Simulation of a Single Mode Wave Generation in Cylindrical Systems Applying Numerical Methods

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Abstract

The paper presents simulations of a selected single mode generation in systems containing duct-like elements applying the finite element method (FEM). Simulations were carried out for values of the Helmholtz number exceeding the plane-wave propagation, *i.e.* for a multimode wave. The presented results constitute the first step to analyse propagation of the multimode wave through more complicated systems such as mufflers, even in a form of a cascade. Propagation of the incident wave in the form of a single mode greatly simplifies solutions of many problems, to mention only derivation of the transmission or the scattering matrices.

The results obtained can be applied to analyse the effectiveness of attenuation of acoustic silencers or elements of heating, ventilation and air conditioning systems (HVAC) at the design stage. Results of simulations are compared with already published experimental data obtained in a measurement set-up containing the self-designed single mode synthesizer.

Keywords: cylindrical duct, multi-mode wave propagation, single mode generation

1. Introduction

Acoustic waveguides are systems in the form of pipes in which an acoustic wave can propagates. They are used in heating, ventilation and air conditioning systems (HVAC) where the duct outlet is often a source of noise inside rooms or the outdoor environment [1, 2]. The waveguide theory is also applying in the analysis of noise coming from the jet engines placed in the cylindrical housing.

The purpose of this paper is numerical simulations related to the generation of a single selected mode inside a rigid cylindrical infinite waveguide using the finite element method (FEM). Simulations enable analysis of the acoustic phenomena occurring in the duct at the design stage without the necessity of construction time-consuming and expensive measuring setup.

When the frequency of the acoustic wave propagating in the duct exceeds the cut-off frequency of the Bessel mode (1,1), (indexing according to the Wejnshtejn monograph [3]) the sound pressure field is the result of the sum of component amplitudes of individual modes that can propagate without attenuation. Determination of the scattering matrix **S** [4] for multimode wave in the case of acoustic muffler is associated with the determination of $4N^2$ coefficients (N - is the number of propagating modes in inlet and outlet pipes [5]). In the case of a wave in the form of a single mode, only $4N$ coefficients of the scattering matrix must be determined [5]. The ability to generate a single mode greatly simplifies the analysis of the sound pressure field and its comparison with theoretical calculations or measurements.

The paper presents the first step, which is simulation of single mode generation in a cylindrical infinite waveguide with a constant cross-section. The required acoustic pressure field distribution was obtained by means of a set of point source models located on the cross-section of the waveguide. Verification of calculations was carried out using the previously published results for analogical acoustic measurements of real waveguide [6]. The results of the work can then be used to analyse more complex acoustic systems and elements of ventilation systems, such as acoustic mufflers.

2. Theory

Propagation of sound in the air is quite well described by the wave equation

$$\Delta p(\bar{r},t) - \frac{1}{c^2}\frac{\partial^2 p(\bar{r},t)}{\partial t^2} = 0 \tag{1}$$

where p is the acoustic pressure and c is the speed of sound.

The calculations in the frequency domain were performed using the Helmholtz equation being a Fourier transform from the wave equation.

$$\Delta p(\bar{r}) + k^2 p(\bar{r}) = 0 \tag{2}$$

where $k = \frac{\omega}{c} = \frac{2\pi f}{c}$ is the wave number, ω is the angular frequency, f is the frequency.

In theoretical considerations, taking into account the solution of equation (2) in the cylindrical coordinate system and the Neumann boundary condition, we obtain the expression for the sound pressure of a single mode

$$p_{mn}(\rho,\varphi,z) = A_{mn} e^{im\varphi} J_m\left(\frac{\mu_{mn}\rho}{a}\right) e^{i(\gamma_{mn}z - \omega t)} \tag{3}$$

where (ρ,φ,z) is the coordinates of the cylindrical system, a - waveguide radius, A_{mn} - amplitude of single mode sound pressure, J_m – Bessel functions of first kind, μ_{mn} – n-th root of derivative of the Bessel function $J'_m(\,)$, $\gamma_{mn} = \sqrt{k^2 - \left(\frac{\mu_{mn}}{a}\right)^2} = \frac{1}{a}\sqrt{(ka)^2 - \mu_{mn}^2}$ – longitudinal wave number. In the case when a larger number of modes propagate within the waveguide, the total sound pressure of the waveform is a superposition of the allowed due to the parameter ka, modes

