Vibrations of Sandwich Plates - Comparison of Chosen Modelling Approaches

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Abstract

Sandwich plates are certain specific type of composites, which are widely used in modern engineering. Due to their complicated multi-layered structure, in literature one can find many different modelling approaches: from a simple double-plate system to a complex theories based on complicated deformation hypothesis. In this paper the modelling of vibrations of sandwich plates is considered, as a brief resume of assumptions of several models along with their governing equations of motion are presented. In the calculation example the free vibrations of a certain sandwich plate strip are investigated with the use of all described models and additionally with the use of FEM model. Eventually, basing on the presented comparison of the obtained results, the discussion on the correctness of each modelling assumption is made.

Keywords: sandwich plates, vibrations, modelling approaches

1. Introduction

The modelling of composite structures is within the field of interests of many researchers. The results of their work can be found in an enormous quantity of articles, where different complicated modelling assumptions, modelling procedures and numerical methods of solving complicated systems of equations are presented. In this article let us focus on modelling of sandwich plates, which are very specific type of composites, that has a lot of applications in modern engineering.

A typical sandwich structure consists of external layers, which are made of materials characterised by high mechanical properties, and an inner layer, so called *core*, which is usually a light-weight, porous material, standing for thermal and acoustic isolation. Moreover, by increasing the thickness of the whole structure, the core plays a significant role in determining structure's bending stiffness. As a result, we obtain a highly durable structure, with a favourable strength-to-mass ratio when compared with classic, homogeneous structures. Hence, properly designed, sandwich structures can be used in many branches of engineering, such as aviation or even space ship construction.

Sandwich structures can be modelled with the use of different approaches. The most simple one assumes that the outer layers are the main bearing parts of the structure, hence they can be modelled as beams/plates/shells, while the core is a certain elastic medium, which connects outer layers. In literature among the assumed models of elastic medium one can find one-parametric Winkler's type material (cf. Szcześniak [1, 2], Oniszczuk [3]), Murakami's type material (cf. Chonan [4]) or Vlasov's type material (cf. Navarro [5]). More complicated models are based on certain deformation hypothesis, such as: classic Euler-Bernoulli linear deformation hypothesis, broken line hypothesis, higher order deformation hypothesis or Zig-Zag hypothesis. Every each of those hypothesis is described in details in literature, for example in work of Magnucki et al. [6] or Carrera and Brischetto [7].

In this article four different models of sandwich plates will be presented and discussed: double-plate system with Winkler's type core, double-plate system with Vlasov's type core, model based on broken line hypothesis and model derived from FEM analysis. As a result of this analysis, the comparison of free vibration frequencies and a discussion of the correctness of assumptions made in every each model will be presented.

In all formulations and calculations below, it is assumed, that the structure is being modelled according to the Cartesian coordinate system $0x_1x_2x_3$, which centre is situated in the midplane of the core of the considered structure. Moreover, every each layer of the structure is assumed to be homogeneous, made of isotropic materials and to have a constant thickness. Eventually, let us introduce following denotations: $\boldsymbol{x} \equiv (x_1, x_2)$, $\partial_\alpha \equiv \frac{\partial}{\partial x_\alpha}$, $\alpha = 1,2,$ and an overdot as a time derivative.

2. Model I: double-plate system with Winkler's type core

In this modelling procedure, the whole three-layered structure is being treated as a system of two Kirchhoff's type thin plates connected with each other with the elastic, one-parametric Winkler's type material. As a result of such assumptions, the outer layers of the structure are capable of carrying only σ_{11}, σ_{22} and τ_{12} stress, while the core of the structure - only σ_{33}, cf. Fig. 1.

