone for ρ = 0.005 and q_1=0.01, q_2=1.0 (b)

If the saturation control is applied by setting q_1 = 0.01, q_2 = 1.0 then the resonance peak is reduced. The final effect of the nonlinear saturation control is presented in Fig. 4(b). The middle part of the resonance curve becomes unstable and in this zone, if the controller is activated, vibrations are suppressed almost to zero.

3. Conclusions

Importance of a specific nonlinear phenomena for fixed and rotating structure has been presented in the paper. In a case of nonlinear oscillator with self, parametric and external excitation the occurrence of a loop inside the resonance curve has been shown. However, by the added time delay signals the existence of the loop can be controlled.

In a case of the rotating structure with active elements, the nonlinear effect has been included into the controller. Thus, due to specific nonlinear coupling, the nonlinear saturation control has been applied and vibrations of the rotor around the resonance zone have been effectively suppressed.

Acknowledgments

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