On the Combined Asymptotic-Tolerance Modelling of Dynamic Problems for Thin Biperiodic Cylindrical Shells

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Abstract

The objects of consideration are thin linearly elastic Kirchhoff-Love-type circular cylindrical shells having a periodically microheterogeneous structure in circumferential and axial directions (biperiodic shells). The aim of this contribution is to formulate and discuss a new averaged *general asymptotic-tolerance model* for the analysis of selected dynamic problems for the shells under consideration. This model is derived by applying the combined modelling which includes two techniques: the asymptotic modelling procedure and a certain extended version of the known tolerance non-asymptotic modelling technique based on a new notion of *weakly slowly-varying function*. Contrary to the starting exact shell equations with highly oscillating, non-continuous and periodic coefficients, governing equations of the averaged combined model have constant coefficients depending also on a cell size. The differences between *the general combined model* proposed here and the corresponding *known standard combined model* derived by means of the more restrictive concept of *slowly-varying functions* are discussed.

Keywords: micro-periodic cylindrical shells, dynamics, tolerance modelling, length-scale effect

1. Introduction

There are considered thin linearly elastic Kirchhoff-Love-type circular cylindrical shells with a periodically microheterogeneous structure in circumferential and axial directions (*biperiodic shells*), cf. Fig. 1.

The dynamic problems of such shells are described by partial differential equations with highly oscillating, non-continuous, periodic coefficients. Hence, the direct application of these equations to investigations of engineering problems is noneffective even using computational methods. That is why there exists a number of various modelling methods leading to simplified averaged equations with constant coefficients. Periodic shells (plates) are usually described using *homogenized models* derived by means of *asymptotic methods*, cf. [1]. Unfortunately, in the models of this kind *the effect of a cell size* (called *the length-scale effect*) on the overall shell behaviour is neglected.

This effect can be taken into account using *the tolerance averaging technique*, cf. [2]. Some applications of this method to the modelling of mechanical and thermomechanical problems for various periodic and tolerance-periodic structures are shown in many works. The extended list of papers and books on this topic can be found in [2]. We mention here monograph by Tomczyk [3], where the length-scale effect in dynamics and stability of periodic cylindrical shells is investigated. In the last years the tolerance averaging approach was adopted for non-stationary problems of functionally graded structures, e.g. for vibrations of functionally graded thin plates by Kaźmierczak and Jędrysiak [4], for dynamics of transversally graded shells by Tomczyk and Szczerba [5].

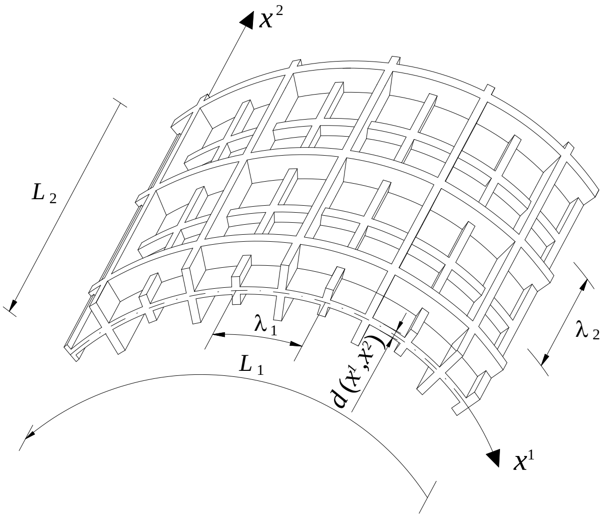

Figure 1. Example of biperiodic cylindrical shell

A certain extended version of the tolerance modelling technique has been proposed by Tomczyk and Woźniak in [6]. This version is based on a new notion of *weakly slowly-varying functions* which is a certain extension of the well known concept of *slowly-varying functions*, cf. [2, 3]. Applying the concept of *weakly slowly-varying functions*, the new *general tolerance and general combined asymptotic-tolerance models* of dynamic problems for thin cylindrical shells with micro-periodic structure in circumferential direction (*uniperiodic shells*) have been proposed by Tomczyk and Litawska in [7, 8]. Moreover, a new *general tolerance model* of dynamics for thin *biperiodic shells* derived by means of the notion of *weakly slowly-varying functions* has been presented by Tomczyk and Litawska in [9]. The models mentioned above are certain generalizations of the corresponding *standard models* proposed in [3], which have been obtained by using the classical concept of *slowly-varying functions*.

