Application of the Wavelet Multifractal Analysis of Vibration Signal for Rotating Machinery Diagnosis

Andrzej PUCHALSKI
University of Technology and Humanities in Radom,
ul. Malczewskiego 29, 26-600 Radom, andrzej.puchalski@uthrad.pl

Iwona KOMORSKA
University of Technology and Humanities in Radom,
ul. Malczewskiego 29, 26-600 Radom, iwona.komorska@uthrad.pl

Abstract

The paper proposes the WLMF (Wavelet Leaders Multifractal Formalism) method enabling the adoption of multifractal parameters mapped by vibration signal log-cumulants as diagnostic features, in the procedure of automatic classification of assembly errors and wear of demonstration gearbox. In the analysis of vibration time signals, initially, a multifractal formalism was used based on the study of time series local regularity, which is measured by Holder exponents. The presented test results relate to time-frequency multifractal analysis, the starting point of which was a continuous wavelet transform. Discrete wavelet transform allowed for much better grounded multifractal formalism and more accurate estimation of multifractal parameters using wavelet leaders, which are determined based on wavelet coefficients and are representatives of Holder exponents.

Keywords: rotating machines, multifractal analysis of vibration signal

1. Introduction

In the era of industry digitization and its transformation to Industry 4.0 standards, making diagnostic decisions involves the analysis of large databases from earlier registers as well as data downloaded from machines in real time via the industrial internet of things. More and more often a better option than searching for specific models is a direct analysis and diagnostics based on experimental data. The process requires advanced methods of stochastic analysis and solutions from the field of machine learning. Models and diagnostic features stop being physically interpreted, giving way to the statistical indicators.

This way of modeling and quantitative analysis of the dynamics of complex systems, consisting of many non-linear interacting systems operating at variable loads, is a big challenge for modern diagnostics of rotating machines. The solution to the task becomes closer due to the study of monitored real vibration signals using advanced numerical algorithms and the increasing computing power of computers.

Multifractal analysis, which is mainly based on estimates of the scaling exponents of the recorded signal, has become a popular tool for statistical analysis of empirical data. The observed properties of time series scaling measures can be used to characterize various states of a complex system [1].

In the analysis of vibration time signals, initially, a multifractal formalism was used based on the study of their local regularity, which is measured by Holder exponents [2]. Trend elimination from the studied time series in the multifractal detrended fluctuation analysis (MF-DFA) leads to the determination of diagnostic features in the form of multifractal spectrum parameters. Detrended fluctuation analysis is an important tool in the study of variable-scale and long-term properties as well as the selection and classification of diagnostic features of vibration signals generated by complex rotating machinery [3-6]. The large computational complexity of the algorithm for the approximation of the time series of N samples, with a polynomial of the order m, is estimated at $2(m+2)^2N$.

Time-frequency analysis of signals based on discrete wavelet transform allowed for much better grounded multifractal formalism and more accurate estimation of multifractal parameters using wavelet leaders, which are determined based on wavelet coefficients and are representatives of Holder's exponents [7]. The algorithm implemented by Mallat pyramid scheme shows a much lower computational complexity than the fluctuation analysis and is estimated to be $N\log N$.

Chapter 2 highlights the theoretical basis and algorithms diagram of time-frequency multifractal formalism. Chapter 3 discusses and verifies the WLMF (Wavelet Leaders Multifractal Formalism) method enabling the adoption of multifractal parameters mapped by vibration signal log-cumulants as diagnostic features, in the procedure of automatic classification of assembly errors and wear of demonstration gearbox. Chapter 4 provides a summary of the studies and the results obtained.

2. Theoretical background. Time-frequency multifractal formalism

Time-frequency signal analysis methods have allowed a new look at the problem of estimating local scaling exponents as a way of testing the regularity of time series and their multifractality. From both a conceptual and practical point of view, the wavelet transform played a special role [8].

