Analysis of Doppler Tomography in Circular Geometry as a Novel Method of Imaging Tissue Cross-Sections in vivo

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Abstract

Currently, methods such as conventional ultrasound B-mode scanning (US), computerized X-ray tomography (CT), magnetic resonance imaging (MRI), standard X-ray diagnostics, radiosotope imaging and thermography are used to visualize the internal structure of tissue in vivo and to diagnose the patient. Doppler tomography (DT) is an innovative method of reconstructing the image of the tissue section using ultrasonic waves and Doppler effect. In contrast to the currently applied solutions (US), this method uses a continuous wave, which, in theory, allows one to operate with higher energy and to detect smaller inclusions within the examined tissue. This study focuses on the analysis of DT simulation in circular geometry, where a two-transducer ultrasonic probe circulating around the tested object is used to measure the useful signal. In this paper, the influence on the tested object’s cross-section imaging quality of both the simulated Doppler signal’s registration parameters, and the calculation algorithm’s parameters, were analyzed.

Keywords: Doppler tomography, Doppler signal, continuous wave ultrasonic tomography, image reconstruction

1. Introduction

Currently, the most popular use in medicine of both ultrasound wave and Doppler effect is the measurement of blood flow velocity in blood vessels. The Doppler Tomography method also uses these two elements but for a completely different kind of measurement. The main task of this method is the reconstruction of the image of the tissue section. Just like in the classical measurement of blood velocity, also in this case, an ultrasonic two-transducer probe generating a continuous wave is used. The difference is that in DT this probe is put in motion to trigger the Doppler effect, while the imaged tissue section is stationary.

Due to the way the probe moves, we distinguish two data archiving geometries: linear geometry and circular geometry [1, 2]. In this paper, we focused on the second geometry in which the probe moves around the object being examined.
2. The principle of data acquisition in the DT method for circular geometry

As already mentioned in the DT method, we use a two-transducer probe that transmits and receives an ultrasonic wave. This probe generates a continuous wave with the frequency $f_{T}$ and moves around the tissue being examined as shown in Figure 1a. At a given rotation angle $\theta$, the wave is reflected from the stationary inclusions and returns to the probe with the changed frequency $f_{R}$. For a single inclusion, we register the appropriate Doppler frequency. In the DT method, the image is reconstructed based on the measurement of this frequency during the rotation of the probe. This frequency can be calculated using formula:

$$f_{d} = \frac{2 \cdot f_{T} \cdot v \cdot \cos(\theta)}{c}$$

\[(1)\]

where: $f_{T}$ – ultrasonic wave frequency generated by the probe, $v$ – linear component of the scatterer instantaneous velocity moving toward the ultrasonic wave propagation, $\theta$ – rotation angle, $c$ – ultrasonic wave speed in the tissue (the cross-section of the examined object). This formula can also be converted to the following form:

$$f_{d} = \frac{2 \cdot f_{T} \cdot \omega_{m} \cdot r \cdot \cos(\theta)}{c}$$

\[(2)\]

where $\omega_{m}$ is the angular speed of the object under examination (the cross section of the tissue), $r$ is the distance between the inclusion and the centre of rotation.

It should also be noted that in the DT measurement system, the imaging tissue and the probe should be immersed in water.

Figure 1. The method of data acquisition for DT in the circular geometry (a) and the nature of velocity distribution $v$ and Doppler frequencies $f_{d}$ in the imaging area (b)
Data acquisition in the DT method requires the registration of data from the rotating probe. However, to understand the essence of image reconstruction it will be easier when we assume that the object being imaged rotates around its own axis, while the probe remains stationary. From the point of view of the Doppler effect, the Doppler frequency is the same when the source moves, and the object remains stationary or the object moves, and the source is immovable.