The aim of this contribution is to formulate and discuss a new averaged *general combined asymptotic-tolerance model* for the analysis of selected dynamic problems for *the biperiodic shells* under consideration. The model will be derived by applying the combined modelling which includes two techniques: *the consistent asymptotic modelling procedure* given by Woźniak [2] and the *extended tolerance non-asymptotic modelling technique* proposed by Tomczyk and Woźniak [6]. Governing equations of *the combined model* have constant coefficients depending also on a cell size. The differences between *the general combined model* proposed here and the corresponding *known standard* *combined model* presented by Tomczyk in [3] and derived by means of the more restrictive notion of *slowly-varying functions* will be discussed.

2. Starting equations

We assume that x^1 and x^2 are coordinates parametrizing the shell midsurface *M* in circumferential and axial directions, respectively. We denote $\mathbf{x} \equiv (x^1, x^2) \in \Omega \equiv (0, L_1) \times (0, L_2)$, where L_1, L_2 are length dimensions of *M*, cf. Fig. 1. Let $O\bar{x}^1\bar{x}^2\bar{x}^3$ stand for a Cartesian orthogonal coordinate system in the physical space R^3 and denote $\bar{\mathbf{x}} \equiv (\bar{x}^1, \bar{x}^2, \bar{x}^3)$. A cylindrical shell midsurface *M* is given by $M \equiv \left\{ \bar{\mathbf{x}} \in R^3 : \bar{\mathbf{x}} = \bar{\mathbf{r}}\left(x^1, x^2\right), \left(x^1, x^2\right) \in \Omega \right\}$, where $\bar{\mathbf{r}}(\cdot)$ is the smooth function such that $\partial\bar{\mathbf{r}}/\partial x^1 \cdot \partial\bar{\mathbf{r}}/\partial x^2 = 0$, $\partial\bar{\mathbf{r}}/\partial x^1 \cdot \partial\bar{\mathbf{r}}/\partial x^1 = 1$, $\partial\bar{\mathbf{r}}/\partial x^2 \cdot \partial\bar{\mathbf{r}}/\partial x^2 = 1$. It means that on *M* the orthonormal parametrization is introduced. Sub- and superscripts $\alpha, \beta, \ldots$ run over 1,2 and are related to x^1, x^2, summation convention holds. Partial differentiation related to x^α is represented by ∂_α. Moreover, it is denoted $\partial_{\alpha\ldots\delta} \equiv \partial_\alpha \ldots \partial_\delta$. Let $a^{\alpha\beta}$ stand for the midsurface first metric tensor. The time coordinate is denoted by $t \in \mathrm{I} = [t_0, t_1]$. Let $d(\mathbf{x})$, r stand for the shell thickness and the midsurface curvature radius, respectively.

Let λ_1 and λ_2 be the period lengths of the shell structure respectively in x^1- and x^2 - directions. Define *the basic cell* $\Delta \equiv [-\lambda_1/2, \lambda_1/2] \times [-\lambda_2/2, \lambda_2/2]$. The diameter $\lambda \equiv [(\lambda_1)^2 + (\lambda_2)^2]^{1/2}$ of Δ is assumed to satisfy conditions: $\lambda / d_{\max} >> 1$, $\lambda / r << 1$ and $\lambda / \min(L_1, L_2) << 1$. Hence, the diameter will be called *the microstructure length parameter*.

