The Subspace Iteration Method for Eigenproblems of Frames with Viscoelastic Dampers Described Using Internal Variables

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Abstract

In the paper, the subspace iteration method is used to solve quadratic eigenproblems written for structures with viscoelastic dampers. This allows a certain previously assumed number of eigenvalues and corresponding eigenvectors to be determined. A shear frame with mass lumped on the storey level with built-in dampers is considered. Dampers are modelled with the classical Zener model. Equations of motion are written in the matrix form and include the internal variables resulting from built-in dampers. The subspace iteration method starts from adopting the number of sought solution for the quadratic eigenproblem, then the starting point of the iteration is determined and the iterative procedure is initiated. In each iterative loop the reduced quadratic eigenproblem with Hermitian matrices is solved by rearranging it into a state space form.

Keywords: subspace iteration method, Hermitian quadratic eigenproblem, dampers, Zener model

1. Introduction

The calculation of eigenvalues and corresponding eigenvectors is a very important issue in many engineering problems. However, knowledge of all eigenvalues is not necessary from an engineering point of view, especially when the problem dimension is very large. In this case, finding the solution to eigenproblem would be time-consuming or sometimes even impossible, and it would be appropriate to reduce the dimension of the considered task. In the case of a linear eigenproblem, to reduce its dimension, the Lanczos method [1], the Arnoldi method [2] or the subspace iteration method [3] can be used.

The subspace iteration method is applied in this paper. This method can be treated as a combination of the vector iteration method and the Rayleigh-Ritz method. It is extensively described for linear eigenproblems, but its application to nonlinear eigenproblems is included in few papers. In [4], the subspace iteration method is used for complex problems and in [5] for quadratic eigenproblems.

In this paper, the solution presented in [5] is extended to eigenproblems formulated for systems with damping elements described using rheological models. The internal variables are included in the description and are associated with the model used for the damping elements. This work is continuation of an earlier one [6] co-authored by prof. Lewandowski, in which the subspace iteration method was used to solve eigenproblems for frames with dampers, but internal variables were not included.

This paper considers a shear frame with viscoelastic dampers. The formulation of the eigenproblem for such a structure is presented in Section 2. The application of the subspace iteration method for solving quadratic eigenproblems is described in Section 3. A numerical example is calculated in Section 4 and the paper ends with conclusions in Section 5.

2. Equation of motion for frames with dampers

The equation of motion for a structure with dampers with *n* degrees of freedom is written in the following form:

$$\mathbf{M}_s\ddot{\mathbf{q}}_s(t)+\mathbf{C}_s\dot{\mathbf{q}}_s(t)+\mathbf{K}_s\mathbf{q}_s(t)=\mathbf{p}_s(t)+\mathbf{f}_s(t)\,, \tag{1}$$

where $\mathbf{M}_s$, $\mathbf{C}_s$ and $\mathbf{K}_s$ denote the mass matrix, damping matrix and stiffness matrix, respectively. $\mathbf{q}_s(t)$ is the global vector of displacements, $\mathbf{p}_s(t)$ is the vector of excitation forces and $\mathbf{f}_s(t)$ is the vector of the interaction between the structure and dampers (Fig. 1).

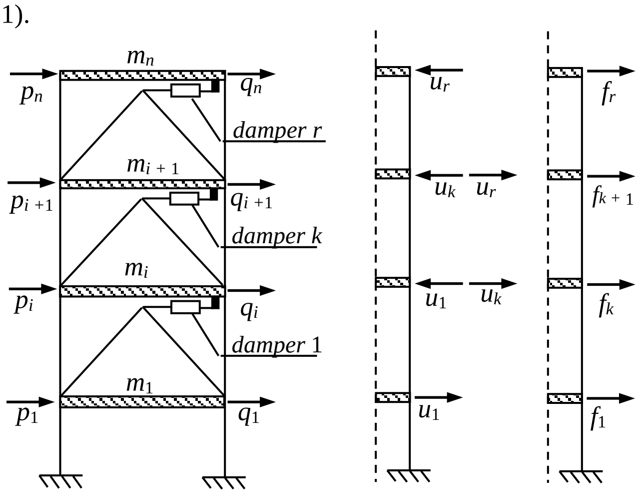

Figure 1. Frame with dampers

In this paper, the Zener model of damper is used (Fig.2) with stiffnesses k_0 and k_1 and damping coefficient c_1. Using the notion of internal variables, the behaviour of the considered model of damper can be described as follows:

