The Investigations of Suspension Model and Its Experimental Characteristics for the Air Springs of Truck Trailer

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Abstract
Air springs applied as shock damping elements are often found in the design of variety modern truck and trailer suspensions. They can also be found as damping and stabilizing suspension elements in the passenger cars and other machines. The advantage of air springs, compared to steel coil springs or leaf springs, is a better damping quality in a wide range of frequencies. The air springs stiffness can be regulated according to the requirements and working conditions. The applied air springs also allow to stabilize the distance between the vehicle body and road level in function of loading. Some proposals of vehicle suspension models can be found in technical literature where the air spring is the main elastic subassembly. Mathematical model descriptions of the suspension with air spring for vehicles apply the thermodynamic laws and relationships between the mechanical forces of cooperating suspension elements, parts geometry (suspension arms), material stiffness (reinforced rubber) and other properties (damping). The results of own investigations on the suspension model of an air spring for cargo trailer have been taken into consideration in this paper. The presented suspension model was applied to design the frame construction of a light stanchion trailer where aerodynamic drag and construction mass were reduced. The suspension model of air springs for a trailer was applied for frame loads evaluation of the light trailer. It was also used for the strength analysis of the frame construction with the reduced mass. The estimated frame loads such as torque, normal forces and bending moments were used for strength estimation of the upgraded trailer frame.

Keywords: Air springs characteristics, Damping elements of truck suspension, Light truck trailer suspensions, Suspension analysis of stanchion trailer, Suspension research of light truck trailer, Frame loads of air spring suspension, Air spring experimental research.
1. Introduction

Air springs as damping elements are widely used in automotive industry as suspension elastic subassemblies. The typical air spring is made of a rubber shell and additional steel parts (plates, mountings, valves and bolts). The most popular types are single cylindrical rubber sleeve (the rolling lobe) air springs which can also be found in double air springs or with more of segments on the rubber sleeve. The designers of vehicle suspensions with air springs use the thermodynamic laws, force relationships and mechanical properties of applied materials.

