Dynamics Control of an Autoparametric System with the Spherical Pendulum Using MR Damper

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Abstract

Dynamic properties of the three degrees of freedom autoparametric system with spherical pendulum including the magnetorheological (MR) damper are investigated. It was assumed that the spherical pendulum is suspended to the oscillator excited harmonically in the vertical direction. The influence of damping force described by Bingham's model on the energy transfer can be modified by magnetic field. The equation of motion have been solved numerically. In this type system one mode of vibration may excite or damp another one, and for except different kinds of periodic vibrations there may also appear chaotic vibration. Results show that MR damper can be used to change the dynamic behavior of the autoparametric system with spherical pendulum giving semiactive control possibilities.

Keywords: spherical pendulum, energy transfer, semiactive damping, chaos

1. Introduction

The presented work deal with nonlinear dynamics of a three degree of freedom system with a spherical pendulum when magnetorheological damping is using to semiactive control. The spherical pendulum is similar to the simple pendulum, but it moves in 3-dimensional space. This system was presented by the authors in the previous paper by Sado et al [3, 4] where the position of the pendulum was described by the coordinate z and two angles: θ and φ where the angle θ was the deflection of the pendulum measured from the vertical position and angle φ describes the rotation of the pendulum in the horizontal space. Another model where the position of the main body is described by the coordinate z and position of the pendulum is described two angles: θ and φ in the vertical planes (Leung and Kuang [2], Witkowski [7], Sado and Bobrowska [5]). When we change the generalized coordinates from typically modelled spherical pendulum to fixed spherical coordinates, we get interesting results of the motion. Magnetorheological (MR) damper used to investigate semiactive control of an autoparametric vibration absorber with simple pendulum is presented by Kecik and Warmiński [1]. In this paper we investigate influence of MR damper on semiactive control of an autoparametric system with a spherical pendulum.

2. System description and equation of motion

The investigated system is shown in Figure 1. The position of the oscillator of mass m_1 and element characterized by linear elasticity and magnetorheological damping of modified Bingham model used by Tang et al. [6], with viscotic damping coefficients c_1 and dry friction coefficient c_2 is described by coordinate z and position of the pendulum of mass m_2 and length l is describe by coordinates: z, θ, φ. Coordinate z is the vertical displacement of the body of mass m_1 measured from the static position of equilibrium. The angle θ is the angle between the vertical axis and the deflection of the pendulum on the space xz . The angle φ is the angle between the deflections of the pendulum on the space xz and the pendulum The body of mass m_1 is subjected to the harmonic vertical excitation $F(t) = P_1 \cos \nu_1 t$.

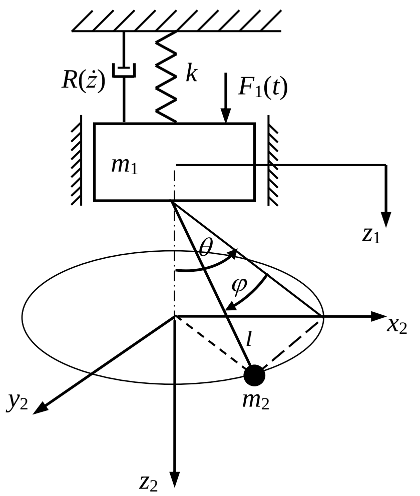


Figure 1. Schematic diagram of the system

The Cartesian coordinates have the form:

$$\begin{aligned} x_2 &= l\cos\varphi\sin\theta \\ y_2 &= l\sin\varphi \\ z_2 &= l\cos\varphi\cos\theta + z_1 \\ z_1 &= z + z_{st} \end{aligned} \tag{1}$$