Denote by $u_\alpha = u_\alpha(\mathbf{x}, t)$, $w = w(\mathbf{x}, t)$, $\mathbf{x} \in \Omega$, $t \in \mathrm{I}$, the shell displacements in directions tangent and normal to M , respectively. Elastic properties of the shell are described by shell stiffness tensors $D^{\alpha\beta\gamma\delta}(\mathbf{x})$, $B^{\alpha\beta\gamma\delta}(\mathbf{x})$. Let $\mu(\mathbf{x})$ stand for a shell mass density per midsurface unit area. The external forces will be neglected.

It is assumed that the behaviour of the shell under consideration is described by the action functional determined by lagrangian *L* being a highly oscillating function with respect to **x** and having the well-known form

$$L = -\frac{1}{2}(D^{\alpha\beta\gamma\delta}\partial_\beta u_\alpha \partial_\delta u_\gamma + \frac{2}{r} D^{\alpha\beta 11} w \partial_\beta u_\alpha + \frac{1}{r^2} D^{1111} ww + \\ + B^{\alpha\beta\gamma\delta}\partial_{\alpha\beta} w \partial_{\gamma\delta} w - \mu a^{\alpha\beta}\dot{u}_\alpha \dot{u}_\beta - \mu \dot{w}^2). \tag{1}$$

Applying the principle of stationary action we arrive at the system of Euler-Lagrange equations, which can be written in an explicit form as

$$
\begin{aligned}
&\partial_{\beta}(D^{\alpha\beta\gamma\delta}\partial_{\delta}u_{\gamma}) + r^{-1}\partial_{\beta}(D^{\alpha\beta 11}w) - \mu a^{\alpha\beta}\ddot{u}_{\beta} = 0\,,\\
&r^{-1}D^{\alpha\beta 11}\partial_{\beta}u_{\alpha} + \partial_{\alpha\beta}(B^{\alpha\beta\gamma\delta}\partial_{\gamma\delta}w) + r^{-2}D^{1111}w + \mu\ddot{w} = 0\,.
\end{aligned}
\tag{2}
$$

It can observed that equations (2) coincide with the well-known governing equations of Kirchhoff-Love theory of thin elastic shells. For periodic shells, coefficients $D^{\alpha\beta\gamma\delta}(\mathbf{x})$, $B^{\alpha\beta\gamma\delta}(\mathbf{x})$, $\mu(\mathbf{x})$ of (1) and (2) are highly oscillating, non-continuous and λ-periodic functions. Applying the combined asymptotic-tolerance modelling technique to lagrangian (1), we will derive the averaged model equations with constant coefficients depending also on a cell size. The combined modelling under consideration includes two techniques: *the consistent asymptotic modelling procedure* given in [2] and *an extended version of the known tolerance non-asymptotic modelling technique* based on a new notion of *weakly slowly-varying function* proposed by Tomczyk and Woźniak in [6].

3. Modelling procedure, equations of combined model

The combined modelling technique used to starting lagrangian (1) is realized in two steps. The first step is based on *the consistent asymptotic averaging* of lagrangian (1) under *the consistent asymptotic decomposition* of fields u_{α}, w in $\Delta(\mathbf{x})\times \mathrm{I}$, $\mathbf{x}\in\Omega$

$$
\begin{aligned}
&u_{\varepsilon\alpha}(\mathbf{z},t) \equiv u_{\alpha}(\mathbf{z}/\varepsilon,t) = u^{0}_{\alpha}(\mathbf{z},t) + \varepsilon h_{\varepsilon}(\mathbf{z})U_{\alpha}(\mathbf{z},t)\,,\\
&w_{\varepsilon}(\mathbf{z},t) \equiv w(\mathbf{z}/\varepsilon,t) = w^{0}(\mathbf{z},t) + \varepsilon^{2} g_{\varepsilon}(\mathbf{z})W(\mathbf{z},t)\,, \quad \mathbf{z}\in\Delta_{\varepsilon}(\mathbf{x}),\ t\in \mathrm{I},
\end{aligned}
\tag{3}
$$

where $\varepsilon = 1/m$, $m = 1,2,...,$ $\Delta_{\varepsilon}(\mathbf{x}) \equiv \mathbf{x} + \Delta_{\varepsilon}$, $\Delta_{\varepsilon} \equiv (-\varepsilon\lambda_{1}/2, \varepsilon\lambda_{1}/2)\times(-\varepsilon\lambda_{2}/2, \varepsilon\lambda_{2}/2)$.