$$p(\rho,\varphi,z) = \sum_{m,n} p_{mn}(\rho,\varphi,z) \tag{4}$$

3. Waveguide geometry and simulation conditions

The simulations were carried out for a waveguide model with a radius of a = 103.25 mm and 1.75 m length (Fig. 1). The geometrical dimensions are consistent with the real object on which previously published measurements were carried out. For a such waveguide geometry, the cut-off frequency of the first Bessel mode (1,1) is 976 Hz. That means that only a plane wave can propagate below this cut-off frequency. The simulations included in the article concern the generation of the first four modes, namely (0,0), (1,1), (2,1) and (0,1). The excitation frequency above the cut-off frequency of mode (0.1) for which the Helmholtz number is ka = 3.83 is selected and is equal 2116 Hz. The mesh size of finite elements corresponds to 1/6 of the wavelength of the excitation frequency and is 0.02 m. For more accurate calculations, the element size was ten times smaller. A 3D mesh has been created from 2.13173e+06 tetrahedral elements included 372219 nodes for which calculations were carried out.

In order to fulfill the assumptions of the infinite waveguide on the surface of the outlet on both sides of the waveguide, the boundary condition was set as totally absorbing acoustic waves, creating so-called anechoic terminations. The surface of the waveguide was treated as rigid, therefore the Neumann boundary condition is associated with this surface. This means that the normal component of the acoustic velocity on the surface of the waveguide is zero $v_n|_\Sigma = 0$.

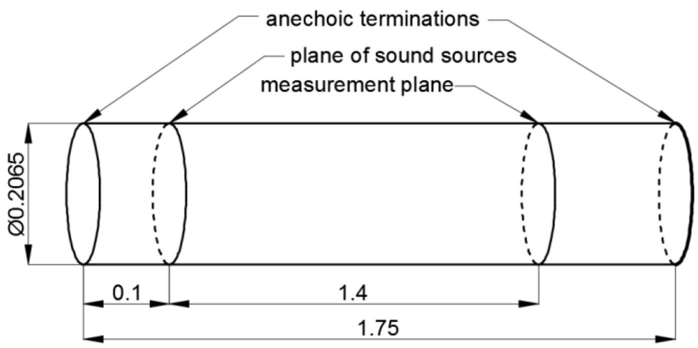

Figure 1. Geometry of the waveguide (dimensions in [m])

The location of the point sound sources in the FEM model corresponded to the distribution of sources as in the measurement set-up. The sources were located on a constant cross-section at a distance of 0.1 m from the anechoic termination on an equilateral triangle plan. Using the Green function and the shape function of a individual mode (0,0), (1,1), (2,1), (0,1), complex amplitudes of each sound sources were determined. The location of the sound sources is shown in Figure 2. Table 1 presents the determined amplitudes of sources for the generation of acoustic wave in the form of a single selected mode.

The distribution of sound pressure field was analysed on a cross-section at a distance of 1.4 m from the plane of the point sound sources (Fig. 2). For practical reasons, sound pressure values (in Pa) are presented in the form of sound pressure levels (SPL) in dB.

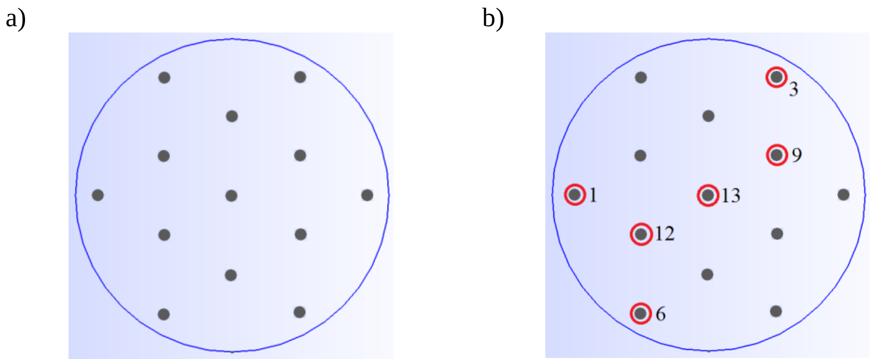

Figure 2. Location of point sound sources on the cross section of the duct; a) all sources, b) sources selected for the generation of single modes (marked with a red envelope)