$$u_0(t)=k_0\left(q_{i+1}(t)-q_i(t)\right), \tag{2a}$$

$$u_{sp}(t)=k_1\left(q_d(t)-q_i(t)\right)\quad u_d(t)=c_1\left(\dot{q}_{i+1}(t)-\dot{q}_d(t)\right), \tag{2b}$$

where $u_0(t)$ denotes the force in the spring with stiffness k_0, $u_{sp}(t)$ and $u_d(t)$ denote the force in the spring with stiffness k_1 and the force in dashpot element, respectively. The equation of motion (1), as presented in [7], could be written in the following form:

$$\begin{aligned}\mathbf{M}_{ss}\ddot{\mathbf{q}}_s(t)+\mathbf{C}_{ss}\dot{\mathbf{q}}_s(t)+\mathbf{C}_{sd}\dot{\mathbf{q}}_d(t)+\mathbf{K}_{ss}\mathbf{q}_s(t)+\mathbf{K}_{sd}\mathbf{q}_d(t)=\mathbf{p}_s(t)\\ \mathbf{C}_{ds}\dot{\mathbf{q}}_s(t)+\mathbf{C}_{dd}\dot{\mathbf{q}}_d(t)+\mathbf{K}_{ds}\mathbf{q}_s(t)+\mathbf{K}_{dd}\mathbf{q}_d(t)=\mathbf{0}\end{aligned}\,, \tag{3}$$

where $\mathbf{q}_s(t)$ and $\mathbf{q}_d(t)$ are the global vector of nodal displacements of structure and the global vector of internal variables, respectively. $\mathbf{M}_{ss}$, $\mathbf{C}_{ss}$ and $\mathbf{K}_{ss}$ are mass, damping and stiffness matrices with dimension *n* x *n* related to the structure supplemented with elements associated with dampers. Matrices $\mathbf{C}_{dd}$ and $\mathbf{K}_{dd}$ with dimension *r* x *r* describe damping properties and the stiffness of dampers (*r* – number of internal variables that are necessary to describe built-in viscoelastic dampers). Matrices $\mathbf{C}_{sd}$ and $\mathbf{K}_{sd}$ with dimension *n* x *r* illustrate the effects of the coupling of structure and dampers ($\mathbf{C}_{ds}^T = \mathbf{C}_{sd}$, $\mathbf{K}_{ds}^T = \mathbf{K}_{sd}$). The method describing the building of matrices occurring in equations of motion is presented in detail in [7].

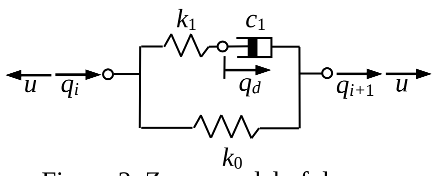

Figure 2. Zener model of damper

After applying the Laplace transformation with zero initial conditions and for $\mathbf{p}_s(t) = \mathbf{0}$, equations of motion can be written as follows:

$$\begin{gathered}\left(s^2\mathbf{M}_{ss} + s\mathbf{C}_{ss} + \mathbf{K}_{ss}\right)\bar{\mathbf{q}}_s(s) + \left(s\mathbf{C}_{sd} + \mathbf{K}_{sd}\right)\bar{\mathbf{q}}_d(s) = \mathbf{0} \\ \left(s\mathbf{C}_{ds} + \mathbf{K}_{ds}\right)\bar{\mathbf{q}}_s(s) + \left(s\mathbf{C}_{dd} + \mathbf{K}_{dd}\right)\bar{\mathbf{q}}_d(s) = \mathbf{0}\end{gathered}, \tag{4}$$

where $\bar{\mathbf{q}}_s(s) = \mathcal{L}\{\mathbf{q}_s(t)\}$, $\bar{\mathbf{q}}_d(s) = \mathcal{L}\{\mathbf{q}_d(t)\}$ and *s* is the Laplace variable. For further analysis, it is more convenient to write equations (4) in the following form:

$$\left(s^2\mathbf{M} + s\mathbf{C} + \mathbf{K}\right)\bar{\mathbf{q}}(s) = \mathbf{0}, \tag{5}$$

where

$$\mathbf{M} = \begin{bmatrix}\mathbf{M}_{ss} & \mathbf{0} \\ \mathbf{0} & \mathbf{0}\end{bmatrix} \quad \mathbf{C} = \begin{bmatrix}\mathbf{C}_{ss} & \mathbf{C}_{sd} \\ \mathbf{C}_{ds} & \mathbf{C}_{dd}\end{bmatrix} \quad \mathbf{K} = \begin{bmatrix}\mathbf{K}_{ss} & \mathbf{K}_{sd} \\ \mathbf{K}_{ds} & \mathbf{K}_{dd}\end{bmatrix} \quad \bar{\mathbf{q}}(s) = \begin{bmatrix}\bar{\mathbf{q}}_s(s) \\ \bar{\mathbf{q}}_d(s)\end{bmatrix}.$$