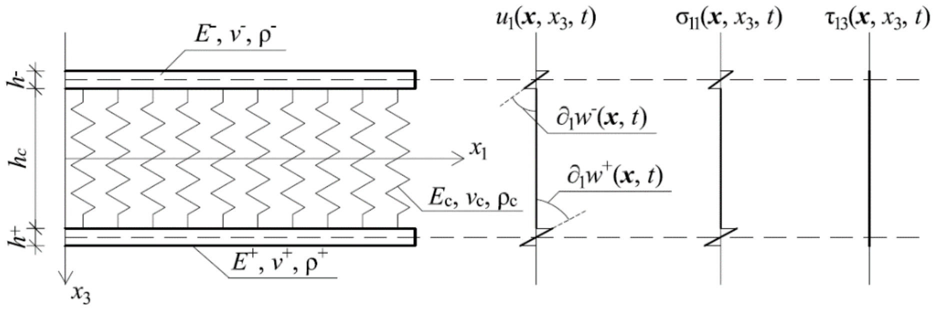

Figure 1. Assumed deflection along x_1-axis direction and stress σ_{11} and τ_{13} in Model I

Governing equations of considered structure according to Model I are obtained in the form of two equations of motion of thin plates, which take into considerations the influence of Winkler's type elastic core:

$$
\begin{aligned}
&\partial_{\alpha\beta}[B^-_{\alpha\beta\gamma\delta} w^-(\boldsymbol{x},t)] + \mu^- \ddot{w}^-(\boldsymbol{x},t) + k[w^-(\boldsymbol{x},t) - w^+(\boldsymbol{x},t)] = p^-(\boldsymbol{x},t),\\
&\partial_{\alpha\beta}[B^+_{\alpha\beta\gamma\delta} w^+(\boldsymbol{x},t)] + \mu^+ \ddot{w}^+(\boldsymbol{x},t) + k[w^+(\boldsymbol{x},t) - w^-(\boldsymbol{x},t)] = p^+(\boldsymbol{x},t),\\
&k = E_c / h_c,
\end{aligned}
\tag{1}
$$

where $B^{-}_{\alpha\beta\gamma\delta}, B^{+}_{\alpha\beta\gamma\delta}$ are bending stiffness tensors of upper and lower outer layers, respectively, μ^{-}, μ^{+} are mass densities per unit area of the outer layers, k is a modulus of the elastic core, $p^{-}(\boldsymbol{x},t), p^{+}(\boldsymbol{x},t)$ are external loadings and $w^{-}(\boldsymbol{x},t), w^{+}(\boldsymbol{x},t)$ are deflections along x_3-axis of the outer layers and basic unknowns. In order to solve the system of equations (1) one should formulate 4 boundary conditions and 2 initial conditions for every each of unknown function $w^{-}(\boldsymbol{x},t), w^{+}(\boldsymbol{x},t)$. It should be emphasised, that in general case, where $w^{-}(\boldsymbol{x},t) \neq w^{+}(\boldsymbol{x},t)$, the deflections along x_3-axis are not constant for the whole cross-section of the sandwich structure, which is often a basic assumption of much more sophisticated models.

3. Model II: double-plate system with Vlasov's type core

Similarly to Model I, this modelling procedure also transforms the three-layered structure into the system of two Kirchhoff's type thin plates. This time, however, the core of the structure is being treated as a Vlasov's two-parametric elastic medium. The main assumption of modelling is that the upper and lower edge of the medium take known displacements along x_3-axis, equal to the displacements of upper and lower outer layer $w^{-}(\boldsymbol{x},t), w^{+}(\boldsymbol{x},t)$, respectively, while the displacements of the rest of the core vary on its whole thickness according to a certain, chosen function. Additionally, displacements of the core along x_1- and x_2-axis are neglected.

In our calculations let us assume, that the function of displacements of the core along x_3-axis direction takes the form of function:

$$w_c(\boldsymbol{x}, x_3, t) = \left(\frac{x_3}{h_c} - \frac{1}{2}\right)^2 w^{-}(\boldsymbol{x},t) + \left(\frac{x_3}{h_c} + \frac{1}{2}\right)^2 w^{+}(\boldsymbol{x},t). \tag{2}$$

As a result of such assumption, we obtain a model of elastic medium, which is capable not only of carrying stress σ_{33}, but also shear stress τ_{13}, τ_{23}. The displacements along x_1-axis and internal stress in the cross-section of the whole sandwich structure is presented in Figure 2.