Assuming that:

$$R(\dot{z}) = c_1\dot{z} + c_2\tanh(10\dot{z}) \tag{2}$$

The equations of the motion of the system derived as Lagrange's equations are as follow:

$$
\begin{aligned}
&(m_1+m_2)\,\ddot z \; - m_2 l\ddot\varphi\sin\varphi\cos\theta - m_2 l\dot\varphi^2\cos\varphi\cos\theta - m_2 l\ddot\theta\cos\varphi\sin\theta + \\
&+ 2m_2 l\dot\varphi\dot\theta\sin\varphi\sin\theta - m_2 l\dot\theta^2\cos\varphi\cos\theta + kz + c_1\dot z + c_2\tanh(10\dot z) = P_1\cos\nu_1 t \\
&m_2 l^2\ddot\varphi - m_2 l\ddot z\;\sin\varphi\cos\theta + m_2 l^2\dot\theta^2\cos\varphi\sin\varphi + m_2 g l\sin\varphi\cos\theta = 0 \\
&m_2 l^2\ddot\theta\cos^2\varphi - 2m_2 l^2\dot\theta^2\cos\varphi\sin\varphi - m_2 l\ddot z\;\cos\varphi\cos\theta + m_2 g l\cos\varphi\sin\theta = 0
\end{aligned}
\tag{3}
$$

The following dimensionless time and dimensionless parameters have been introduced:

$$
\begin{aligned}
&\tau=\omega_1 t,\quad \omega_1^2=\frac{k}{m_1+m_2},\quad \omega_2^2=\frac{g}{l},\quad \beta=\frac{\omega_2}{\omega_1},\quad \gamma_1=\frac{c_1}{(m_1+m_2)\omega_1},\quad \bar z=\frac{z}{l} \\
&a=\frac{m_2}{m_1+m_2},\quad A_1=\frac{P_1}{(m_1+m_2)\omega_1^2},\quad \mu_1=\frac{\nu_1}{\omega_1},\quad \gamma_2=\frac{c_2}{(m_1+m_2)\omega_1}
\end{aligned}
\tag{4}
$$

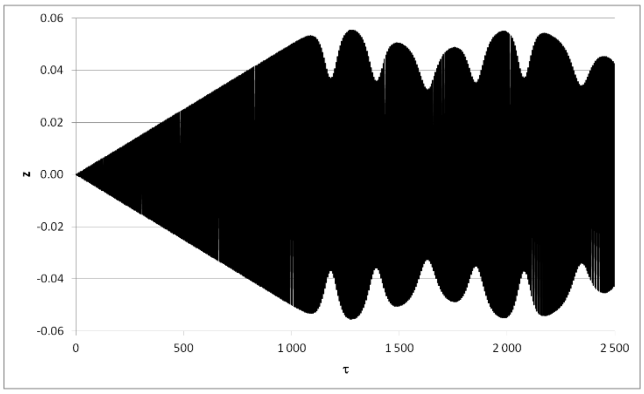
After transformation, the equations of motion can be written in the dimensionless form:

$$
\begin{aligned}
&\ddot z - a\ddot\theta\cos\varphi\sin\theta - a\ddot\varphi\sin\varphi\cos\theta = a(\dot\varphi^2\cos\varphi\cos\theta - 2\dot\varphi\dot\theta\sin\varphi\sin\theta \\
&+\dot\theta^2\cos\varphi\cos\theta) - z - \gamma_1\dot z - \gamma_2\tanh(10\dot z) + A_1\cos\mu_1\tau \\
&-\ddot z\cos\varphi\sin\theta + \ddot\theta\cos^2\varphi = 2\dot\theta\dot\varphi\cos\varphi\sin\varphi - \beta^2\cos\varphi\sin\theta \\
&-\ddot z\sin\varphi\cos\theta + \ddot\varphi = -\dot\theta^2\cos\varphi\sin\varphi - \beta^2\sin\varphi\cos\theta
\end{aligned}
\tag{5}
$$

3. Numerical results

The equations of motion in dimensionless form (5) have been solved numerically using Runge-Kutta method with a variable step length. The calculations are carried out for different values of parameters of the MR damper. Exemplary time histories of displacements z, θ and φ of forced vibrations without damping for parameters of the system: $a=0.5$, $\beta=0.482$, $\gamma_1=\gamma_2=0$, $A_1=0.0001$, $\mu_1=1$ and for the initial conditions: $z(0)=0,\ \theta(0)=\varphi(0)=0.005^\circ,\ \dot z(0)=\dot\theta(0)=\dot\varphi(0)=0$ are presented in the Fig.2.