Table 1. Amplitudes of individual sound sources located on the cross-section inside the waveguide

	amplitude of the modes [Pa]			
Point	mode (0,0)	mode (1,1)	mode (2,1)	mode (0,1)
1	0.46+0*i*	0.00+0*i*	0.05+0*i*	-0.46+0*i*
6	0.46+0*i*	-1.00+0*i*	0.51+0*i*	-0.46+0*i*
12	-1.00+0*i*	0.80+0*i*	-1.00+0*i*	1.00+0*i*
13	0.57+0*i*	0.00+0*i*	0.87+0*i*	0.03+0*i*
9	0.10+0*i*	-0.80+0*i*	-0.89+0*i*	-0.10+0*i*
3	0.00+0*i*	1.00+0*i*	0.46+0*i*	0.00+0*i*

The grid of points in which the sound pressure level was determined was obtained by rotating points distributed along the waveguide radius with a step of 5mm in the range of full angle 360 with a step equal to 15. The total number of calculation points is 580 (Fig. 3).

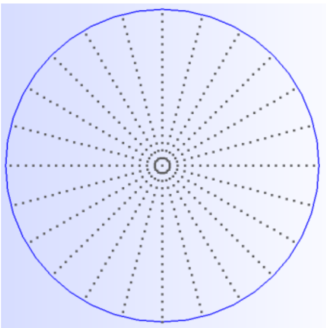

Figure 3. Arrangement of calculation points on the cross-section of the waveguide at distance 1.4 meters from the plane of the sound sources

4. Results and their interpretation

For the above-described rigid model of the infinite cylindrical waveguide, calculations of sound pressure level in points were carried out. It can be seen that only in the case of mode (2,1) all six point sound sources are involved in the generation of acoustic single-mode wave. In other cases, all sources are not needed. The results obtained are presented below (Fig. 4).

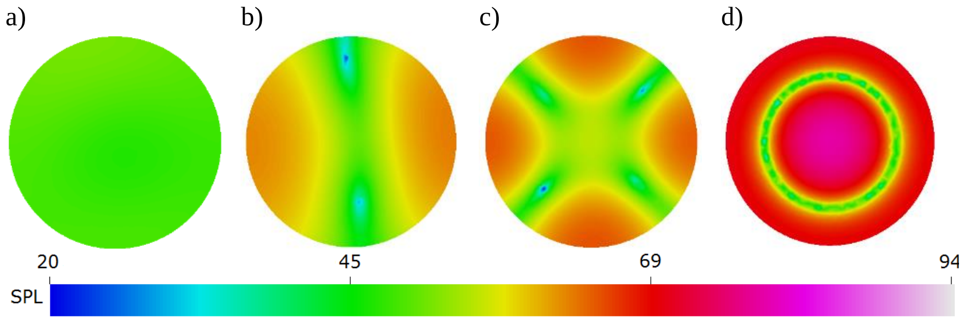

Figure 4. results of simulation single-mode generation using FEM: a) mode (0,0), b) mode (1,1), c) mode (2,1), d) mode (0,1)

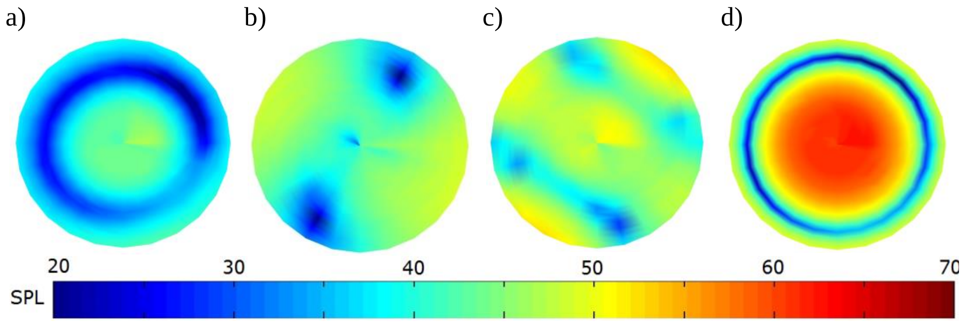

Figure 5. Results of single mode generation measurements using a rotary matrix of sound sources [6]: a) mode (0,0), b) mode (1,1), c) mode (2,1), d) mode (0,1)

