Fatigue Crack Detection Using Diffused Lamb Wave Field – Damage Index vs. Crack Length Estimation

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**Abstract**

Fatigue crack detection is one of the major problems in maintenance of engineering structures. Ultrasonic guided waves are used for fatigue crack detection in metallic plate components. The application involves Lamb wave propagation and the analysis of the diffused wave field. The cross-correlation function is used to obtain a damage index that reveals the initiation of fatigue crack. The major focus of the paper is on crack length estimation. The results demonstrate that crack length estimation based on Lamb wave propagation is challenging but not impossible.

**Keywords:** Fatigue crack detection, Lamb waves, crack length estimation

1. Introduction

Many engineering structures rely on the damage-tolerant design concept. This concept assumes that structural damage is inevitable and therefore reliable damage detection and monitoring methods are required to guarantee safe operation. Various fatigue crack detection methods have been developed for the last decades. Maintenance of critical structures has benefited a lot from these developments. Non-Destructive Testing and Evaluation (NDT/E) techniques – such as visual inspection eddy current, acoustic emission, ultrasonic testing – are well established and widely used for crack detection in aerospace structures [1]. More recently a number of Structural Health Monitoring (SHM) approaches have been developed. These methods are less time consuming, less costly and offer real-time crack detection thanks to network of sensors that are permanently attached to monitored structures, as discussed in [1-5]. Ultrasonic techniques based on Lamb wave propagation belong to SHM methods that can be used for crack detection and monitoring. Despite many research efforts and literature reports engineering applications of Lamb waves are still limited. This is mainly due to the physical complexity of wave propagation that requires good monitoring strategies and experience with respect to data interpretation, as discussed in [1-8]. The vast majority of reported research developments in this field involve the application of low-profile, surface-bonded piezoelectric transducers and rely on the analysis of Lamb wave responses [6-8].

The application of Lamb wave techniques usually does not require complex data processing. Simple damage indices based on amplitude and/or phase of ultrasonic responses can provide some indication about crack initiation and growth. It is well known that the amplitude of the incident wave passing through the crack decreases and the amplitude of the wave reflected from the crack increases with the crack length.
However, for complex structures propagating wave components overlap, making the entire analysis difficult for the interpretation. Some effort has been undertaken to analyse the so-called diffused Lamb wave field - i.e. the wave field generated by multiple scattering and reflection of Lamb wave components – as demonstrated in [9]. This approach can lead to data signatures and even images can be used to detect and monitor the severity of damage. Nevertheless the estimation of real crack length still remains a difficult problem. Previous research work in this field indicates data processing of Lamb waves can be combined with fatigue and fracture analysis to estimate the crack length, as shown in [10]. The former involves the phase analysis of Lamb wave responses whereas the latter utilises the crack growth equation. This pioneering work has been recently extended to the probabilistic framework in [11] to provide damage prognosis, i.e. to predict the crack growth rate and possible failure.

The paper recalls the work presented in [10] and addresses the important problem of crack length estimation based on Lamb waves. In contrast to the previous research in this field – based on simple plate components – the current work focuses on the experimental work that involves a lap joint, multi-riveted aerospace component. The structure of the paper is as follows. The application of Lamb wave propagation for damage detection is briefly discussed in Section 2 for the sake of completeness. Section 3 demonstrates the component used in the damage detection investigation and the experimental procedure undertaken for crack detection. The experimental results are given in Section 4. Finally, the paper is concluded in Section 5. Although, the paper does not offer a unique solution to the challenging crack length estimation problem, the results indicate that Lamb wave responses can be used to achieve this goal when further research work needs to be focused on more experimental investigations.

2. Lamb Waves for Structural Damage Detection

It is well known that various types of ultrasonic waves can propagate in solids. Waves travelling in an unbounded bulk of elastic material are often called bulk waves. From the basic wave theory transverse (shear or S) waves exhibit particle movement in the direction perpendicular to the wave propagation. In contrast, longitudinal (or P) waves are wave in which particles of the medium move in the direction parallel to the wave propagation. Waves propagating in bounded media are known as guided waves. Lamb waves are guided ultrasonic waves that propagate in traction-free plates and are bounded by the upper and lower plate boundaries known as the waveguide. Lamb waves have a very complex mechanism of wave propagation. These waves are formed by the combined, bouncing P and SV (Shear Vertical) waves and are multimodal in nature. The thickness of the plate and the excitation frequency determine the wave propagation velocity of infinite number of symmetric and anti-symmetric modes. It is well known that these waves are also dispersive, i.e. the velocity of each mode varies with respect to its frequency. Lamb wave properties can be obtained from the classical elastodynamic wave equation given as
\[ \rho \frac{\partial^2 u}{\partial t^2} - \nabla \cdot \tau = f \]  

(1)

where \( u \) is the displacement, \( \tau \) is the stress tensor, \( \rho \) is the density of the material, \( t \) is the time variable and \( f \) is the excitation. The relevant strain tensor can be defined as

\[ \varepsilon_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \]  

(2)

and is coupled to \( \tau \) by the Hook’s law. When the traction-free boundary condition are applied for the plate (i.e. the relevant components of stress at the top and bottom of the plate are zero) Equation (1) can be solved using the displacement potential and the partial wave technique, as demonstrated in [12]. The solution leads to the characteristic equation that is well known as the Rayleigh-Lamb frequency relations given as [12]

\[ \tanh(0.5\beta d) = \frac{4\alpha \beta k^2}{(k^2 + \beta^2)^2} \]  