Unknown functions u^{0}_{α}, w^{0} and U_{α}, W in (3) are assumed to be continuous and bounded in Ω. Unknowns u^{0}_{α}, w^{0} and U_{α}, W are called *macrodisplacements* and *fluctuation amplitudes*, respectively. They are independent of ε. Functions $h_{\varepsilon}(\mathbf{z}) \equiv h(\mathbf{z}/\varepsilon)$ and $g_{\varepsilon}(\mathbf{z}) \equiv g(\mathbf{z}/\varepsilon)$ in (3) are highly oscillating Δ-periodic *fluctuation shape functions*. They are assumed to be known in every problem under consideration. In this work, they have to satisfy conditions: $h\in O(\lambda)$, $\lambda\partial_{\alpha}h\in O(\lambda)$, $g\in O(\lambda^{2})$, $\lambda\partial_{\alpha}g\in O(\lambda^{2})$, $\lambda^{2}\partial_{\alpha\beta}g\in O(\lambda^{2})$, $\frac{1}{|\Delta|}\int_{\Delta(\mathbf{x})}\mu h d\mathbf{z} = \frac{1}{|\Delta|}\int_{\Delta(\mathbf{x})}\mu g d\mathbf{z} = 0$.

Introducing decomposition (3) into (1), under weak limit passage $\varepsilon\to 0$ we obtain the averaged form of lagrangian (1). Then, applying the principle of stationary action we arrive at *the governing equations of consistent asymptotic model for the periodic shells under consideration*. These equations consist of partial differential equations for macrodisplacements u^{0}_{α}, w^{0} coupled with linear algebraic equations for fluctuation amplitudes U_{α}, W. After eliminating fluctuation amplitudes from the governing equations by means of

$$U_\gamma = -(G^{-1})_{\gamma\eta}[< \partial_\beta h D^{\beta\eta\mu\vartheta} > \partial_\vartheta u_\mu^0 + r^{-1} < \partial_\beta h D^{\beta\eta 11} > w^0] \;,$$
$$W = -E^{-1} < \partial_{\alpha\beta} g B^{\alpha\beta\gamma\delta} > \partial_{\gamma\delta} w^0 \,, \qquad (4)$$

where $G_{\alpha\gamma} =< \partial_\beta h\, D^{\alpha\beta\gamma\delta} \partial_\delta h >$, $E =< \partial_{\alpha\beta} B^{\alpha\beta\gamma\delta} \partial_{\gamma\delta} g >$, we arrive finally at *the asymptotic model equations expressed only in macrodisplacements* u_α^0, w^0

$$D_h^{\alpha\beta\gamma\delta} \partial_{\beta\delta} u_\gamma^0 + r^{-1} D_h^{\alpha\beta 11} \partial_\beta w^0 - < \mu > a^{\alpha\beta} \ddot{u}_\beta^0 = 0 \,,$$
$$B_g^{\alpha\beta\gamma\delta} \partial_{\alpha\beta\gamma\delta} w^0 + r^{-1} D_h^{11\gamma\delta} \partial_\delta u_\gamma^0 + r^{-2} D_h^{1111} w^0 + < \mu > \ddot{w}^0 = 0 \,, \qquad (5)$$

where

$$D_h^{\alpha\beta\gamma\delta} \equiv < D^{\alpha\beta\gamma\delta} > - < D^{\alpha\beta\eta\chi}\, \partial_\chi h > (G^{-1})_{\eta\zeta} < \partial_\chi h\, D^{\chi\zeta\gamma\delta} > \,,$$
$$B_g^{\alpha\beta\gamma\delta} \equiv < B^{\alpha\beta\gamma\delta} > - < B^{\alpha\beta\mu\zeta} \partial_{\mu\zeta} g > E^{-1} < \partial_{\mu\zeta} g\, B^{\mu\zeta\gamma\delta} > . \qquad (6)$$