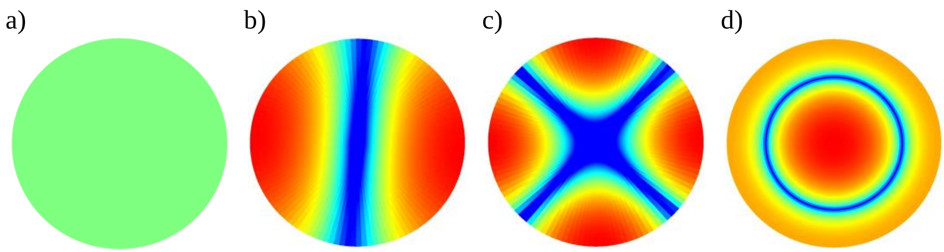

Figure 6. Theoretical shape functions of modes: a) mode (0,0), b) mode (1,1), c) mode (2,1), d) mode (0,1)

In the figures 4 – 6, it can be seen that the shape of the sound pressure level distribution obtained by the FEM calculations (Fig. 4) shows similarity to the theoretical shape functions (Fig. 6). Figure 5 shows the measurements results on a logarithmic scale. Fig. 5a) shows a significant contribution of mode (0,1), so it can be concluded that for ka = 4 it was not possible to generate a plane wave without the participation of the other permitted modes.

5. Conclusions

The presented simulation results of the single mode generation show that the shape of the sound pressure level distribution obtained by the FEM method is similar to the shape of the modes obtained by means of measurements. Even better similarity is visible comparing simulations with theoretical calculations. For quantitative analysis, decomposed of the modes should be carried out, *i.e*. their complex amplitudes should be determined applied the Fourier-Lommel [7] transform and the properties of the shape function. Finally, it can be concluded that these preliminary FEM calculations are accurate enough and the results can be used to analyse more complex acoustic systems such as silencers or flow regulators.

Acknowledgments

The study reported in this paper was financed by the Dean's of the Faculty of Mechanical Engineering and Robotics AGH University of Science and Technology – research project number 16.16.130.942.

References

1. P. Joseph, P. A. Nelson, M. A. Fisher, *Active control of fan tones radiated from turbofan engines. I. External error sensors,* J. Acoust. Soc. Am., **106** (1999) 766 – 778.
2. A. Snakowska, J. Jurkiewicz, E*fficiency of energy radiation from an unflanged cylindrical duct in case of multimode excitation,* Acta Acustica united with Acustica, **96** (2010) 416 – 424.
3. L. A. Vainshtein, *The theory of diffraction and the factorization method*, Golem Press, 1969.
4. A. Sitel, J. M. Ville, F. Foucart, *Multiload procedure to measure the acoustic scattering matrix of a duct discontinuity for higher order mode propagation conditions*, J. Acoust. Soc. Am., **120** (2006) 2478 – 2490.
5. A. Snakowska, K. Kolber, Ł. Gorazd, J. Jurkiewicz, *Derivation of an acoustic two-port scattering matrix for a multimode wave applying the single-mode generator*, Acoustics 2018 proceedings of joint conference, IEEE (2018), 294 – 298.
6. A. Snakowska, Ł. Gorazd, J. Jurkiewicz, K. Kolber, *Generation of a single cylindrical duct mode using a mode synthesizer,* Applied Acoustics, **114** (2016) 56 – 70.
7. J. M. Auger, J. M. Ville, *Measurement of linear impedance based on the determination of duct eigenvalues by a Fourier-Lommel's transform.* J. Acoust. Soc. Am., **88**(1) (1990) 19 – 22.