Eigenproblem (5) has 2*n* + *r* number of solutions (2*n* conjugate complex eigenvalues and eigenvectors and *r* real eigenvalues and eigenvectors [8]).

3. Application of the subspace iteration method to solve eigenproblems for frames with dampers

Equation (5) is rewritten to include all solutions:

$$\mathbf{MQ\Lambda}^2 + \mathbf{CQ\Lambda} + \mathbf{KQ} = \mathbf{0}, \tag{6}$$

where $\mathbf{Q} = \left[\mathbf{q}_1, \mathbf{q}_2, ..., \mathbf{q}_{2n}, ..., \mathbf{q}_{2n+r}\right]$, $\mathbf{\Lambda} = diag\left(s_1, s_2, ..., s_{2n}, ..., s_{2n+r}\right)$.

The subspace iteration method starts with the assumptions of *m* solutions to be determined. In formula (6), matrices $\mathbf{Q}$ and $\mathbf{\Lambda}$ take the following form: $\mathbf{Q} = \left[\mathbf{q}_1, \mathbf{q}_2, ..., \mathbf{q}_m\right]$ and $\mathbf{\Lambda} = diag\left(s_1, s_2, ..., s_m\right)$. The next step is to determine the zero point of iteration as the solution to the linear eigenproblem:

$$\left(s^2\mathbf{M}_{ss} + \mathbf{K}_{ss}\right)\bar{\mathbf{q}}_s(s) + \mathbf{K}_{sd}\bar{\mathbf{q}}_d(s) = \mathbf{0}. \tag{7a}$$

$$\mathbf{K}_{ds}\bar{\mathbf{q}}_s(s) + \mathbf{K}_{dd}\bar{\mathbf{q}}_d(s) = \mathbf{0}. \tag{7b}$$

Equation (7b) is used for static reduction:

$$\bar{\mathbf{q}}_d(s) = -\mathbf{K}_{dd}^{-1}\mathbf{K}_{ds}\bar{\mathbf{q}}_s(s)\,. \tag{8}$$

Substituting (8) into (7a) leads to:

$$\left(\tilde{\mathbf{K}}_{ss} - \omega^2\mathbf{M}_{ss}\right)\bar{\mathbf{q}}_s(s) = \mathbf{0}\,, \tag{9}$$

where $\tilde{\mathbf{K}}_{ss} = \mathbf{K}_{ss} - \mathbf{K}_{sd}\mathbf{K}_{dd}^{-1}\mathbf{K}_{ds}$. The solution of eigenproblem (9) is natural frequencies ω_i and corresponding eigenvectors $\bar{\mathbf{q}}_{si}(s)$ (for $i = 1,..,n$). Next, the vectors of internal variables $\bar{\mathbf{q}}_{di}(s)$ are calculated using formula (8) and vector $\bar{\mathbf{q}}_i(s)$ is written in the following form:

$$\bar{\mathbf{q}}_i(s) = \begin{bmatrix}\bar{\mathbf{q}}_{si}(s)\\ \bar{\mathbf{q}}_{di}(s)\end{bmatrix}. \tag{10}$$

The zero point of iteration is assumed as $s_i^{(0)} = \mathrm{i}\omega_i$ and $\mathbf{q}_i^{(0)} = \bar{\mathbf{q}}_i(s)$. Two matrices are formed:

$$\mathbf{\Lambda}_0 = diag\left(\mathrm{i}\omega_1, \mathrm{i}\omega_2, ..., \mathrm{i}\omega_m\right) \quad \mathbf{X}_0 = \left[\mathbf{q}_1^{(0)}\ \mathbf{q}_2^{(0)}\ ...\ \mathbf{q}_m^{(0)}\right]. \tag{11}$$