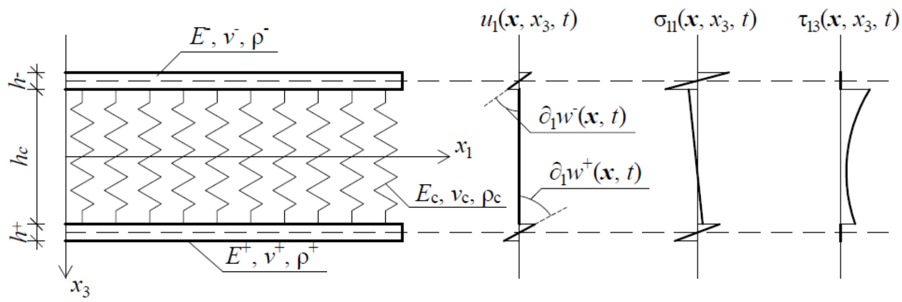

Figure 2. Assumed deflection along x_1-axis direction and stress σ_{11} and τ_{13} in Model II

Governing equations of Model II can be presented as a system of two equations of motion of thin plates, both affected by an internal stress σ_{33}, τ_{13} and τ_{23}, which are caused by deformations of the core:

$$\partial_{\alpha\beta}[B^{-}_{\alpha\beta\gamma\delta}w^{-}(\boldsymbol{x},t)] - K_t\partial_{\alpha\alpha}w^{-}(\boldsymbol{x},t) + k[w^{-}(\boldsymbol{x},t) - w^{+}(\boldsymbol{x},t)] + \hat{\mu}^{-}\ddot{w}^{-}(\boldsymbol{x},t) = p^{-}(\boldsymbol{x},t),$$

$$\partial_{\alpha\beta}[B^{+}_{\alpha\beta\gamma\delta}w^{+}(\boldsymbol{x},t)] - K_t\partial_{\alpha\alpha}w^{+}(\boldsymbol{x},t) + k[w^{+}(\boldsymbol{x},t) - w^{-}(\boldsymbol{x},t)] + \hat{\mu}^{+}\ddot{w}^{+}(\boldsymbol{x},t) = p^{+}(\boldsymbol{x},t), \quad (3)$$

$$K_t = \frac{1}{10}\frac{h_c E_c}{1+v_c},\ k = \frac{4}{3}\frac{E_c(1-v_c)}{h_c(1-2v_c)(1+v_c)},\ \hat{\mu}^{-} = \mu^{-} + \frac{1}{5}h_c\rho_c,\ \hat{\mu}^{+} = \mu^{+} + \frac{1}{5}h_c\rho_c,$$

where all denotations from Section 2 are held. In order to describe the behaviour of the three-layered structure, the formulated model of sandwich double-plate system with Vlasov's type core requires 4 boundary conditions and 2 initial conditions per both $w^{-}(\boldsymbol{x},t)$ and $w^{+}(\boldsymbol{x},t)$.

4. Model III - Broken Line Hypothesis

The Broken Line Hypothesis is the basic deformation hypothesis used to model the behaviour of sandwich structures, which outer layers are identical. According to this hypothesis, the whole sandwich structure is being treated as a multi-layered plate, which deflections along x_1- and x_2-axis directions can be defined with a proper, linear, piecewise functions, while the deflections along x_3-axis direction are independent of the x_3 coordinate. As a result we obtain:

$$u_1(\boldsymbol{x},x_3,t) = \begin{cases} -x_3\partial_1 w(\boldsymbol{x},t) - h_c\psi_1(\boldsymbol{x},t) & \textit{for} \quad x_3 \in \langle -h_f - h_c/2, -h_c/2) \\ -x_3\partial_1 w(\boldsymbol{x},t) + 2x_3\psi_1(\boldsymbol{x},t) & \textit{for} \quad x_3 \in \langle -h_c/2, h_c/2) \\ -x_3\partial_1 w(\boldsymbol{x},t) + h_c\psi_1(\boldsymbol{x},t) & \textit{for} \quad x_3 \in \langle h_c/2, h_c/2 + h_f \rangle \end{cases},$$