For example, let’s assume that as shown in Fig. 1a, we have three inclusions marked with the letters a, b, c, in the section of the imaged tissue. It should be noted that for the rotation angle equal to 0° the component linear velocity in the direction of wave propagation for these three inclusions is $v_a$, $v_b$ and $v_c$ (Fig. 1a). Each of these velocities will be related to the generated Doppler frequency in accordance with the formula (2). In this case, Doppler frequencies will be $f_{da}$, $f_{db}$, $f_{dc}$, respectively. Suppose that at a given rotation angle, the incidence moves in the direction from the probe. For such a case, the Doppler frequency is taken into account with the plus sign (+). If the inclusions move towards the probe, we take the frequency with the minus sign (–).

To understand the DT method thoroughly one should notice two facts. First, as we move away from the center of rotation, the linear velocity component in the direction of wave propagation increases to the maximum value of $v_{max}$ (Fig. 1b). This velocity determines the maximum Doppler frequency. Assuming the given diameter of the imaging area, this frequency can be calculated using the formula (2). The second fact is that on lines perpendicular to the direction of wave propagation (vertical lines in Fig. 1b), the velocity components $v$ have the same values. In Fig. 1b this can be seen on the example for $v_c$ velocity. In this case, by using the formula (1) it can be concluded that the inclusions on the vertical lines (Fig. 1b) will generate the same Doppler frequency at a given rotation angle $\theta$.

To reconstruct the image in the DT method, the maximum Doppler frequency should be calculated and the range from $-f_{max}$ to $+f_{max}$ should be created. Next, the range should be divided into equal bands with the width $\Delta f$. These bands are called Doppler ranges. At a given rotation angle, each of these bands represents a portion of the imaged zone as shown in Fig. 1b. If one or more inclusions appear in a given zone, they will generate Doppler frequencies of a given value. In the DT method, at each rotation angle to a given band, we write the sum of the amplitudes of the Doppler frequencies. As a result, we get a matrix called a sinogram, where in the following lines the angles of rotation change, while in the columns we have the sum of Doppler frequency amplitudes in the given Doppler bands. An example of such a sinogram is shown in Fig. 2a. Doppler frequencies have been calculated from the equation (2). The calculations were carried out for the case in which we simulate one inclusion in the position of 4 cm from the center of rotation and set at an angle of 0° in relation to the ultrasound probe. The inclusion is infinitely small and scatters the ultrasonic wave in the same way in every direction. The remaining parameters of the experiment are: $f_T = 4.7$ MHz, $\omega_{turn} = 2$ turns per second, $c = 1482$ m/s, the diameter of the imaging area is equal to 10 cm, the number of read angles for half rotation is equal to 500, the number of Doppler bands is 99. After the creation of sinogram, the image should be reconstructed by means of the algorithm used in ultrasound tomography. In our case, it is a fast algorithm under the name of Filtered Back Projection (FBP) [4]. An additional advantage of
the algorithm is the ability to reconstruct the entire image from data recorded only from the half of the rotation of the tested object (or ultrasound probe). An example of the image of an inclusion reconstructed by FBP algorithm from a previously calculated sinogram (Fig. 2a) is shown in Fig 2b.

Figure 2. Simulation of the sinogram (a) and the reconstructed image of a single inclusion (b) placed at a distance of 4 cm from the center of rotation and at an angle of 0° in relation to the probe

3. Doppler signal simulation

Doppler signal contains only Doppler frequencies. Because this signal is used to reconstruct the image, its simulation and thorough examination is necessary for a better understanding of the DT method.

It is worth noting that this is a chirp type signal. Starting from the definition of this signal, it is possible to determine the formula:

$$s(t) = A \sin \left( \frac{4 \cdot \pi \cdot f_{\text{rot}} \cdot r_0}{c} \left[ \sin(2 \pi \cdot f_{\text{rot}} \cdot t + \alpha_0) - \sin(\alpha_0) \right] \right)$$

(3)

which allows to simulate a Doppler signal from the incident being at a distance $r_0$ from the center of rotation, at an angle $\alpha_0$ in relation to the ultrasound probe [4]. In this equation, $f_{\text{rot}}$ is the rotation frequency of the object, $t$ is time, and $A$ is the amplitude of the signal. To be able to simulate more than one inclusions, we calculate the Doppler signal for each incident separately, and then add their values for the appropriate time samples.