The iterative loop *j* starts with the calculation of the set of equations:

$$\mathbf{K}\tilde{\mathbf{X}}_j = \mathbf{P}^{(j-1)}\,, \tag{12}$$

where $\mathbf{P}^{(j-1)} = -\mathbf{M}\mathbf{X}_{j-1}\mathbf{\Lambda}_{j-1}^2 - \mathbf{C}\mathbf{X}_{j-1}\mathbf{\Lambda}_{j-1}$. The solution is complex matrix $\tilde{\mathbf{X}}_j$ that can be written as follows:

$$\tilde{\mathbf{X}}_j = \begin{bmatrix}\tilde{\mathbf{X}}_s^{(j)}\\ \tilde{\mathbf{X}}_d^{(j)}\end{bmatrix} = \begin{bmatrix}\tilde{\mathbf{x}}_{s1}^{(j)} & \tilde{\mathbf{x}}_{s2}^{(j)} & ... & \tilde{\mathbf{x}}_{sm}^{(j)}\\ \tilde{\mathbf{x}}_{d1}^{(j)} & \tilde{\mathbf{x}}_{d2}^{(j)} & ... & \tilde{\mathbf{x}}_{dm}^{(j)}\end{bmatrix}. \tag{13}$$

It is assumed that the new approximation of eigenvectors for eigenproblem (5) is as follows:

$$\bar{\mathbf{q}}_i^{(j)} = \tilde{\mathbf{X}}_j\mathbf{z}_i^{(j)}\,, \tag{14}$$

where $\mathbf{z}_i^{(j)}$ is understood as an unknown Ritz coordinate vector. After substituting (14) into (5) and pre-multiplying by $\tilde{\mathbf{X}}_j^H$ ($\tilde{\mathbf{X}}_j^H$ is complex conjugate of matrix $\tilde{\mathbf{X}}_j$), the following formula is obtained:

$$\tilde{\mathbf{X}}_j^H\left(s^{(j)2}\mathbf{M} + s^{(j)}\mathbf{C} + \mathbf{K}\right)\tilde{\mathbf{X}}_j\mathbf{z}^{(j)} = \mathbf{0}\,, \tag{15}$$

Equation (15) is a reduced Hermitian eigenproblem with dimension *m* x *m* that can be written in the following form:

$$\left(s^{(j)2}\tilde{\mathbf{M}}_j + s^{(j)}\tilde{\mathbf{C}}_j + \tilde{\mathbf{K}}_j\right)\mathbf{z}^{(j)} = \mathbf{0}\,, \tag{16}$$

where

$\tilde{\mathbf{M}}_j = \tilde{\mathbf{X}}_s^{(j)H}\mathbf{M}_{ss}\tilde{\mathbf{X}}_s^{(j)}$

$$\tilde{\mathbf{C}}_j = \tilde{\mathbf{X}}_s^{(j)H}\mathbf{C}_{ss}\tilde{\mathbf{X}}_s^{(j)} + \tilde{\mathbf{X}}_d^{(j)H}\mathbf{C}_{ds}\tilde{\mathbf{X}}_s^{(j)} + \tilde{\mathbf{X}}_s^{(j)H}\mathbf{C}_{sd}\tilde{\mathbf{X}}_d^{(j)} + \tilde{\mathbf{X}}_d^{(j)H}\mathbf{C}_{dd}\tilde{\mathbf{X}}_d^{(j)}$$

$$\tilde{\mathbf{K}}_j = \tilde{\mathbf{X}}_s^{(j)H}\mathbf{K}_{ss}\tilde{\mathbf{X}}_s^{(j)} + \tilde{\mathbf{X}}_d^{(j)H}\mathbf{K}_{ds}\tilde{\mathbf{X}}_s^{(j)} + \tilde{\mathbf{X}}_s^{(j)H}\mathbf{K}_{sd}\tilde{\mathbf{X}}_d^{(j)} + \tilde{\mathbf{X}}_d^{(j)H}\mathbf{K}_{dd}\tilde{\mathbf{X}}_d^{(j)}.$$

$\tilde{\mathbf{M}}_j$, $\tilde{\mathbf{C}}_j$ and $\tilde{\mathbf{K}}_j$ are Hermitian matrices. In order to solve quadratic eigenproblem (16), a state vector is introduced $\mathbf{v} = col\left(\mathbf{z},\ s\mathbf{z}\right)$ and an additional equation is added $s\tilde{\mathbf{M}}\mathbf{z} - s\tilde{\mathbf{M}}\mathbf{z} = \mathbf{0}$. In this way, quadratic eigenproblem (16) is reduced to a linear eigenproblem in the following form:

$$\left(\mathbf{A} - s\mathbf{B}\right)\mathbf{v} = \mathbf{0}, \tag{17}$$

where

$$\mathbf{A} = \begin{bmatrix} -\tilde{\mathbf{K}} & \mathbf{0} \\ \mathbf{0} & \tilde{\mathbf{M}} \end{bmatrix} \text{ and } \mathbf{B} = \begin{bmatrix} \tilde{\mathbf{C}} & \tilde{\mathbf{M}} \\ \tilde{\mathbf{M}} & \mathbf{0} \end{bmatrix}.$$