$$u_2(\boldsymbol{x},x_3,t) = \begin{cases} -x_3\partial_2 w(\boldsymbol{x},t) - h_c\psi_2(\boldsymbol{x},t) & \textit{for} \quad x_3 \in \langle -h_f - h_c/2, -h_c/2) \\ -x_3\partial_2 w(\boldsymbol{x},t) + 2x_3\psi_2(\boldsymbol{x},t) & \textit{for} \quad x_3 \in \langle -h_c/2, h_c/2) \\ -x_3\partial_2 w(\boldsymbol{x},t) + h_c\psi_2(\boldsymbol{x},t) & \textit{for} \quad x_3 \in \langle h_c/2, h_c/2 + h_f \rangle \end{cases}, \quad (4)$$

$$u_3(\boldsymbol{x},x_3,t) \equiv w(\boldsymbol{x},t),$$

where h_f is a thickness of both outer layers, h_c is a thickness of the core and functions $\psi_1(\boldsymbol{x},t), \psi_2(\boldsymbol{x},t)$ are certain dimensionless displacements along x_1- and x_2-axis directions, respectively (cf. Figure 3). One can observe, that by neglecting functions $\psi_1(\boldsymbol{x},t), \psi_2(\boldsymbol{x},t)$ we obtain a classic hypothesis of deformation of cross-section of thin plate.

Let us assume the stress-strain relation according to Hooke's law for isotropic materials. As a result of all assumptions, we obtain a model of sandwich structure, within which the core of the structure is an elastic medium in 3D state of stress, while the outer layers are capable of carrying only σ_{11}, σ_{22} and τ_{12}. By formulating a system of equations of equilibrium, we arrive at a following system of governing equations for the presented model:

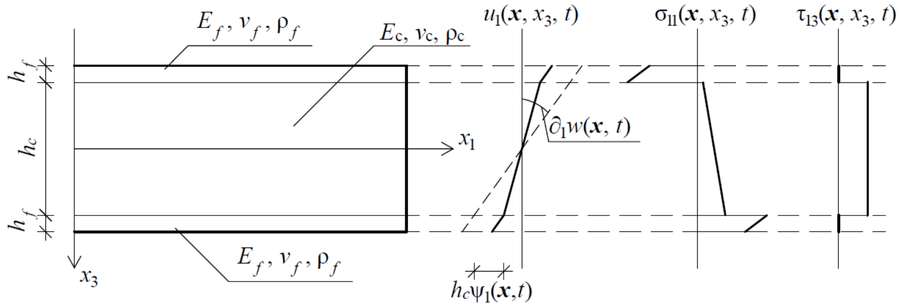

Figure 3. Assumed deflection along x_1-axis direction and stress σ_{11} and τ_{13} in Model III