A few proposals of air springs, experimental, theoretical and modelling research as well as complex functional analysis can be found in technical literature. The problems described in literature survey include typical static cases of load for air springs. Research of dynamic cases as well as few papers and books which include practical guidelines for the design engineers and scientific workers have also been presented. The authors [5] point out the accurate air springs model of the suspension system as a main key factor for the correct vehicle performance. In paper [2] the researchers have investigated the damping effectiveness of air spring suspension for the systems using control throttle and additional capacity of air reservoir. The authors of the paper [13] presented a numerical finite element model of the rolling lobe air spring and obtained results which have been compared to the existing analytical model. The authors [13] also carried out an experimental research of rolling lobe air spring with the respect to the vertical stiffness of shell made of reinforced rubber. The finite element method for the processes of air compression and air extension is based on the virtual work principle – it is generally applied for the analytical description of the rigid body. The presented nonlinear FE simulation results give a good correlation with an analytical model which is a reference point for the researchers. The analytical model was built, as a reference, on a base of the thermodynamic laws with the following assumption: the thermodynamic parameters do not depend on the position inside the air spring bag. In work [14], the general analytical model of an air spring was presented. The author [14] proposed an analytical model which represents the main stiffness and hysteresis characteristics of an air spring. The described analytical model of the air spring was connected to the pneumatic model of the system designated for the level position control of the air spring. The mathematical assumption of the model proposed by [14] was based on the same thermodynamics laws as those presented by the authors of paper [13], where it has also been established that the thermodynamic parameters do not vary in relationship to the position inside the air spring space. It was assumed that the compressed and expanded air has properties of the ideal gas. The author of paper [14] omitted the kinetic and potential energies of the air inside the air spring sleeve. It was shown that the air spring stiffness depends on the gas volume changes, heat transfer, air mass fluctuation and the so called effective area. The stability analysis of the analytical model was presented as well as simulation and experimental results were applied for evaluation of the proposed analytical model. The described experimental research of the air spring concerns the stiffness characteristics and hysteresis loops in air spring axial direction only. In paper [4] the authors focused on the dynamic stiffness and damping properties of the air spring with an orifice and auxiliary pressure vessel. The authors also described the theoretical model.
The analytical model of [4] was based on the equations of the energy conservation, gas state and orifice gas flow rate. In addition, the proposed theoretical model of the air spring was investigated with the additional pressure reservoir. During a dynamic experiment the researchers observed the fact that harmonic displacement and load increase are the main cause of the dynamic stiffness phenomenon. The authors also controlled the orifice gas flow of the air spring and observed the effect of the damping properties of the dynamic stiffness. The simulation results were validated by experimental research. As a result of research work [3] the authors proposed a suspension control system for improving the travel comfort and driving manoeuvres. The stability of the control system was numerically simulated by applying the Lyapunov theory. The presented simulation results of the proposed nonlinear control model of the suspension were compared with the linear reference model. In work [17] the authors described the air spring based suspension control system by applying the fuzzy adaptive controller. The active suspensions are becoming increasingly popular in passenger cars nowadays. The results of the presented investigation of the active air spring suspension can be a good basis for the engineers who are dealing with suspension properties modelling, damping or nonlinear material properties modelling [10], [16]. In paper [18] the authors presented the orthogonal analysis and experimental results of the dual-chamber air spring applied as the damping elements in marine structures. The stiffness and damping characteristics of the dual-chamber air spring were investigated and discussed. The volume changes of the dual-chamber air spring were also investigated. Gas flow orifice ratio, excitation amplitude and frequency were researched. The authors [18] proposed the practical model which can be useful in the design of dual-chamber air springs as vibration absorbers. The active suspension system was also investigated by authors in paper [12]. The behaviour of air spring based suspension is controlled by the use of the proportional integral derivative (PID) controller. The mathematical model was prepared for the dynamic model of a single wheel suspension where the passenger seat and numerical simulation were created by using the LABVIEW software. The experimental and simulation results were compared and a new better solution with the PID control was proposed and discussed. In paper [6] the damping properties of the hydropneumatic suspension strut were investigated for static loads. On the basis of the obtained experimental results the authors assumed that the static load changes as well as the speed changes in the investigated ranges do not have any influence on the damping force of the hydropneumatic assembly. The paper [10] deals with the pneumatic elastic clutch where the static nonlinear load characteristics of the investigated coupling were presented. The presented work results contain the mathematical and physical model of the pneumatic clutch and the required physical values were determined by the experimental measurements under static loads. The experimental results were compared to the analytical model and it showed a good correlation. The authors of work [15] examined the mathematical model of the air spring connected to the additional air-pressure absorber. The problem was defined as an optimization of two criteria (acceleration and displacement) and the equations were solved by the Matlab/Simulink system. The flow of air mass between the air spring chamber and the additional air-pressure absorber was described by the Mietluk-Awtuszko model. Applied and described mathematical model was a base for preparing the computational optimization model.
The authors [15] discuss the mathematical problems during the two-criterion optimization process. The results obtained by the same engineering system Matlab/Simulink were compared with the experimental investigation and presented by [8] where the tractor suspension was equipped with hydro-pneumatic elements. The experimental results in paper [7] are interesting for the engineers who are responsible for air-spring operation conditions. Vibrations were measured on the real bus frame with the application of acceleration sensors. On the basis of registered data the authors [7] discussed and showed the possibilities of application of the obtained results as the diagnostic indicators. According to the authors [7] the stability of the bus frame and the technical conditions of the air-spring suspension can be verified. In the chapter of work [9] the authors presented the practical mathematical models and the air springs characteristics which depend on the construction of the double bellows and rolling lobe air spring. The air spring stiffness generally depends on the pressure and capacity changes inside the air spring chamber which allows to adapt adequately the load changes. The distinction between the pressure and the chamber volume changes is described as a polytropic process. The authors of book [9] also presented the valuable practical guidelines for the design engineers who are responsible for the suspension construction where air springs and combined systems are applied. The proposed design methodology can also be useful for the engineers who are interested in analytical modelling of the existing air spring suspension systems. In paper [3] the author described the analytical model of airplane pneumatic suspension under impact load. The obtained results were compared with the numerical simulations and experimental research. Interesting investigation can be found in paper [1] where interaction between the human body and vehicle seat is the base for a new method of vibration analysis. The experimental test and signal analysis are described in one step at time. The authors [1] efficiently applied the wavelet transformations for the signal analysis.

The experimental investigations on the influence of the initial inner pressure and the cooperation requirements (with and without the absorber) on the air spring static and dynamic stiffness have been presented in this paper. The conclusions have been applied in developing the characteristic approximation of the air spring suspension. The authors used the discussed approximation results for improving the boundary support conditions of the numerical model of reduced mass frame in modern trailer.