We can observe the energy transfer. As it can be seen from the presented diagrams, the amplitude z diminishes almost to zero and the amplitudes angles θ and φ grow. After the fixed period the opposite effect occurs.



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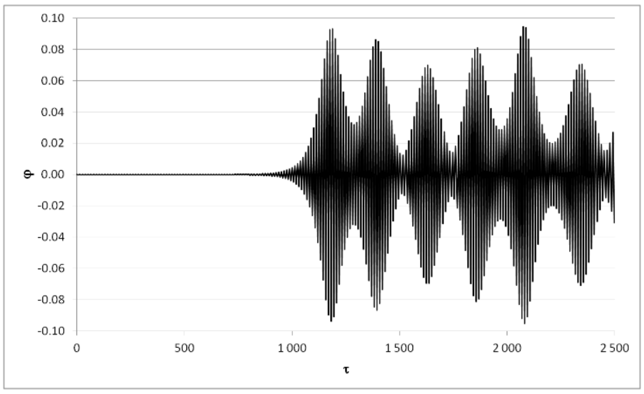

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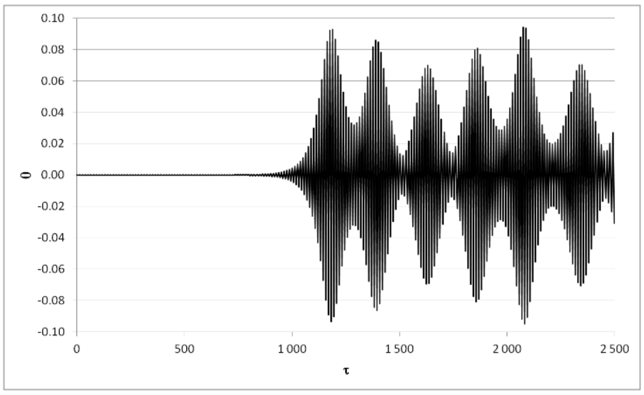
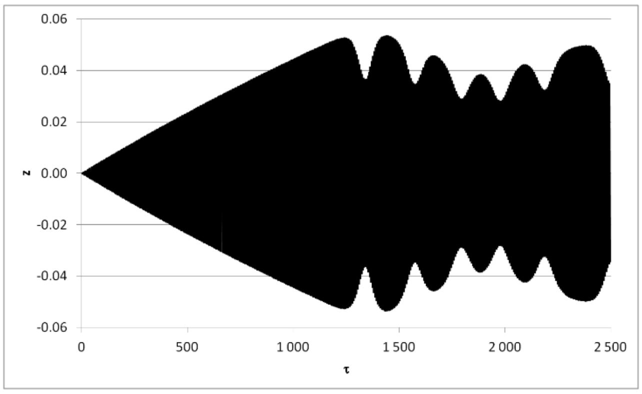


c

Figure 2. Time histories for: $a = 0.5$, $\beta = 0.482$, $\gamma_1 = \gamma_2 = 0$, $A_1 = 0.0001$, $\mu_1 = 1$, $z(0) = 0$, $\theta(0) = \varphi(0) = 0.005°$, $\dot{z}(0) = \dot{\theta}(0) = \dot{\varphi}(0) = 0$