The WTMM (Wavelet Transform Modulus Maxima) method is based on the continuous wavelet transform (CWT). It consists in determining wavelet skeleton defined by the set of all maxima lines, summation executed along q-th power of maxima chains and its scaling exponents and Legendre transforms. Practical implementations of such an algorithm have shown some disadvantages that make it impossible to carry out tests for some types of real signals. For wavelet coefficients centered around zero values, it is difficult to guarantee numerical stability. This problem does not appear in the case of the WLMF method, in which the base is the wavelet coefficients obtained as a result of the discrete wavelet transform (DWT) according to the Mallat pyramid scheme. The next steps of the algorithm include the selection of coefficients called wavelet leaders, the determination of the structural function and scaling exponents as well as the multifractal spectrum $D(h)$ or the direct determination of spectrum parameters using a log-cumulants (Figure 1).

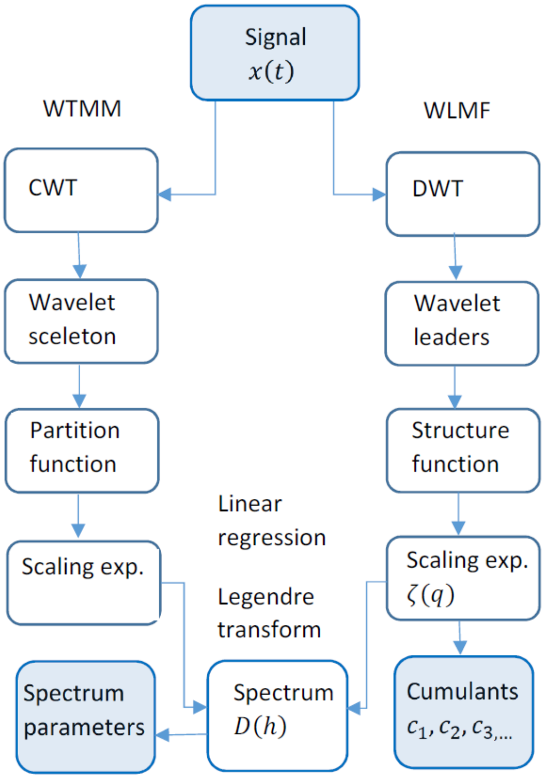

Figure 1. Block diagram of time-frequency multifractal analysis according to the WTMM and WLMF scheme

Wavelet leaders are local maxima of discrete wavelet coefficients $d_{\lambda'}$: $L_x(j,k) = sup_{\lambda' \in 3\lambda}|d_{\lambda'}|$, where $3\lambda := 3\lambda_{j,k} = \lambda_{j,k-1} \cup \lambda_{j,k} \cup \lambda_{j,k+1}$ and $\lambda := \lambda_{j,k} = [k2^j, (k+1)2^j]$ at any scale. Wavelet leaders are representatives of the Holder exponents h: $L_x(j,k) \sim 2^{jh}$. Structure-function $Z_L(q,j)$ is defined as a spatial average of the q-th order of the leaders. It can be shown that: $Z_L(q,j) \sim 2^{j\zeta(q)}$ in the limit $2^j \to 0$ [9]. Besides, the Legendre transform of the scaling exponent $\zeta(q)$ of the structure-function, provides an upper bound for the multifractal spectrum .

Knowledge of the scaling exponent $\zeta(q)$ also allows direct estimation of multifractal spectrum $D(h)$ parameters using a log-cumulants c_p of order $p \geq 1$, obtained as a result of Taylor series expansion.

Log-cumulant c_1 describes the location of the highest multifractal spectrum value, while c_2 and c_3 describe the level of multifractality: spectrum width and asymmetry, respectively. The scaling exponents of the structural function are not dependent on the choice of the wavelet, provided that the number of zero moments of the wavelet is two times greater than the largest exponent of the Holder exponent. The use of a discrete wavelet transform has also reduced the time costs of calculations.

3. Implementation of the diagnostic algorithm

Measurements were carried out on a demonstration stand (Figure 2). The influence of assembly errors and wear of gear teeth on vibrations was investigated. The electric motor allows speed control in the range of 100 - 3000 rpm (no load). The load is pressure regulated using an overflow valve up to 5 MPa. The acceleration of vibrations was measured utilizing an accelerometer screwed to the gear bearing housing in a vertical direction. The optimal backlash was set to 0.1 mm.