2. Constructional features of truck trailer and the principle of air spring operation

The single and two-axle cargo trailers are produced by the REDOS company based on the different cross section shapes and steel beam constructions with stanchions and tilt. The support elements of the main trailer frame are two steel beams with a double T cross-section being a base for the other steel elements such as steel bars connected by welding. Trailer structure parts are connected by welding technics. The additional open and closed section profiles, such as steel bars and beams, are the base for the supporting structure for stanchions and trailer floor made of wood sandwich panel. The manufacturer uses the welded closed section steel profiles for trailer fixed shaft construction. The trailer shaft is connected to the trailer frame by the dedicated cold
formed and welded steel brackets. All main trailer structure elements i.e. shaft, floor bars and brackets are connected to the double T-section stringer beams by welding. The offered trailer construction from the REDOS company so far does not differ from the presented standards applied by the other producers of the cargo truck trailers. The whole steel frame construction is protected from the corrosion by zinc coating process. Wheel axis, arms and suspension elements such as air springs and other parts applied in the trailer suspension are commercial solutions offered by specialized producers – Fig. 1. Currently the REDOS company uses two kinds of pneumatic springs which are adequate for single and two-axle trailers. A dependent suspension system, being an integral product, consists of a transverse beam connected to the longitudinal arms by special bolts. The longitudinal suspension arms are connected to the frame brackets and the air springs by the rubber bushing. Bearing journals of the brake drums and wheels are connected to the transverse suspension beams. Two telescopic shock absorbers are damping elements in the trailer suspension.

Figure 1. Dependent suspension equipped with air spring truck trailer axle – applied by the REDOS company: 1 – air spring assembly, 2 – suspension arm, 3 – shock absorber, 4 – frame support, 5 – brake drum assembly

The pneumatic elements, applied in single and two-axle truck trailer suspensions of the REDOS company, are used as elastic parts. The main objective of using these parts is to maintain a constant distance between the trailer frame and road level, stabilize a tilt of the vehicle as well as dissipate the vibration energy with significant action of the hydraulic shock absorbers. The air spring cooperates with the pressure air tank reservoir, the control valve and air compressor which is powered by the truck engine. Two different air springs were tested during the experimental research. The same air springs were applied in the production of the cargo trailers by the REDOS company. Conducted laboratory research consisted of the static and dynamic investigations. The main aim was to determine the static and dynamic characteristics of the investigated air springs which were loaded with the different air pressure levels and gas volume inside of the air spring. As a results of the research, the stiffness characteristics for the different air pressure
values and the different capacities for two different air springs were performed. The experimental results were presented. In addition, the results were used in the application for a new trailer frame design. The presented research results are especially useful as the stiffness coefficient characteristics for different air pressure and volume of two different air springs.

3. Experimental study

Static and dynamic stiffness research of the air springs was made on the test stand equipped with:
- hydraulic impulse generator from MTS,
- computer with special dedicated software for MTS impulse generator control,
- tested smaller air spring installed to MTS impulse generator,
- greater air spring acting as a pressure air reservoir,
- air compressor with the pressure range 0 – 0,5 MPa,
- pneumatic hoses and valves connecting air compressor, the tested air spring and pressure air reservoir.

Air spring test stand is presented below in Fig 2.

![Figure 2. Hydraulic impulse generator MTS with additional equipment](image)

The displacement of the MTS spreader beam was blocked in order to show the actual working conditions of the air springs. The upper handle of the hydraulic machine was connected with the tested air spring with the use of upper plate. The lower movable MTS machine handle, connected to the lower plate of the air spring, was set in a position which allowed to achieve an initial working height \( h_0 \) [mm]. Two variants of initial air spring height namely 300 mm and 381 mm were investigated. The selected air spring
height level during the test was in the range of operating field. Air spring operating force \( Q \) from loaded trailer was done by the air spring pressure under the inner pressure \( p_0 \) and balanced reaction force \( Q \). In the experimental sense the equilibrium state between the external forces and internal air spring pressure was kept adequately. The result of the load application to the air springs was the same as during normal operation of the trailer where the air spring stiffness has an influence on the vibration damping range and the dynamic vibration coefficient value, e.g. the overloads \( \Delta Q \) in vertical direction. The described interpretation is schematically illustrated in Fig. 3.