4. Modifications of the DT method due to the improvement of the imaging quality

In the DT method, one should record the Doppler signal section for each probe rotation at angle $\theta$ and calculate its spectrum. The next step is to divide the frequency range of the determined spectra into individual Doppler bands in which we have the sum of
amplitudes of Doppler frequencies derived from inclusions. However, we come to a disturbing conclusion. In order to register Doppler signals for particular angles of rotation $\theta$, the recording of this signal should be switched on in a proper measuring time even hundreds of times. This leads to a relatively complicated measurement system. The second important problem is the limited time period and the number of Doppler signal samples for particular angles of rotation. In this approach, the signal spectrums have a low resolution, which translates into low accuracy in determining Doppler frequencies. This is due to the fact that the spectral resolution $\Delta f$ is equal to the sampling frequency $f_s$ divided by the number of samples $N$. It seems natural that if we increase the number of signal recording samples, the spectrum resolution should increase. However, it turns out that by increasing the number of $N$ samples we also increase the sampling frequency $f_s$ and consequently their ratio, i.e. resolution, remains the same. This fact means that we cannot determine the right amount of Doppler bands. Please note that the higher the number of bands, the better the image resolution [3].

The solution to both the first and the second problem can be registering the Doppler signal from the entire turnover and applying the signal overlap procedure for individual angles of rotation $\theta$. This procedure is illustrated in Fig. 3.

Figure 3. Example of Doppler signal determination for acquisition angle: (a) $\theta_1$ and (b) $\theta_2$ with $\alpha$ length in the overlay procedure

The step forward for this procedure is the division of the Doppler signal recorded from the total turnover into the individual acquisition angles $\theta$. For example, for 36 reading angles, these angles will be: $\theta_1 = 0^\circ$, $\theta_2 = 10^\circ$, ..., $\theta_{36} = 350^\circ$. In Fig. 3, the angles $\theta_1$ and $\theta_2$ are presented respectively. Then, each Doppler signal sample should be given a rotation angle at which it was recorded instead of time. In the next step, for determined angles $\theta$ we are calculating the middle sample of the Doppler section of the $\alpha$ length which is taken for analysis. For each angle $\theta$ is designated Doppler range of $\alpha$. Then we calculate the spectrum of this signal and Doppler frequencies, as a result we create the sinogram and reconstruct the image.

The influence of the $\alpha$ parameter on the imaging quality was examined on the example of rotation of an infinitely small inclusions placed on the rotating platform
4 cm from the center of rotation and at an angle of 0° in relation to the ultrasound probe. The value of the α parameter is given in relation to the length of the Doppler signal recorded from full rotation, i.e. 360°. During the experiment, it took values α = 3.6°, 7.2°, ..., 36°. The other simulation parameters are: \( f_T = 4.7 \ \text{MHz} \), \( \omega_{\text{turn}} = 2 \ \text{turns per second} \), \( c = 1482 \ \text{m/s} \), the diameter of the imaging area is 10 cm, the number of acquisition angles for half rotation is 500, the number of Doppler samples per half of the rotation is 25000.

Figure 4 shows images of inclusions reconstructed on the basis of accurately calculated Doppler frequencies, i.e. without determining these frequencies from the Doppler signal. Each image has a dimension of 15 mm x 15 mm. This simulation later in the article will be called an perfect case. It should be noted that this method cannot be applied in practice. This simulation gives the opportunity to examine the changes in the imaging that introduces the inaccuracy of Doppler frequency determination. For each of the values of the α parameter, the corresponding number of Doppler bands was determined. These are values of 21, 39, 59, ..., 199, respectively. This information allows to create appropriate sinograms and, as a result, to determine the image of the inclusion. Figure 4 clearly shows that the higher the value of α corresponding to the larger number of Doppler bands, the better the resolution.