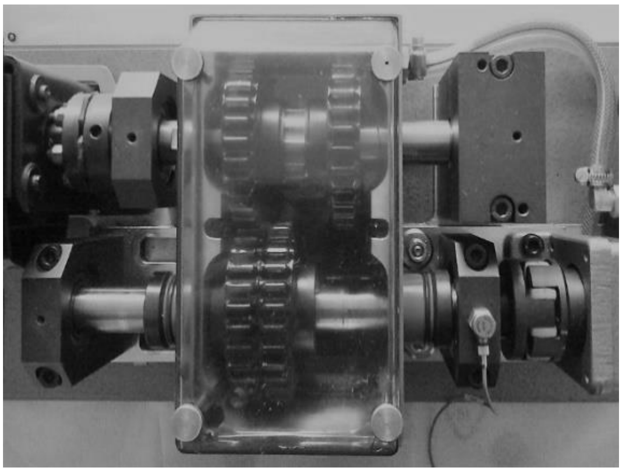

Figure 2. The test stand

Measurements were carried out for the following 5 states:

- fault-free (new gears, the optimal backlash, parallel shaft location),
- new gears and misalignment by an angle 1/3°,
- new gears and increased backlash +0.2 mm,
- worn teeth,
- worn teeth and increased backlash +0.2 mm.

Vibration acceleration signals were recorded for a rotational speed of 1365 rpm and a load of 12% - pressure 0.6 MPa (Figure 3). Each sample included a time series with a length of N = 10.000, recorded at a sampling frequency of 10 kHz.

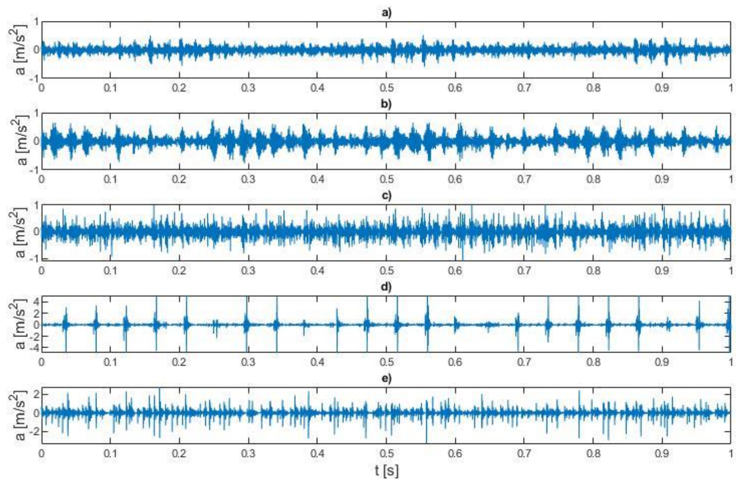

Figure 3. Waveforms of vibration signals recorded in 5 states of the gearbox:
a) fault-free state, b) misalignment, c) increased backlash, d) worn teeth,
e) worn teeth, and increased backlash

Figure 4 shows the maps of wavelet leaders designated for two states: fault-free and worn teeth. For the classification of the tested operating states of the propulsion system, mapped using multifractal spectra (Figure 5), log-cumulants of the 1st and 2nd order were selected, determined based on wavelet leaders of the vibration acceleration signal.

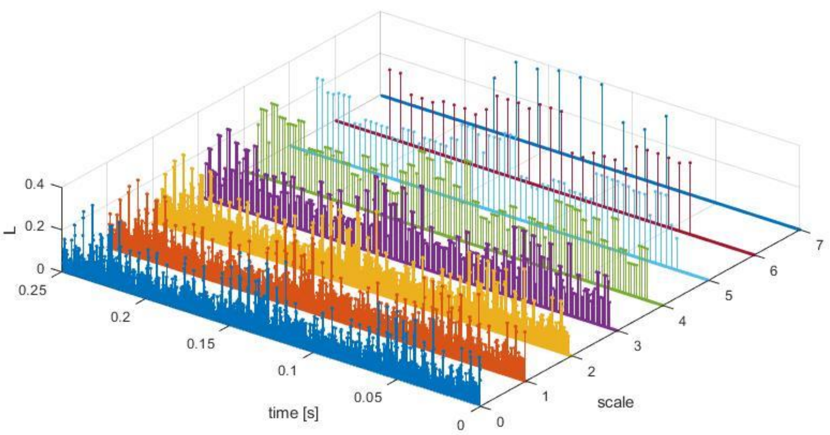

Figure 4a. Sample values of vibration signals wavelet leaders
for gearbox in fault-free state