![Figure 3. Air spring under load – working scheme](image)

During the experimental tests the air pressure inside the reservoir was regulated by the change of air spring height. The tests have been made during the experiments with the use of two settings of the absorber air spring height: 294 mm and 374 mm. As before, the air spring operating level was in a working range. The received test results are compared in Table 1.

In range of both static and dynamic loads two cases were taken into consideration - namely a smaller air spring deflection with and without the connected additional pressure air reservoir. In both experimental cases the dependence between the force \( \Delta Q \) [kN] and the axial air spring displacement \( \Delta l_0 \) [mm] was designated. Due to the sampling rate the dependence between the force and deflection takes each time a discrete character and the angular coefficient of the received regression where the straight line represents the air spring stiffness \( C_M \) [kN/mm].
### Table 1. Experimental parameters for the prepared air springs tests

<table>
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<tr>
<th>TEST TYPE</th>
<th>$p$ [bar]</th>
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The higher air spring was connected as an additional pressure reservoir and locked in one position (axial displacement was blocked). The deformation of the additional pressure reservoir (rolling sleeve type) was possible only in radial direction.

The following external parameters were under control:
- initial pressure $p_0$ [bar] inside the air spring during the test only or in the tested system: air spring - pressure reservoir (additional absorber),
- loading speed (10 mm/60 s and 10 mm/0.05 s, respectively in case of static and dynamic loading),
- initial height of the air spring $l_0$ [mm],
- capacity of the additional pressure reservoir $V_A$ [mm$^3$/S].

4. Experimental results and their interpretation

As an effect of the prepared experimental tests, the obtained results and diagrams are presented below in Fig. 4 and Fig. 5.

![Figure 4. Characteristics of 300 mm height working level of the air spring in relationship to the initial inner air pressure](image)

In Fig. 7 the experimental characteristics of the vertical stiffness for air spring are presented where the initial height of working level was equal $l_0 = 300$ mm - depending on the initial pressure $p_0$ with or without the additional air reservoir connected where the initial height of additional reservoir was equal to 374 mm. The increase of the air spring stiffness can be clearly observed as a function of the initial internal pressure and a
correlation between the initial internal pressure and air spring deflection speed (static test signed as S and dynamic test signed as D).

In case when the tested air spring was connected with the additional pressure reservoir (marked by A374) the air spring stiffness also depends on the reservoir state i.e. the pressure and capacity. When the additional pressure reservoir is connected to the air hoses and air spring then the dynamic stiffness values of the tested air spring is decreasing from D300 (green points on the chart Fig. 4) to D300A374 (violet points on the chart Fig. 4).

The static stiffness values of the tested air spring also decrease from S300 (blue points Fig. 4) to S300A374 (red points Fig. 4). The observed intense decrease of the air spring stiffness has implications in the static load case. This situation can be explained by the longer time required for air flow between the tested air spring and the additional pressure reservoir. In the dynamic load case the inertia of air flow process determines the suspension properties while the stiffness reduction is not effective and significant.

In Fig. 5 the relationship between the air spring stiffness as a function of initial inner air pressure for initial height of the air spring 381mm is presented. The other parameters during the test concur with the research cycle compared in Fig. 5. The results of the conducted analysis show that the dynamic stiffness (Fig. 5 - violet points on the chart marked D381_A374) is significantly greater (more than twice) than the static stiffness (Fig. 5 - red points on the chart marked S381_A374). Comparison of
red and violet points on the charts in Fig. 4 and 5 concludes that the static and dynamic stiffness decrease when the initial height of the air spring increases. It can be seen that the presented plots are linear, however the highest accuracy is given by using of third degree polynomial.

The authors did not define the damping effects because the experimental tests were static and performed slowly. The dynamic test run very slowly like the quasi static process. The damping coefficient is crucial for the dynamic analysis obviously and this type of analysis requires to investigate the problem with speed dependent and changeable damping ratio.

5. Analytical description of forces in suspension

The analytical investigations were basically limited to research cases of uniformly, symmetrically loaded and uniformly displaced for the single-axle trailer suspension. In agreement with the above mentioned the analytical scheme for both wheels is identical. The case of unsymmetrical loads, such as torsional load and deflection of the investigated trailer axle, is a different issue. Another similar problem is the investigation of the behaviour of the strictly dynamic suspension.

For the presented analytical investigation it was assumed that each load distribution affects the air spring suspension parts under the suspension axle load which equals to force 2F, what can be shown in accordance with the scheme in Fig. 6.

