Vibration Properties of Auxetic Beam

Agata MATUSZEWSKA
Poznan University of Technology, Institute of Applied Mechanics
ul. Jana Pawła II 24, 60-965 Poznan
agata.k.matuszewska@doctorate.put.poznan.pl

Tomasz STREK
Poznan University of Technology, Institute of Applied Mechanics
ul. Jana Pawła II 24, 60-965 Poznan
tomasz.strek@put.poznan.pl

Abstract

This study presents vibration analysis for a beam with an auxetic cross-section. In order to verify damping properties of auxetic materials, the numerical results were compared with classical H-beam which has basic geometry. The response of analyzed models was considered with taking into account the Rayleigh damping of the internal material structure. Performed calculations comprise deformation of the certain beam, selected points displacement and vibration transmission loss coefficient. The analysis was carried out by means of Finite Element Method using Comsol Multiphysics software.

Keywords: auxetic, vibration transmission loss, Rayleigh damping, finite element method

1. Introduction

Auxetics are special kind of materials which exhibit a negative value of Poisson's ratio. This fact is related to their unusual behavior under uniaxial tension. During longitudinal stretching their cross-section increases the dimensions. On the other hand, in case of material compressing it comes to shrink of the cross-section. In comparison with the conventional materials, auxetics exhibit enhanced mechanical properties such as strengthened hardness, toughness and stiffness [1] as well as greater damping and acoustic isolation [2, 3].

The materials with auxetic structure exist in nature as natrolite and analcime zeolite, a certain crystalline phase of SiO_2, zeolites, and metals [4]. It could be also artificially manufactured including honeycombs structure, polymeric and metallic foams, and microporous polymers [5].

In applied mechanics, there are many applications of auxetic materials [6]. Their unique properties allow using auxetics in vibration isolation and sound transmission reduction. Their damping properties and material behavior due to harmonic excitation are widely considered by many authors [7-17].

Damping of sound pressure in acoustics is classified by two categories: sound absorption and sound insolation [2]. Sound absorption is described as the conversion of sound energy into the heat energy. Whereas the sound isolation is measured by transmitted sound power between two spaces. The behavior of sound absorption and acoustic isolation depends on the frequency domain.

In this paper, the frequency response and vibration isolation behavior of the auxetic beam were investigated. The cross-section of beam is the auxetic anti-tetra-chiral structure type or H-type. In calculations, the damping performance of beam with auxetic and H-type cross-section were analyzed. This study was conducted with respect to particular load distributions on the top of the certain beam. The vibration analysis contains frequency response for both beam types.

2. Model geometry and governing equations

There are considered two types of cross-section geometry. The figures below show unit cell geometries for anti-tetra-chiral auxetic structure (Fig. 1a) and H-beam shape (Fig. 1b) which the last one exhibit non-auxetic behavior. The height and width of a single cell are the same in both cases as well as the length of the cross-section. These models are featured as a shell component. Established material for two types of the beam is structural steel.

a) b)

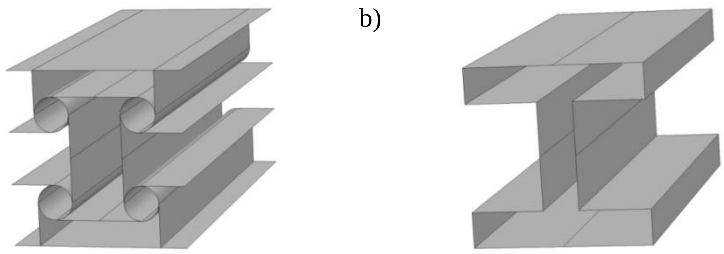

Figure 1. The model geometry of beam's structure a) anti-tetra-chiral structure and b) H-beam