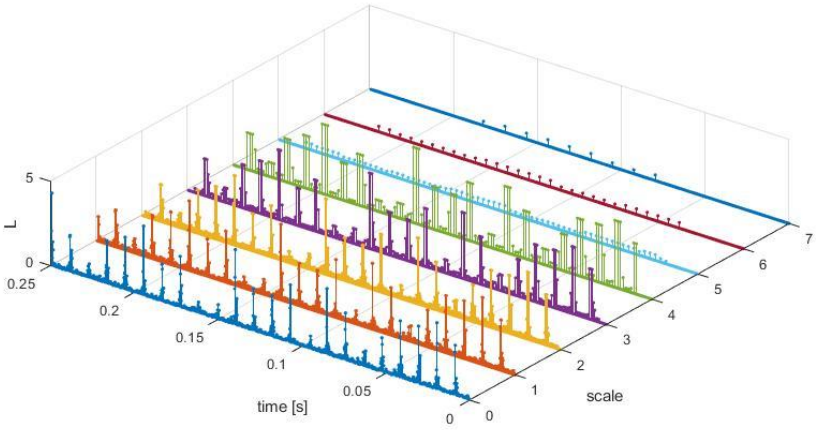

Figure 4b. Sample values of vibration signals wavelet leaders
for gearbox in worn teeth

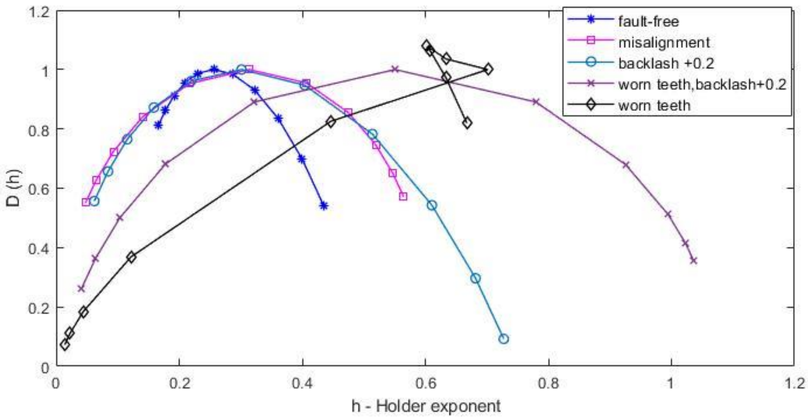

Figure 5. Multifractal spectra of vibration signals recorded
in 5 states of the gearbox

A series of 30 measurements were taken in each state. Classification of the tested state of the system to the appropriate class and analysis of the quality of the classification was carried out using the method of the nearest neighbors. The cross-validation technique was used to estimate accuracy. Classification accuracy was assessed based on the ratio of the number of correctly classified results of the experiments to their total number. All tested states of the system were separated with 92% accuracy of classification. The classification errors referred to the states: fault-free, misalignment, and increased backlash.

For improving the classification efficiency, other signal measures were determined: RMS, skewness, kurtosis. The scattering analysis of the tested parameters showed that achieving 100% accuracy in the classification of the tested operating states of the transmission is enabled by a feature vector whose third coordinate, in addition to the two log-cumulants, is skewness (Figure 6).