Coefficients of equations (5) are *constant* but they are *independent of the microstructure cell size*. Hence, this model is not able to describe the length-scale effect on the overall shell dynamics and it will be referred to as *the macroscopic model*.

In the first step of combined modelling it is assumed that within the asymptotic model, solutions u_α^0, w^0 to the problem under consideration are known. Hence, there are also known functions $u_{0\alpha} = u_\alpha^0 + hU_\alpha$ and $w_0 = w^0 + gW$, where U_α, W are given by means of (4).

The second step is based on *the tolerance averaging* of lagrangian (1) under so-called *superimposed decomposition*.

The fundamental concepts of the tolerance approach under consideration are those of *two tolerance relations between points and real numbers determined by tolerance parameters, weakly slowly-varying functions, tolerance-periodic functions, fluctuation shape functions and the averaging operation*, cf. [2, 3, 6].

A continuous, bounded and differentiable function $F(\cdot)$ defined in $\overline{\Omega} \equiv [0, L_1] \times [0, L_2]$ is called *weakly slowly-varying of the R-th kind* with respect to cell Δ and tolerance parameters δ, $F \in WSV_\delta^R(\Omega, \Delta)$, if it can be treated (together with its derivatives up to the *R-th* order) as constant on an arbitrary cell. Nonnegative integer *R* is assumed to be specified in every problem under consideration. Note, that the main difference between *the weakly slowly-varying* and the well-known *slowly-varying functions* is that the products of derivatives of *weakly slowly-varying functions* and *microstructure length parameter* λ are not treated as negligibly small.

An integrable and bounded function $f(\cdot)$ defined in $\overline{\Omega}$ is called *tolerance-periodic of the R-th kind* with respect to cell Δ and tolerance parameters δ, $f \in TP_\delta^R(\Omega, \Delta)$, if it can be treated (together with its derivatives up to the *R-th* order) as periodic on a cell.

Let $f(\mathbf{x})$, $\mathbf{x} \in \Omega$, be an integrable and bounded function in Ω. *The averaging operation of* $f(\cdot)$ is defined by

$$< f > (\mathbf{x}) \equiv \frac{1}{|\Delta|} \int_{\Delta(\mathbf{x})} f(\mathbf{z}) d\mathbf{z}, \quad \mathbf{z} \in \Delta(\mathbf{x}), \quad \mathbf{x} \in \Omega. \tag{7}$$

It can be seen that if $f(\cdot)$ is Δ-periodic then $< f >$ is constant,

The tolerance modelling is based on two assumptions. The first of them is called *the tolerance averaging approximation* (tolerance relations which make it possible to neglect terms of an order of tolerance parameters δ), cf. [6]. The second one is termed *the micro-macro decomposition.* In the problem under consideration, we introduce *the extra micro-macro decomposition* superimposed on the known solutions $u_{0\alpha}, w_0$ obtained within the macroscopic model

$$u_{c\alpha}(\mathbf{x},t) = u_{0\alpha}(\mathbf{x},t) + c(\mathbf{x})Q_\alpha(\mathbf{x},t), \quad w_b(\mathbf{x},t) = w_0(\mathbf{x},t) + b(\mathbf{x})V(\mathbf{x},t), \tag{8}$$