![Figure 6. Distribution of forces which are acting on the suspension (load applied to one wheel)](image)

The stability state is fulfilled when:

\[ \sum M_B = 0 \iff -R_A(l_1 + l_2) + Fl_1 + M_w = 0 \]  

and after transformations it gives:

\[ R_A = F \frac{l_1}{l_1 + l_2} + M_w \frac{l_1}{l_1 + l_2} \]

where F is one half of the axial load which is equal to the force of wheel reaction and half of pressure for one axial - the reaction force for one wheel. The symbol of \( M_w \) means the reactive moment of suspension arm of rubber bushing.
Moment $M_w$ depends on the rotation angle of the rubber bushing which can be described as:

$$M_w = f(\varphi)$$

particularly when:

$$f(\varphi) = a\varphi$$

where $a$ is a coefficient of the proportionality between the torque $M_w$ and the rotation angle of rubber bushing $\varphi$. The obtained relationship has a linear character. In this situation it is necessary to notice that the value of reactive torque $M_w$ of rubber bushing changes and reduces during the operation time of the suspension which causes the possibility to skip the torque value $M_w$ in further calculations. The displacement of suspension arm around the rotation centre in normal operation conditions causes a reduction of the torque value $M_w$ so significantly that its importance has no practical relevance even from formal analytical dependencies.

Based on force balance in vertical directions it can be described:

$$R_B = F - R_A = F \frac{l_1}{l_1 + l_2} - a\varphi \frac{l_1}{l_1 + l_2}$$

Denoting by $\Delta z_M$ the increase of the air spring vertical displacement caused by the increment of wheel load $\Delta F$ which can be described by the situation when the investigated trailer drives on a rough road.

$$\Delta z_M = (l_1 + l_2) \cdot \tan \varphi$$

by granting a small value of rotation angle $\varphi$ of suspension arm, $\tan \varphi \approx \varphi$, it gives

$$\varphi = \frac{\Delta z_M}{l_1 + l_2}$$

Based on the balance of torques in relation to trailer suspension arm of frame support:

$$M_w = \Delta F l_1 - \Delta R_A (l_1 + l_2)$$
Then by providing the relations (4) and (7)

\[ a \frac{\Delta z_M}{l_1 + l_2} = \Delta F l_1 - C_M \cdot \Delta z_M (l_1 + l_2) \]  

where \( C_M \) is a static stiffness of the air spring.

The value of the rotation angle of the suspension arm \( \varphi \) is low which consequently gives:

\[ \Delta z_M = \Delta z_2 + \frac{\Delta z_2}{l_1} l_2 = \frac{l_1 + l_2}{l_1} \Delta z_2 \]  

By substituting the dependence (10) to the formula (9), we obtain:

\[ a \frac{1}{l_1 + l_2} \frac{l_1 + l_2}{l_1} \Delta z_2 = \Delta F l_1 - C_M \cdot \frac{l_1 + l_2}{l_1} \Delta z_2 (l_1 + l_2) \]  

Bilateral divide achieved equality by \( \Delta z_2 \), we obtain:

\[ a \frac{1}{l_1} l_1 = \frac{\Delta F}{\Delta z_2} l_1 - C_M \cdot \frac{(l_1 + l_2)^2}{l_1} \]  

By defining the suspension stiffness of II-stage as \( C_2 = \frac{\Delta F}{\Delta z_2} \), the formula can be described in the following form:

\[ C_2 = \frac{a}{l_1} + C_M \cdot \frac{(l_1 + l_2)^2}{l_1^2} \]  

Denoting the stiffness (I-stage of suspension) and tire deflection adequately by \( C_1 \) and \( \Delta z_1 \), it results in:

\[ \Delta F = C_1 \Delta z_1 \Rightarrow \Delta z_1 = \frac{\Delta F}{C_1} \]  

Total suspension deflection can be described as:

\[ \Delta z = \Delta z_1 + \Delta z_2 = \frac{\Delta F}{C_1} + \frac{\Delta F}{C_2} \]  

which after transformation gives:

\[ C_z = \frac{\Delta F}{\Delta z} = \left( \frac{1}{C_1} + \frac{1}{C_2} \right)^{-1} \]

Right side of the equation (16) denoted by \( C_z \) can be called the replacement suspension stiffness.