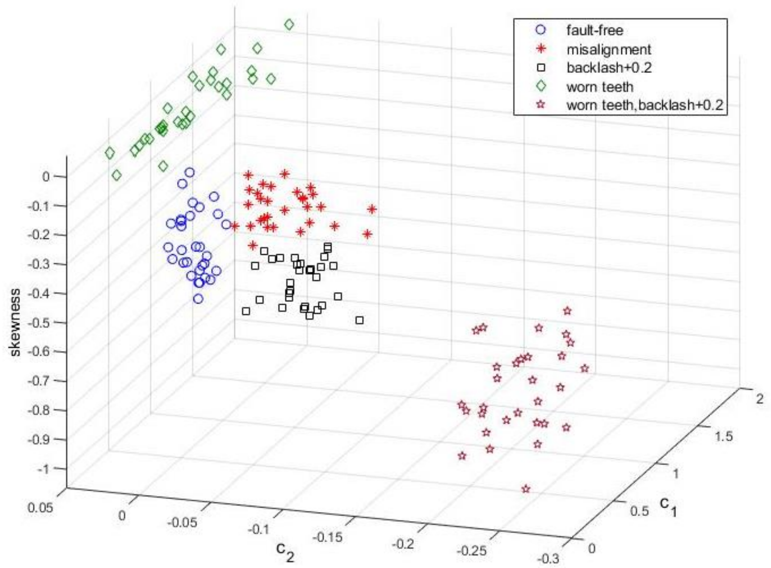

Figure 6. Scattering plot of selected diagnostic features: log-cumulants c_1, c_2, and skewness for 30 signal samples in each of the tested states

4. Conclusions

Most of the real vibration time series exhibit properties well described by local scaling exponents, which can act as diagnostic features. Multifractal formalism makes it possible to describe such signals that potentially contain an infinite number of singularities. There are several methods to study time series for their fractality. The possibilities of time and time-frequency algorithms were compared. Considering the greatest universality in real signal analysis and the lowest numerical complexity (time and requirements for memory resources) of the wavelet leader algorithm, its operation has been verified for use in vibration diagnostics of rotating systems. Multifractal parameters estimated using the log-cumulants were adopted as diagnostic features in the classification procedure by the method of nearest neighbors.

As part of the continuation of research, tests are carried out on multidimensional feature vectors of low-energy damage to rotating machines defined based on log-cumulants and the other values of scaling exponents of the structural function and multifractal spectra for a selected range of moments.

References

1. I. W. Kantelhardt, *Fractal and Multifractal Time Series*, Mathematics of Complexity and Dynamical Systems, Springer-Verlag, New York (2011) 463 – 487.
2. S. J. Loutridis, *An algorithm for the characterization of time-series based on local regularity*, Physica A, **381** (2007) 383 – 398.
3. S. Zhang, Y. He, J. Zhang, Y. Zhao, *Multi-fractal based fault diagnosis method of rotating machinery,* Applied Mechanics and Materials, **130**(2) (2012) 571 – 574.
4. A. Puchalski, I. Komorska, *A generalised entropy in multifractal time signals analysis of mechanical vibration,* JVE Journal of Vibroegineering, **20**(4) (2018) 1667 – 1675.
5. H. Liu, X. Wang, C. Lu, *Rolling bearing fault diagnosis based on LCD-TEO and multifractal detrended fluctuation analysis*, Mechanical Systems and Signal Processing, **60-61** (2015) 273 – 288.
6. A. Puchalski, I. Komorska, *Stable Distributions and Fractal Diagnostic Models of Vibration Signals of Rotating Systems*, Appl. Condition Monitoring (Springer Int. Pub. AG, **9** (2018) 91 – 101.
7. A. J. Chen, Z Li, J. Pan, G. Chen, Y. Zi, J. Yuan, B. Chen, Z. He, *Wavelet transform based on inner product in fault diagnosis of rotating machinery*, Mechanical Systems and Signal Processing, **70-71** (2016) 1 – 35.
8. W. Du, J. Tao, Y. Li, C. Liu, *Wavelet leaders multifractal features based fault diagnosis of rotating mechanism,* Mechanical Systems and Signal Processing, **43** (2014) 57 – 75.
9. H. Wendt, S. G. Roux, S. Jaffard, P. Abry, *Wavelet leaders and bootstrap for multifractal analysis of images,* Signal Processing, **89**(6) (2009) 1100 – 1114.