![Figure 4](image)

Figure 4. Image reconstruction for a single inclusion in the perfect case simulation for different values of α parameter

Figure 5 shows the results of image reconstruction for calculating the frequency from the Doppler signal and applying the overlapping procedure in similar way as in the case of real measurement method. This simulation will be referred to as the real case later in the article. It can be seen that as for the previous case, as the α parameter increases, the resolution of the image improves. However, it is clearly visible that for α = 21.6° the resolution in the direction of the OX axis deteriorates and the image of the object is extended. To explain this phenomenon, the values of the Doppler signal spectrum resolution for the read angle \( \Delta f \) were calculated depending on the given α. The values of \( \Delta f \) are equal to: 398.39 Hz, 199.59 Hz, 133.15 Hz, 99.90 Hz, 79.93 Hz, 66.62 Hz, 57.14 Hz, 49.97 Hz, 44.42 Hz, 39.98 Hz. When calculating the maximum Doppler frequency for this simulation equal to 3984.27 Hz, both the previous reconstruction of images can be used to determine the number of Doppler bands. Considering only the \( \Delta f \)
spectrum resolution, one should expect that the accuracy of Doppler frequency determination and the image quality should improve with the increase of α.

![Image reconstruction for a single inclusion in real case simulation for different values of α parameter](image)

**Figure 5.** Image reconstruction for a single inclusion in real case simulation for different values of α parameter

**Figure 6.** Inclusion image (a) resolutions and (b) blurs in direction of the OX axis and inclusion image (c) resolutions and (d) blurs in direction of the OY axis for the real case and perfect case simulation for different α parameter

It should be noted that by increasing the width of overlapping α, we also increase the path on which we observe the moving inclusions for one angle θ. In this causes we register a greater number of velocities for inclusions (in the direction of wave...
propagation), and this leads to a greater number of Doppler frequencies for a single acquisition angle $\theta$. Therefore, an additional elongation and blurring of the object appears on the image caused by averaging. In order to investigate this phenomenon carefully and determine which value $\alpha$ gives the best resolution and the smallest blur in the direction of the $OX$ and $OY$ axes, the values for individual pixels of the images were examined. Point resolution was determined at 3 dB decrease in pixel values and blur at 90 % of maximum value. The results are shown above, in Figure 6. Four additional values of the $\alpha$ parameter were considered: 54°, 72°, 90° and 108°. The charts in Fig. 6c and Fig. 6d show that both the resolution and image blur in the direction of the $OY$ axis for both simulations for the perfect and real case remain practically the same. It can therefore be concluded that the overlap procedure has no significant effect on imaging in the referenced direction. In contrast, charts in Fig. 6a and Fig. 6b clearly show a deterioration in both resolution and blur for $\alpha$ above 21.6°.

5. Conclusions

Doppler tomography is a promising method of reconstructing the image of the tissue in vivo. The advantage of this method is the use of ultrasonic waves that can be safe and the use of a single head with only two transducers, which makes it possible to design a relatively cheap device. But in order for it to be used in practice, modifications are necessary. It should be remembered that the Doppler signal should be registering for the full rotation of the probe (or imaging object). It is not necessary to record this signal for each rotation angle $\theta$ separately during measurements. Secondly, a method should be used to accurately determine the Doppler frequencies from the recorded Doppler signal. The solution may be the overlapping procedure, where the length of the signal for a particular rotation angle $\theta$ is $\alpha$. It is possible to set the $\alpha$ parameter value so that the image resolution of a single point will be approximately 2 mm, and the image blur will not exceed 7 mm. These results suggest that one of the practical applications of this method in the future may be the diagnosis of female breasts in detection of cancerous changes.

References