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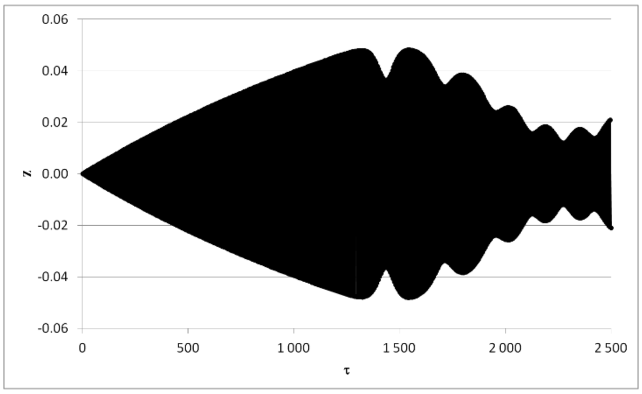
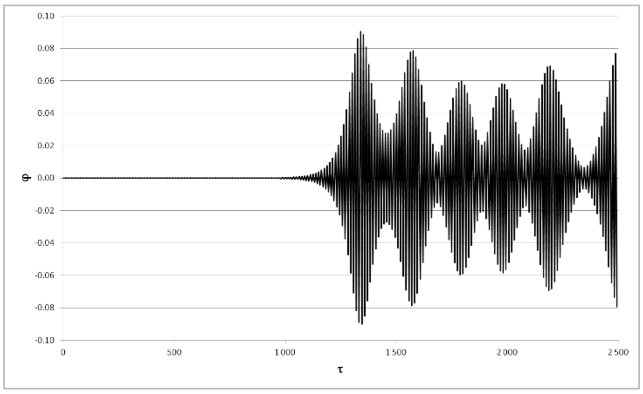


b

Figure 3. Time histories for z: $a = 0.5$, $\beta = 0.482$, $\gamma_1 = 0$, $A_1 = 0.0001$, $\mu_1 = 1$, $z(0) = 0$, $\theta(0) = \varphi(0) = 0.005°$, $\dot{z}(0) = \dot{\theta}(0) = \dot{\varphi}(0) = 0$ and $\gamma_2 = 0.00005$ (a), $\gamma_2 = 0.0001$(b)



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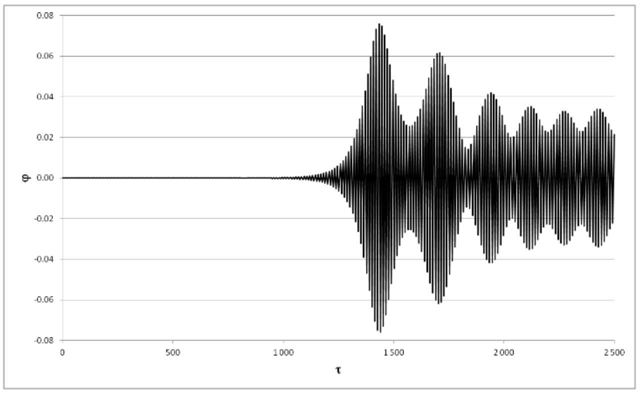

b

Figure 4. Time histories for φ: a = 0.5, β = 0.482, γ_1=0, A_1 = 0.0001, μ_1 = 1, $z(0) = 0$, $\theta(0) = \varphi(0) = 0.005°$, $\dot{z}(0) = \dot{\theta}(0) = \dot{\varphi}(0) = 0$ and γ_2 = 0.00005 (a), γ_2 = 0.0001(b)

Next the influence of damping force in MR damper on the phenomenon of energy transfer has been studied. Exemplary results for displacement z are presented in Fig. 3 and for displacement φ are presented in Fig. 4 (for displacement θ results are similar to φ).

The motion of the oscillator and of the spherical pendulum may be periodic, quasiperiodic or chaotic. So the Poincaré maps and maximal exponents of Lyapunov for different damping parameters are studied. Exemplary Poincaré maps and maximal exponents of Lyapunov (Fig. 5) are presented for value of damping parameters γ_1= 0.0001 and γ_2= 0.00005.