In this study, the motion of the analyzed structures is assumed as a harmonic motion which allows describing displacement vector as:

$$\mathbf{u}(x, t) = \mathbf{u}(x)e^{i\omega t} \tag{1}$$

The models were calculated in accordance with Navier's equation of motion as:

$$\rho\, \omega^2\mathbf{u} - \nabla \cdot \boldsymbol{\sigma} = \mathbf{F} \tag{2}$$

where **u** denotes displacement vector, ρ is the density of the solid, **σ** is the stress tensor and **F** reflects the body load. Stress tensor σ in this formula is defined as:

$$\boldsymbol{\sigma} = \mathbf{D}\boldsymbol{\varepsilon} = \lambda(\nabla \cdot \mathbf{u})\mathbf{I} + 2\mu\, \boldsymbol{\varepsilon} \tag{3}$$

where **D** is a constitutive matrix, **I** is identity matrix and ε is the strain tensor which is described by the following expression:

$$\boldsymbol{\varepsilon} = \frac{1}{2}\,(\nabla\mathbf{u} + (\nabla\mathbf{u})^{\mathrm{T}}). \tag{4}$$

In previous equation λ and μ denote Lame constants which fulfill equations:

$$\lambda = \frac{E \cdot \nu}{(1 - 2\nu)\,(1 + \nu)}\,, \qquad \mu = G = \frac{E}{2(1 + \nu)} \tag{5}$$

where *E* denotes Young's modulus, G is shear modulus and ν is Poisson's ratio.

In analyzed models, the face harmonic load is assumed on the top boundary of the certain beam. This force causes vibration of the beam material. The response of the bodies is measured by vibration transmission loss (VTL) which can be calculated by the following equation:

$$\mathrm{VTL} = 10\log_{10}\frac{\iint_{Bi}(\omega\, u_3)^2 dBi}{\iint_{Bt}(\omega\, u_3)^2 dBt} \tag{6}$$

where *Bi* is boundary which is loaded by force along the load acting, *Bt* is the boundary on the opposite side, u_3 is the displacement in z-direction (direction of harmonic load with frequency $f = \omega/2\pi$.

In damped systems, it is common to assume classical damping of materials which is called Rayleigh damping [16, 17]. This magnitude could be expressed as a linear combination of the mass and stiffness matrices which is written as follows:

$$[C] = \alpha\,[M] + \beta\,[K] \tag{7}$$

where [*C*], [*M*] and [*K*] denotes damping, mass and stiffness matrix respectively, α and β are pre-defined constants of the Rayleigh damping model.

3. Numerical results

At this study, the frequency domain analysis was performed for two types of beam cross-section: an auxetic beam and a basic H-beam. The computations give information about the frequency response of analyzed models which are subjected to harmonic excitation for various frequencies in the range from 1 to 100 Hz. Figures 2a-b present total displacement values for both types of beam. Beams are loaded on the top boundary of the analysed structures of beam. In both cases, the models are constrained on one side of beam. The others boundary conditions are set as a free boundary, except top boundary of beam with applied harmonic load. Both analyzed type of beam have the same weight. Parameters of materials and geometry of beam are presented in Table 1.

Table 1. Parameters of materials and geometry of the beams

Parameter	Auxetic beam	H-beam
Young's modulus, *E* [Pa]	200e9	200e9
Poisson's ratio, ν [-]	0.33	0.33
Density, ρ [kg/m^3]	7850	7850
Rayleigh damping parameter, α [s^{-1}]	5.36	5.36
Rayleigh damping parameter, β [m/s]	7.46e-5	7.46e-5
Thickness of shell [m]	0.0117	0.0178
Length of beam [m]	5	5
Height of beam [m]	2	2
Width of beam [m]	2	2
Weight of beam [kg]	7850	7850

The results of the analysis are presented for the exemplary frequency of forced vibrations which is equal 12 Hz. For this value of vibrations, it is clearly visible that the beams exhibit different behavior. The maximum magnitude of deformation by auxetic beam equals 0.07 mm and 0.8 mm for H-beam. The observed displacement has a smaller value in auxetic structure. Furthermore, in H-beam occurs the transmission of vibrations into the material structure which causes displacement of the bottom side of the beam.

a) b)