$$
\begin{aligned}
&A_{11}(E_f, E_c)[\partial_{1111} w(\boldsymbol{x},t) + \partial_{2222} w(\boldsymbol{x},t)] + 2A_{11}(E_f \nu_f, E_c \nu_c)[\partial_{1112} w(\boldsymbol{x},t) + \partial_{1222} w(\boldsymbol{x},t)] + \\
&\quad +4A_{11}(G_f, G_c)\partial_{1122} w(\boldsymbol{x},t) + A_{12}(E_f, E_c)[\partial_{111}\psi_1(\boldsymbol{x},t) + \partial_{222}\psi_2(\boldsymbol{x},t)] + \\
&\quad +A_{12}(E_f \nu_f, E_c \nu_c)[\partial_{112}\psi_1(\boldsymbol{x},t) + \partial_{222}\psi_1(\boldsymbol{x},t) + \partial_{122}\psi_2(\boldsymbol{x},t) + \partial_{111}\psi_2(\boldsymbol{x},t)] + \\
&\quad +2A_{12}(G_f, G_c)[\partial_{122}\psi_1(\boldsymbol{x},t) + \partial_{112}\psi_2(\boldsymbol{x},t)] + \\
&\quad +B_{11}[\partial_{11}\ddot{w}(\boldsymbol{x},t) + \partial_{22}\ddot{w}(\boldsymbol{x},t)] + B_{12}[\partial_1\ddot{\psi}_1(\boldsymbol{x},t) + \partial_2\ddot{\psi}_2(\boldsymbol{x},t)] + C_1\ddot{w}(\boldsymbol{x},t) = p(\boldsymbol{x},t), \\
&A_{11}(E_f, E_c)\partial_{111} w(\boldsymbol{x},t) + 2A_{11}(E_f \nu_f, E_c \nu_c)\partial_{112} w(\boldsymbol{x},t) + 2A_{11}(G_f, G_c)\partial_{122} w(\boldsymbol{x},t) + \\
&\quad +A_{12}(E_f, E_c)\partial_{11}\psi_1(\boldsymbol{x},t) + A_{12}(E_f \nu_f, E_c \nu_c)[\partial_{12}\psi_1(\boldsymbol{x},t) + \partial_{11}\psi_2(\boldsymbol{x},t)] + \\
&\quad +A_{12}(G_f, G_c)[\partial_{22}\psi_1(\boldsymbol{x},t) + \partial_{12}\psi_2(\boldsymbol{x},t)] + \\
&\quad +B_{11}\partial_1\ddot{w}(\boldsymbol{x},t) + B_{12}\ddot{\psi}_1(\boldsymbol{x},t) + C_2\psi_1(\boldsymbol{x},t) = 0, \\
&2A_{11}(G_f, G_c)\partial_{112} w(\boldsymbol{x},t) + 2A_{11}(E_f \nu_f, E_c \nu_c)\partial_{122} w(\boldsymbol{x},t) + A_{11}(E_f, E_c)\partial_{222} w(\boldsymbol{x},t) + \\
&\quad +A_{12}(E_f, E_c)\partial_{22}\psi_2(\boldsymbol{x},t) + A_{12}(E_f \nu_f, E_c \nu_c)[\partial_{22}\psi_1(\boldsymbol{x},t) + \partial_{12}\psi_2(\boldsymbol{x},t)] + \\
&\quad +A_{12}(G_f, G_c)[\partial_{12}\psi_1(\boldsymbol{x},t) + \partial_{11}\psi_2(\boldsymbol{x},t)] + \\
&\quad +B_{11}\partial_2\ddot{w}(\boldsymbol{x},t) + B_{12}\ddot{\psi}_2(\boldsymbol{x},t) + C_2\psi_2(\boldsymbol{x},t) = 0, \\
&A_{11}(Y,Z) = -Ya_1 - \tfrac{1}{12}Z, \qquad A_{12}(Y,Z) = Ya_2 + \tfrac{1}{6}Z, \\
&B_{11} = \rho_f a_1 + \tfrac{1}{12}\rho_c, \qquad B_{12} = -\rho_f a_2 - \tfrac{1}{6}\rho_c, \\
&C_1 = -(2\rho_f h_f + \rho_c h_c)h_c^{-3}, \qquad C_2 = -2G_c h_c^{-2} \\
&a_1 = (\tfrac{2}{3}X^2 + X + \tfrac{1}{2})X, \qquad a_2 = X^2 + X, \qquad X \equiv h_f / h_c.
\end{aligned} \tag{5}
$$

The system of equations (5) should be followed by 4 boundary conditions and 2 initial conditions for function $w(\boldsymbol{x},t)$ and additional 3 boundary conditions and 2 initial conditions for every each function $\psi_1(\boldsymbol{x},t), \psi_2(\boldsymbol{x},t)$. Let us emphasise, that the presented system of equations can be used only to analyse sandwich structures, which are symmetric to its midplane. If this condition is not fulfilled, the assumed functions of displacements (4) do not describe properly the behaviour of composite.

5. Calculation example - free vibration analysis of sandwich plate strip

In this section the analysis of vibrations of sandwich plate strip is performed with the use of three presented models. Let us consider a sandwich plate strip with dimensions L_1 and L_2 along x_1- and x_2-axis, respectively, simply supported on edges $x_1 = 0, x_1 = L_1$. Material properties and detailed information about dimensions of the structure are presented below:

$$
\begin{aligned}
&E = E^- = E^+ = E_f = 210\,GPa, && E_c = 5\,GPa,\\
&\nu = \nu^- = \nu^+ = \nu_f = 0.3, && \nu_c = 0.3,\\
&\rho = \rho^- = \rho^+ = \rho_f = 7850\,kg/m^3, && \rho_c = 500\,kg/m^3,\\
&L_1 = 1200\,mm, \quad L_2 = 100\,mm, && h = h^- = h^+ = h_f = 5\,mm,\ h_c = 50\,mm.
\end{aligned}
\tag{6}
$$

The considered structure can be analysed as a 1-dimensional issue. Hence, governing equations of presented models can be rewritten into a simplified forms.