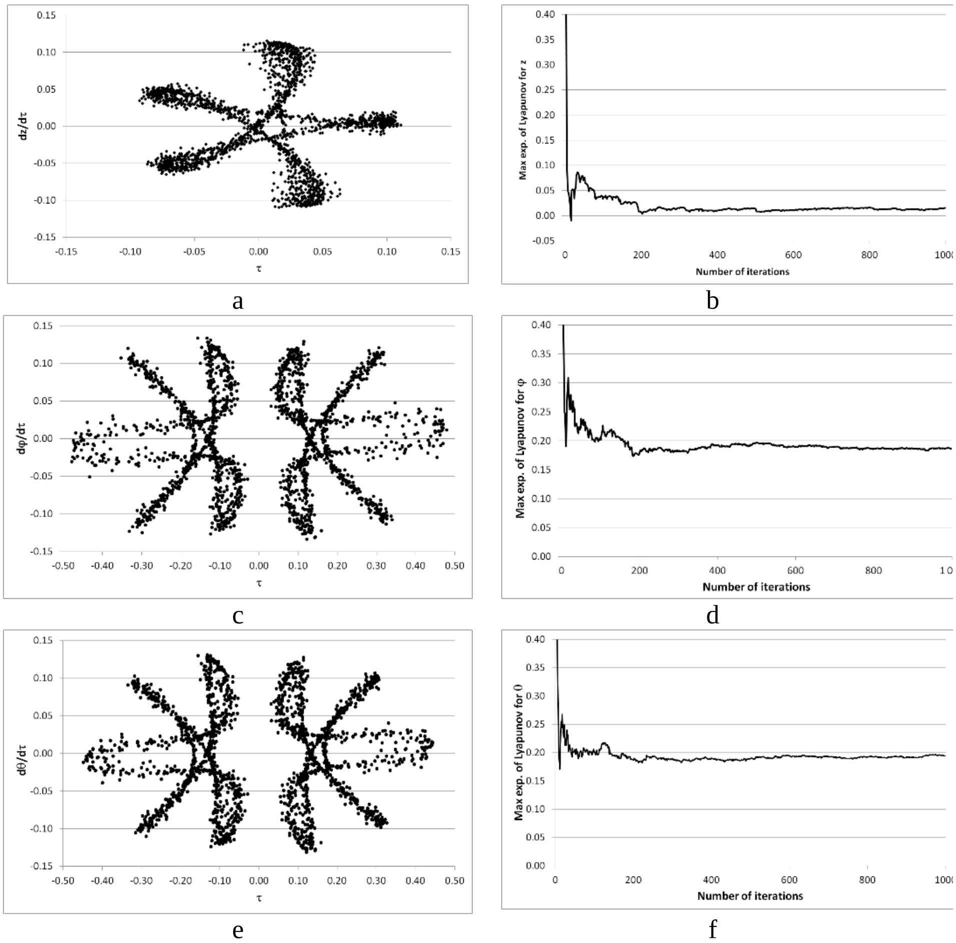

Figure 5. Poincare maps (a, c, e) and max. exp. of Lyapunov (b, d, f) for: $a = 0.5$, $\beta = 0.5$, $\gamma_1 = 0.0001$, $A_1 = 0.00222$, $\mu_1 = 1.01$

As it can be seen from these diagrams the Poincaré maps trace the strange atractors and the maximum exponents of Lyapunov are positive, so the motions of all coordinates are chaotic.

4. Conclusions

The spherical pendulum is more similar to the real system than the simple pendulum. We can observe the energy transfer between all modes of vibration in a closed cycle. The time of this cycle depends on the values of the damper parameters. Besides the regular behaviour of the spherical pendulum, near the internal and external area of the resonance chaotic vibrations may occur for all coordinates. The magnetorheological damper activated by magnetic field can be used to change the dynamic behaviour of the autoparametric system with spherical pendulum giving reliable semiactive control possibilities.

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