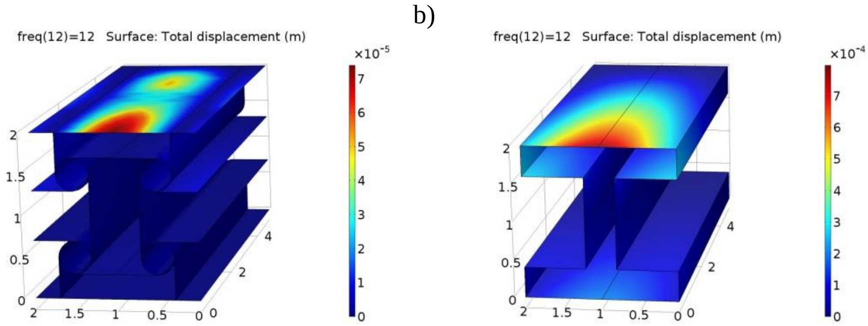

Figure 2. The total displacement of a) auxetic beam and b) H-beam

In the further calculation, the displacement of selected points was studied. For both types of beam, the analyzed points are located on the top and the bottom edge of the cross-section. These points were set in following the way in order to verify intensity of vibration transmission along the cross-section. Figure 3a and 3b depict points displacement for auxetic and H-beam respectively.

a) b)

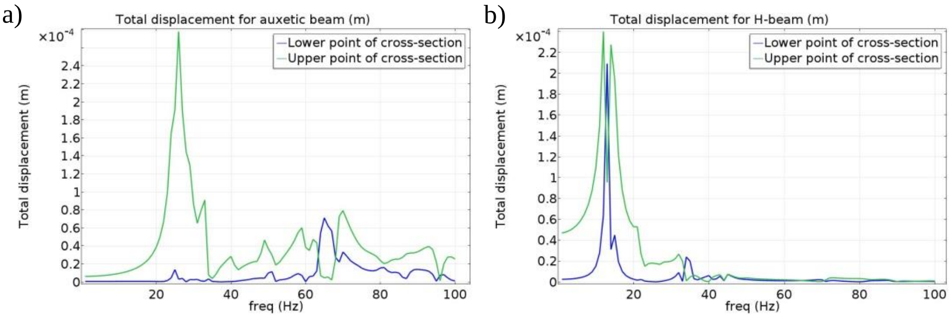

Figure 3. Point displacement for a) auxetic beam and b) H-beam

In lower range of frequencies between 1 and 20 Hz the calculating points displacement has visibly smaller values in an auxetic structure in comparison with conventional H-beam. Higher magnitudes of vibrations from 20 to 30 Hz causes a local increase of points displacement for auxetic structure. In this range, total displacement for H-beam has lower magnitude. These figures show that auxetic beam exhibit good damping properties especially in the smaller range of frequencies. The transmission of loaded vibrations is relatively low in an auxetic structure which is reflected by the small value of lower point displacement mainly in range 1 to 60 Hz.

In order to measure the effectiveness of analyzed models with reference to vibration isolation behavior the vibration transmission loss coefficient (VTL) was calculating. Figures 4a and 4b present VTL characteristics in the frequency domain for two investigated cross-sections.

The VTL characteristics show that damping properties are strongly connected with frequency values. For lower frequencies in the range from 1 to 20 Hz as well as 30-50 Hz the auxetic beam has better damping properties than classical beam . Between 60 and 80 Hz it could be observed the local decrease of VTL for auxetic beam. In this range H-beam is characterized by better damping. However beam which has auxetic cross-section exhibit largely greater damping in wider range of frequencies. Better vibration isolation for auxetic structure occurs also for extremely high frequencies close to 100 Hz. In both cases, the VTL coefficient reaches critical values for different frequencies. In analyzed cases results of VTL presented on Fig. 4a-b damping properties depend on frequency of load applied for beam.

a) b)

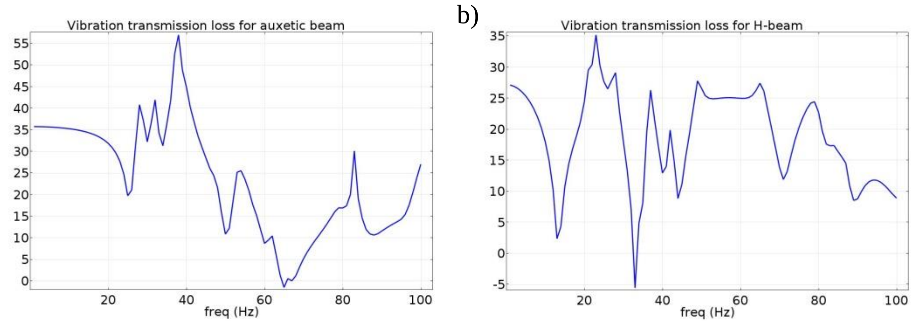

Figure 4. Vibration transmission loss coefficient for a) auxetic beam and b) H-beam

4. Conclusions

The performed vibrating analysis for auxetic beam suggests that in general, the anti-tetra-chiral structure has better damping properties than the classical beam. In analyzed frequency range occur some frequencies in which computing vibration transmission loss reaches greater values for H-beam. However maximum value for VTL coefficient is observed for auxetic cross-section. According to this results, it is really significant to pay attention to the value of harmonic excitation by choosing properly cross-section of the beam.