(3)

for symmetric modes and

\[ \frac{\tanh(0.5\beta d)}{\tanh(0.5\alpha d)} = \frac{(k^2 + \beta^2)^2}{4\alpha \beta k^2} \]  

(4)

for asymmetric modes, where

\[ \alpha^2 = k^2 - \frac{\omega^2}{c_L^2} \quad \text{and} \quad \beta^2 = k^2 - \frac{\omega^2}{c_T^2} \]  

(5)

and \( d \) is the plate thickness, \( \omega \) is the angular frequency, \( k \) is the wave number \( c_L \) and \( c_T \) are the longitudinal and shear wave velocities respectively. Equations (4-5) can be solved numerically to obtain the dispersion curves that relate phase/group wave velocities to the \( \rho d \) product for various frequencies and thicknesses of plates. However, in practice these characteristics are not easy – or often impossible - to obtain and to analyse for complex materials and complex geometries. The so-called diffused Lamb wave field is than used. Lamb wave responses exhibit a number of different modes and wave components (i.e. incident, reflected and scattered waves) that are often overlapped. The diffused Lamb wave field can be also used for structural damage detection, as demonstrated in [9].

3. Fatigue Testing – Experimental Work

A complex metallic specimen – shown in Figure 1 – was used in the fatigue experiment. Two symmetrical aluminium-alloy multi-riveted plates (750 x 300 x 2 mm) were connected through a strap joint. The joint was fastened to either side by three rows of
rivets adding up to a total of 6 rows with 84 rivets. The 1.8 mm long notches on either side of the central rivets were introduced in the centre of the specimen by means of spark erosion. The central rivets were also slightly larger and stiffer to ensure crack propagation. Four piezoceramic SMART Layer® sensors from Acellent Technologies, Inc. were surface-bonded to the specimen for Lamb wave generation and sensing.

The Schenck servo-hydraulic test machine was used for fatigue testing. The specimen was loaded statically with a tensile load of 22 kN. Then a sinusoidal dynamic loading of ±18 kN at a frequency of 6 Hz was applied to initiate and propagate the crack. The crack lengths were determined using a microscope.

Figure 1. Aluminium lap joint used for crack detection investigations:
(a) general view; (b) fatigue crack close-up in the centre of the specimen

4. Crack Length Estimation Based on Lamb Wave Responses

The fatigued specimen was monitored using the Lamb wave propagation approach. The TTI TGA 130 30 MHz arbitrary waveform generator was used for the excitation. Five cycles of the sine wave of 75 kHz frequency and 5 V amplitude were used as the excitation signal. Lamb wave responses were acquired using the LeCroy 9303AM QUAD 200 MHz oscilloscope. The sampling rate was equal to 25 MHz. Each recorded vector consisted of 5000 samples. Altogether 10 data vectors were recorded to produce averaged Lamb wave responses for different fatigue crack lengths. The results exhibited a number of overlapped wave packages, as expected. These wave responses were used for crack detection. The cross-correlation function for the Lamb wave responses representing the undamaged (intact) specimen and the damaged (cracked) specimen was calculated for various crack length to produce the so-called damage index $DI$ defined as

$$DI = 1 - \rho_{xy} = 1 - \frac{C_{xy}}{\sigma_x \sigma_y}$$

(6)

where $\rho_{xy}$ is the cross-correlation coefficient, $C_{xy}$ the cross-correlation – that is used to assess the similarity between two Lamb wave responses - i.e. $x$ (representing intact or undamaged condition) and $y$ (representing the cracked condition) – and $\sigma_x, \sigma_y$ are the
relevant standard deviations that assure the normalization. Since the correlation coefficient is normalized between -1 and 1, the defined damage index \( DI \) increases with the severity of damage (i.e. the crack length). However, the unsolved problem in these damage detection investigations is how to estimate the crack length from the analysed damage index. In other words, the major question is whether Lamb wave responses can be used to reliably estimate the fatigue crack length. An interesting observation can be made when the damage index \( DI \) and the crack propagation curve (i.e. crack length vs. fatigue cycles) are plotted together in logarithmic scales in Figure 2. Interestingly both characteristics are approximately parallel. The Crack Growth Rate (CGR) can be estimated from the well-known Paris-Erdogan equation given as

\[
\frac{da}{dN} = C(\Delta K)^m
\]

(7)

where \( a \) is the crack length, \( N \) is the number of fatigue cycles, \( \Delta K \) is the range of the stress intensity and \( C, m \) are material constants that also depend on stress ratio, specimen size and environmental conditions. The stress intensity factor \( K \) estimates the load around the crack tip. The range of the stress intensity \( \Delta K = K_{max} - K_{min} \) represents the difference between the maximum and minimum stress intensity \( K \) in a fatigue cycle. These results indicate that once material constants are known - for the case investigated - the estimation of crack length is possible from the damage index \( DI \) obtained from Lamb wave responses.

![Figure 2. Crack Growth Rate (from fatigue analysis) and Damage Index (from Lamb wave propagation) characteristics for the tested aluminium lap joint](image)

3. Conclusions

Lamb wave propagation was used in the riveted lap joint for fatigue crack detection. The cross-correlation between diffused field Lamb wave response – representing undamaged and damaged conditions – where used to obtain the damage index that can be used for crack detection. The results demonstrate that this index increases with the crack length. More interestingly, the results also demonstrate that the calculated damage
index can be directly related to the Paris-Erdogan fatigue equation through the crack propagation curve, making the estimation of crack length possible directly from the analysed crack length. The work presented does not offer a unique solution for different materials, components and damage scenarios but at least indicates that the estimation of crack length from Lamb wave responses is possible. It is clear that further theoretical and experimental research studies are required to confirm these findings.

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