- Model I - double plate strip system with Winkler's type core:

$$
\begin{aligned}
&\frac{Eh^3}{12(1-\nu^2)}\partial_{1111}w^-(x_1,t) + \rho h\ddot{w}^-(x_1,t) + \frac{E_c}{h_c}[w^-(x_1,t) - w^+(x_1,t)] = 0,\\
&\frac{Eh^3}{12(1-\nu^2)}\partial_{1111}w^+(x_1,t)] + \rho h\ddot{w}^+(x_1,t) + \frac{E_c}{h_c}[w^+(x_1,t) - w^-(x_1,t)] = 0,
\end{aligned}
\tag{7}
$$

where the solution to system of equations (7) can be assumed as:

$$
w^-(x_1,t) = A_{w^-}\sin(n\pi x_1/L_1)\sin(\omega t), \qquad w^+(x_1,t) = A_{w^+}\sin(n\pi x_1/L_1)\sin(\omega t), \tag{8}
$$

where A_{w^-}, A_{w^+} are vibration's amplitudes and ω is a frequency of vibrations;

- Model II - double plate strip system with Vlasov's type core:

$$
\begin{aligned}
&B\partial_{1111}w^-(x_1,t) - K_t\partial_{11}w^-(x_1,t) + k[w^-(x_1,t) - w^+(x_1,t)] + \hat{\mu}\ddot{w}^-(x_1,t) = 0,\\
&B\partial_{1111}w^+(x_1,t) - K_t\partial_{11}w^+(x_1,t) + k[w^+(x_1,t) - w^-(x_1,t)] + \hat{\mu}\ddot{w}^+(x_1,t) = 0,\\
&B = \frac{Eh^3}{12(1-\nu^2)}, \qquad k = \frac{4}{3}\frac{E_c(1-\nu_c)}{h_c(1-2\nu_c)(1+\nu_c)}, \qquad K_t = \frac{1}{10}\frac{E_c h_c}{1+\nu_c},
\end{aligned}
\tag{9}
$$

where the solution to system of equations (9) can be assumed as:

$$
w^-(x_1,t) = A_{w^-}\sin(n\pi x_1/L_1)\sin(\omega t), \qquad w^+(x_1,t) = A_{w^+}\sin(n\pi x_1/L_1)\sin(\omega t), \tag{10}
$$

where A_{w^-}, A_{w^+} are vibration's amplitudes and ω is a frequency of vibrations;

- Model III - broken line hypothesis:

$$
\begin{aligned}
&A_{11}\partial_{1111}w(x_1,t)+A_{12}\partial_{111}\psi_1(x_1,t)+B_{11}\partial_{11}\ddot{w}(x_1,t)+B_{12}\partial_1\ddot{\psi}_1(x_1,t)+C_1\ddot{w}(x_1,t)=0,\\
&A_{11}\partial_{111}w(x_1,t)+A_{12}\partial_{11}\psi_1(x_1,t)+B_{11}\partial_1\ddot{w}(x_1,t)+B_{12}\ddot{\psi}_1(x_1,t)+C_2\psi_1(x_1,t)=0,\\
&A_{11}=A_{11}(E_f,E_c)=-E_f(\tfrac{2}{3}X^2+X+\tfrac{1}{2})X-\tfrac{1}{12}E_c, \qquad B_{11}=\rho_f a_1+\tfrac{1}{12}\rho_c,\\
&A_{12}=A_{12}(E_f,E_c)=E_f(X^2+X)+\tfrac{1}{6}E_c, \qquad B_{12}=-\rho_f a_2-\tfrac{1}{6}\rho_c,\\
&C_1=-(2\rho_f h_f+\rho_c h_c)h_c^{-3}, \qquad C_2=-2G_c h_c^{-2}, \qquad X\equiv h_f/h_c,
\end{aligned}
\tag{11}
$$

where the solution to system of equations (11) can be assumed as:

$$
w(x_1,t)=A_w\sin(n\pi x_1/L_1)\sin(\omega t), \qquad \psi(x_1,t)=A_\psi\cos(n\pi x_1/L_1)\sin(\omega t), \tag{12}
$$

where A_w, A_ψ are vibration's amplitudes and ω is a frequency of vibrations.