where *fluctuation amplitudes* Q_α, V are *the new weakly slowly-varying unknowns*, i.e. $Q_\alpha \in WSV_\delta^1(\Omega,\Delta)$, $V \in WSV_\delta^2(\Omega,\Delta)$. Functions $c(\cdot)$ and $b(\cdot)$ are *the new* periodic, continuous and highly-oscillating *fluctuation shape functions* which are assumed to be known in every problem under consideration. These functions have to satisfy conditions: $c \in O(\lambda)$, $\lambda\partial_\alpha c \in O(\lambda)$, $b \in O(\lambda^2)$, $\lambda\partial_\alpha b \in O(\lambda^2)$, $\lambda^2\partial_{\alpha\beta} b \in O(\lambda^2)$, $<\mu c> = <\mu b> = 0$.

We substitute the right-hand sides of (8) into (1). The resulting lagrangian is denoted by L_{cb}. Then, we average L_{cb} over cell Δ using averaging formula (7) and applying *the tolerance averaging approximation*. As a result we obtain function $< L_{cb} >$ called *the tolerance averaging of starting lagrangian* (1) *in* Δ *under superimposed decomposition* (8). Then, applying the principle of stationary action, under the extra approximation $1 + \lambda / r \approx 1$, we arrive at the system of Euler-Lagrange equations for Q_α, V, which can be written in an explicit form as

$$\begin{aligned} &\underline{\underline{< D^{\alpha\beta\gamma\delta}(c)^2 >}}\partial_{\beta\gamma}Q_\delta - < \partial_\beta c\, D^{\alpha\beta\gamma\delta}\partial_\gamma c > Q_\delta - \underline{< \mu(c)^2 >} a^{\alpha\beta}\ddot{Q}_\beta = \\ &= r^{-1} < D^{\alpha\beta 11}\partial_\beta c\, w_0 > + < D^{\alpha\beta\gamma\delta}\partial_\delta c\, \partial_\beta u_{0\gamma} >, \end{aligned} \tag{9}$$

$$\begin{aligned} &\underline{\underline{< B^{\alpha\beta\gamma\delta}(b)^2 >}}\partial_{\alpha\beta\gamma\delta}V + [\underline{\underline{2 < B^{\alpha\beta\gamma\delta} b \partial_{\alpha\beta} b >}} - 4\underline{\underline{< \partial_\alpha b\, B^{\alpha\beta\gamma\delta}\partial_\beta b >}}\ ]\partial_{\gamma\delta}V + \\ &+ (\underline{\underline{< D^{1111}(b)^2 >}} + < \partial_{\alpha\beta} b\, B^{\alpha\beta\gamma\delta}\partial_{\gamma\delta} b >)V \underline{+ < \mu(b)^2 >} \ddot{V} = - < B^{\alpha\beta\gamma\delta}\partial_{\gamma\delta} b\, \partial_{\alpha\beta} w_0 >. \end{aligned} \tag{10}$$

Equations (9) and (10) together with *the micro-macro decomposition* (8) constitute *the superimposed microscopic model*. Coefficients of the derived model equations are constant and some of them *depend on a cell size* λ (underlined terms). The right-hand sides of (9) and (10) are known under assumption that $u_{0\alpha}, w_0$ were determined in the first step of modelling. The basic unknowns Q_α, V of the model equations must be *the weakly slowly-varying functions in periodicity directions*, i.e. $Q_\alpha \in WSV_\delta^1(\Omega,\Delta)$, $V \in WSV_\delta^2(\Omega,\Delta)$. This requirement can be verified only *a posteriori and it determines the range of the physical applicability of the model*.