Obtained results show that auxetic structure is characterized by smaller points displacement for lower frequencies than the conventional beam. On the 3D plots, it is clearly visible that the auxetic beam exhibit also smaller deformation from among analyzed models. This analysis could be helpful by determining the geometry of the beam cross-section to the industrial applications. Vibration transmission is a relevant problem in mechanical engineering so it is important to study this in detail.

Acknowledgments

This work was supported by grants of the Ministry of Science and Higher Education in Poland: 02/21/DS MK/3499/2017. The simulations have been made at the Institute of Applied Mechanics, Poznan University of Technology.

References

1. Y. Liu, H. Hu, *A review on auxetic structures and polymeric materials*, Sci. Res. Essays, **5** (2010) 1052 – 1063.
2. H. R. Joshi, *Finite Element Analysis of effective mechanical properties, vibration and acoustic performance of auxetic chiral core sandwich structures*, All Theses., Clemson University, Clemson, South Carolina, 2013.
3. F. Scarpa, L. G. Ciffo, J. R. Yates, *Dynamic properties of high structural integrity auxetic open cell foam*, Smart Mater. Struct., **13**(1) (2004).
4. T. Strek, H. Jopek, M. Nienartowicz, *Dynamic response of sandwich panels with auxetic cores*, Phys. Status Solidi B, **252**(7) (2015) 1540 – 1550.
5. X. Zhang, D. Yang, *Mechanical Properties of Auxetic Cellular Material Consisting of Re-Entrant Hexagonal Honeycombs*, Materials (Basel), **9**(11) (2016) 900.
6. T. C. Lim, *Auxetic Materials and Structures*, Springer-Verlag, Singapur, 2015.
7. A. Spadoni, M. Ruzzene, *Structural and Acoustic Behavior of Chiral Truss-Core Beams*, Journal of Vibration and Acoustics, **128**(5) (2016) 616 – 626.
8. M. Ruzzene, *Vibration and sound radiation of sandwich beams with honeycomb truss core*, Journal of Sound and Vibration, **277**(4-5) (2004) 741 – 763.
9. C. F. Ng, C. K. Hui, *Low frequency sound insulation using stiffness control with honeycomb panels*, Applied Acoustics, **69**(4) (2008) 293 – 301.
10. D. C. Griese, *Finite Element Modeling and Design of Honeycomb Sandwich Panels for Acoustic Performance*, Clemson University, 2012.
11. A. Spadoni, M. Ruzzene, *Elasto-static micropolar behavior of a chiral auxetic lattice*, Journal of the Mechanics and Physics of Solids, **60**(1) (2012) 156 – 171.
12. E. Idczak, T. Strek, *Dynamic Analysis of Optimized Two-Phase Auxetic Structure*, Vibrations in Physical Systems, **28** (2017) 2017003-01-20017003-12.
13. E. Idczak, T. Stręk, *Computational Modelling of Vibrations Transmission Loss of Auxetic Lattice Structure*, Vibrations in Physical Systems, **27** (2016) 124 – 128.
14. F. Scarpa, J. Giacomin, Y. Zhang, P. Pastorino, *Mechanical Performance of Auxetic Polyurethane Foam for Antivibration Glove Applications*, Cellular Polymers, **24**(5) (2005) 253 – 268.
15. M. Bianchi, F. Scarpa, *Vibration transmissibility and damping behaviour for auxetic and conventional foams under linear and nonlinear regimes*, Smart Mater. Struct. **22** (2013) 084010.
16. A. Alipour, F. Zareian, *Study Rayleigh Damping in Structures; Uncertainties and Treatments*, Beijing, China, 2008.
17. Z. Song, C. Su, *Computation of Rayleigh Damping Coefficients for the Seismic Analysis of a Hydro-Powerhouse*, Shock and Vibration, **2017** (2017) 2046345.