Friction process in the presented suspension model was completely omitted because this kind of suspension has not got any friction devices for the energy dissipation. Damping process is mainly done by the shock absorbers and slightly by air and heat flow inside the air springs.
6. Numerical model of the air spring suspension for truck trailer

The prepared experimental characteristics of the air spring were the basis for the preliminary prepared model of the trailer suspension. The presented investigation can be applied to the analysis of both single and two-axle trailer suspension due to the symmetric technical solution. Tire pressure and the load influence of the contact between the wheel and the surface was simulated as a rectangle area. The adopted and uniformly distributed vertical forces are presented in Fig. 8. It means that the contact area between the tire and road surface was a rectangular area (approximately 40 mm x 272 mm per one wheel) with the same pressure value (0.23 [MPa] - it allows to obtain the value of load equal 1 [kN] per one axle) on this area.

Bearing of longitudinal suspension arms with rubber bushing was modelled by blocking the linear displacement with the use of the free area around the pivot and thus gaining the angular degrees of freedom.

The experimental characteristics of the air spring stiffness were simulated by applying the elastic support where the vertical stiffness is consistent with the experimental stiffness and takes the values of 0.08, 0.12, 0.16 and 0.18 respectively [kN/mm].

In Fig. 8 the elastic supports are presented as navy blue cones pointed to the air spring base.

![Figure 8. Sets and loads of dependent suspension of the air spring axle](image_url)
Figure 9. Vertical displacements of dependent suspension of the air spring axle under 10 kN load and air springs vertical stiffness equals to:
a) 0.08 [kN/mm], b) 0.12 [kN/mm], c) 0.16 [kN/mm], d) 0.18 [kN/mm].
Figure 10. Relation between the suspension deflection and vertical stiffness for axle under 10 [kN] load

The simulation results of displacement are presented in Fig. 9 – these ones were gained as an effect of 10 kN input load for the axle with the air springs stiffness defined as 0.08 kN/mm, 0.12 kN/mm, 0.16 kN/mm and 0.18 kN/mm.

All received displacements are, in fact, the rotation of the suspension axis around the pivot point of the longitudinal suspension arm pivot. The increase of the air spring stiffness causes the angular displacement reduction. Using the additional application of the static radial stiffness of the tire on the level of 200 N/mm (most tires have the radial static stiffness about 200 N/mm) we can get the tire deflection of about 12.5 mm for the load 10 kN on the axis. On the basis of the results presented in Fig. 10 it can be found that the trailer body vertical displacement, in terms of the road surface, is equal to 53.65 mm with the air spring stiffness on the level of 0.08 kN/mm and is equal to 30.73 mm when the air spring stiffness is on the level of 0.18 kN/mm. It should be mentioned here that the static radial stiffness of the tire has no influence on the rotation of the axis around the pivot point of the longitudinal suspension arm.

The equivalent stiffness for the suspensions of the single-axle cargo trailer can be found by using the following formula:

\[ C_z = \left( \frac{1}{4C_1} + \frac{1}{2C_2} \right)^{-1} \]  

where \( C_1 \) i \( C_2 \) [N/mm] means respectively the tire stiffness and air spring stiffness. On the basis of the experimental results we get the following values of the equivalent
stiffness $C_z$: 0.13, 0.18, 0.23 and 0.25 [kN/mm] respectively for $C_2 = 0.08, 0.12, 0.16$ and 0.18 [kN/mm].

7. Conclusions

Interpretation of the presented experimental results allows to formulate the following conclusions:

- the vertical air spring stiffness is increasing adequately to an initial value of the inner air pressure $p_0$,
- the increase of load speed causes the increase of the air spring stiffness (close to isentropic process),
- the vertical air spring stiffness decreases if the tested air spring is connected to the additional air pressure reservoir,
- the additional air pressure reservoir has an explicit influence on the stiffness under the static loads.

The additional air pressure reservoir which is dynamically loaded has a low influence on the air spring vertical stiffness. The methodology presented in this paper can be helpful for the design engineers when weak-nonlinear characteristics of the investigated dependencies are important and should be applied in practical applications.

The authors presented and applied the nonlinear model of the investigated suspension characteristic. The proposed procedure can be modified and developed for more advanced problems.

Further plans of the investigations include the measurement of accelerations and strains on the new designed structure of the trailer. On the base of the measured values we can investigate and prepare the more advanced mathematical models of the suspension.

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