Summarizing results obtained above, we conclude that *the general combined asymptotic-tolerance model of selected dynamic problems for the biperiodic shells under consideration* derived here is represented by:

a) *Macroscopic model* defined by equations (5) for u_α^0, w^0 with expressions (4) for U_α, W, formulated by means of *the consistent asymptotic modelling* and being independent of the microstructure length. Unknowns of this model must be continuous and bounded functions in **x**.

b) *Superimposed microscopic model equations* (9), (10) derived by means of *an extended version of the tolerance (non-asymptotic) modelling* and having constant coefficients depending also on a cell size λ (underlined terms) as well as combined with the macroscopic model equations under assumption that in the framework of the asymptotic model the solutions to the problem under consideration are known. Unknown fluctuation amplitudes of this model must be *weakly slowly-varying functions* in **x**.

c) Decomposition

$$\begin{aligned} u_\alpha(\mathbf{x},t) &= u_\alpha^0(\mathbf{x},t) + h(\mathbf{x})U_\alpha(\mathbf{x},t) + c(\mathbf{x})Q_\alpha(\mathbf{x},t), \\ w(\mathbf{x},t) &= w^0(\mathbf{x},t) + g(\mathbf{x})W(\mathbf{x},t) + b(\mathbf{x})V(\mathbf{x},t), \quad \mathbf{x}\in\Omega,\ t\in \mathrm{I}, \end{aligned} \tag{11}$$

where functions $u_\alpha^0, U_\alpha, w^0, W$ have to be obtained in the first step of combined modelling, i.e. in the framework of *the consistent asymptotic modelling*.

It can be shown, cf. [3], that under assumption that fluctuation shape functions $h(\mathbf{x})$, $g(\mathbf{x})$ of macroscopic model coincide with fluctuation shape functions $c(\mathbf{x})$, $b(\mathbf{x})$ of microscopic model, we can obtain *microscopic model equations* (9), (10) in which $c(\cdot)$ and $b(\cdot)$ are replaced by $h(\cdot)$ and $g(\cdot)$, respectively, and in which the right-hand sides are equal to zero. In this case equations (9), (10) *are independent of the solutions obtained in the framework of the macroscopic model*. It means, that an important advantage of the combined model is that it makes *it possible to describe selected problems of the shell micro-dynamics* (e.g. the free micro-vibrations) *independently of the shell macro-dynamics*.

Let us compare *the general combined model* proposed here with the corresponding known *standard combined model* presented and discussed in [3], which was derived under assumption that the unknown fluctuation amplitudes $Q_\alpha(\mathbf{x},t), V(\mathbf{x},t)$ in micro-macro decomposition (8) are *slowly-varying*. We recall that the main difference between *the weakly slowly-varying* and the well-known *slowly-varying functions* is that the products of derivatives of *weakly slowly-varying functions* and *microstructure length parameter* λ are not treated as negligibly small. Following [3], *the standard combined asymptotic-tolerance model* consists of:

a) *Macroscopic model* defined by equations (5) for u_α^0, w^0 with expressions (4) for U_α, W. It is assumed that in the framework of this model the solutions to the problem under consideration are known.

b) *Superimposed microscopic model equations* (9), (10) without the doubly underlined terms.

c) Decomposition (11), in which *the weakly slowly-varying functions* are replaced by *slowly-varying functions*.

From comparison of both the general and the standard combined models it follows that *the general model equations* contain a bigger number of terms depending on the microstructure size than *the standard model equations*. Thus, the general model proposed here makes it possible to investigate the length-scale effect in more detail.

4. Final remarks

The combined asymptotic-tolerance modelling technique based on the notion of *weakly slowly-varying function*, cf. [6], is proposed as a tool to derive a new mathematical averaged model for the analysis of selected dynamic problems for thin cylindrical shells with micro-periodic structure in circumferential and axial directions.

Contrary to exact shell equations (2) with highly oscillating, non-continuous and periodic coefficients, the governing equations (5), (9), (10) of the *general combined asymptotic-tolerance model* have constant coefficients depending also on a cell size. Hence, this model makes it possible *to describe the effect of a length scale on the global shell behaviour*.

The main advantage of the combined model is that it makes it possible to separate the macroscopic description of some special dynamic problems from their microscopic description.

Some applications of this new model will be shown in forthcoming papers.

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