Additionally, the FEM model of the considered structure was developed. The whole structure was modelled as 3D structure with the use of eight-node brick elements with reduced integration (C3D8R). Calculations were performed using ABAQUS calculation's environment. The obtained free vibration frequencies of first 7 modes of vibrations are used as a benchmark for comparison of Models I-III, cf. Table 1.

Table 1. Comparison of free vibration frequencies obtained within all presented models

Mode	Model I	Model II	Model III	FEM
	[Hz]	[Hz]	[Hz]	[Hz]
1	8.54	274.80	130.48	130.02
2	34.15	550.30	458.58	455.55
3	76.83	827.22	879.61	870.03
4	136.59	1106.23	1331.17	1310
5	213.42	1388.04	1788.31	1750.7
6	307.32	1673.32	2243	2185.2
7	418.3	1962.72	2693.27	2613.2

By analysing data in Table 1, one can conclude that free vibration frequencies derived from Model I are significantly lower than those obtained within other models. It is evident, that due to many simplifications, which occur in the modelling procedure, such as: neglecting shear stress in the core of the structure, neglecting in-plane displacements or the influence of inertia of the core, the proposed Model I is not able to describe properly the behaviour of sandwich structure and should be omitted. One can notice, that in special case, when the considered sandwich structure is symmetric to its midplane and solutions to governing equations (1) are assumed in the same form for both upper and lower outer layers, the derived frequency is in fact the frequency of a single outer layer, as the core of the structure has no influence on vibrations of the outer layers.

The results obtained within Model II are ambiguous. The free vibration frequencies connected with the first two modes of vibrations are significantly higher than those obtained within FEM analysis. For the rest of the analysed modes this relation is completely different, as frequencies obtained within FEM are unquestionably higher than those of Model II. One can perceive, that taking into consideration the effect of shear stress in the core of the structure has a remarkable influence on the overall description of the behaviour of considered composite.

Eventually, differences between results of Model III and FEM are negligibly small for lower modes of vibrations. However, even in this modelling approach one can observe, that those differences tend to increase, as a mode number *n* increases. It should be stated, that for the 7th mode the differences do not exceeded 3%.

6. Conclusions

By analysing the results of free vibration frequencies derived from all presented models one can conclude, that simple models of sandwich structures are not able of describing properly the behaviour of the composite. Keeping in mind the fact, that in all presented models outer layers were always capable of carrying only stress σ_{11}, σ_{22} and τ_{12}, the most important difference between analysed models is the state of stress of the core. The results of Model I, where the shear stress in the core is omitted, are far from being correct or comparable with any other model. As a result of assumptions made in Model II we obtain a structure which is much closer to our expectations - basing on the values of coefficients in equations (9) in calculation example, one can observe that the behaviour of the outer layers of the structure is close to the behaviour of membranes, while the core is responsible for carrying shear forces. Still - the difference between the results of Model II and FEM analysis are also far from being acceptable. It should be emphasised, that by assuming a proper function of displacement (2) one can obtain results much closer to those obtained within FEM. However, negligence of the in-plane displacements of both the core and the outer layers (cf. Figure 2) is an assumption, which makes it unable to perform a full-scale analysis of vibrations with the use of Model II. Eventually, Model III can be considered sufficient for such analysis, however, one should perceive its disadvantages, such as an inability to describe the behaviour of sandwich structures, which are not symmetric to its midplane. For such structures the assumption $u_3(\boldsymbol{x}, x_3, t) \equiv w(\boldsymbol{x}, t)$ becomes questionable (let us note, that it was not made in Model I and Model II!), hence further adjustments should be made.

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