The solution to eigenproblem (17) is $2m$ conjugate complex eigenvalues $s_i^{(j)}$ and corresponding eigenvectors:

$$\mathbf{v}_i^{(j)} = \begin{bmatrix} \mathbf{z}_i^{(j)} \\ s\mathbf{z}_i^{(j)} \end{bmatrix}. \tag{18}$$

Only those modes are taken to the next iterative loop whose eigenvalues have a positive imaginary part. Eigenvectors $\mathbf{v}_i^{(j)}$ are normalized according to the condition proposed in [5]:

$$\mathbf{v}^T \begin{bmatrix} -\bar{\mathbf{K}} & \mathbf{0} \\ \mathbf{0} & \bar{\mathbf{M}} \end{bmatrix} \mathbf{v} = 2\mathbf{\Lambda}^2, \tag{19}$$

where $\bar{\mathbf{K}} = \tilde{\mathbf{X}}_j^T\mathbf{K}\tilde{\mathbf{X}}_j$, $\bar{\mathbf{M}} = \tilde{\mathbf{X}}_j^T\mathbf{M}\tilde{\mathbf{X}}_j$. The matrix $\mathbf{Z}^{(j)}$ can be written in the following form $\mathbf{Z}^{(j)} = \left[\mathbf{z}_1^{(j)}\ \mathbf{z}_2^{(j)}\ \dots\ \mathbf{z}_m^{(j)}\right]$.

Linear eigenproblem (17) with Hermitian matrices is solved using the QZ algorithm implemented in MATLAB.

Eigenvalues $s_i^{(j)}$ are the approximation of the solution to eigenproblem (5) and create the following matrix:

$$\mathbf{\Lambda}_j = \mathrm{diag}\left(s_1^{(j)}, s_2^{(j)}, \dots, s_m^{(j)}\right). \tag{20}$$

The new approximation of eigenvectors can be computed as follows:

$$\mathbf{X}^{(j)} = \tilde{\mathbf{X}}_j\mathbf{Z}^{(j)}, \tag{21}$$

where

$$\mathbf{X}^{(j)} = \begin{bmatrix} \mathbf{X}_s^{(j)} \\ \mathbf{X}_d^{(j)} \end{bmatrix} = \begin{bmatrix} \bar{\mathbf{q}}_{s1}^{(j)} & \bar{\mathbf{q}}_{s2}^{(j)} & \dots & \bar{\mathbf{q}}_{sm}^{(j)} \\ \bar{\mathbf{q}}_{d1}^{(j)} & \bar{\mathbf{q}}_{d2}^{(j)} & \dots & \bar{\mathbf{q}}_{dm}^{(j)} \end{bmatrix}.$$

The iteration process is completed when the following conditions are fulfilled:

$$\left| s_i^{(j)} - s_i^{(j-1)} \right| \le \varepsilon_1 \left| s_i^{(j)} \right|, \quad \left\| \bar{\mathbf{q}}_i^{(j)} - \bar{\mathbf{q}}_i^{(j-1)} \right\| \le \varepsilon_2 \left\| \bar{\mathbf{q}}_i^{(j)} \right\|, \quad \text{for } i = 1, 2, ..., m. \tag{22}$$

In order to check the correctness of the obtained solutions, the following condition is calculated:

$$\left\| \mathbf{r}_{1,i} \right\| \le \varepsilon_3 \left\| \mathbf{r}_{2,i} \right\|, \text{ for } i = 1, 2, ..., m, \tag{23}$$

where

$$\mathbf{r}_{1,i} = \left(s_i^2 \mathbf{M} + s_i \mathbf{C} + \mathbf{K} \right) \bar{\mathbf{q}}_i, \quad \mathbf{r}_{2,i} = \mathbf{K} \bar{\mathbf{q}}_i,$$

ε_1, ε_2 and ε_3 are the assumed accuracies of calculation.

4. Numerical example

In order to check the correctness of the presented method, a fifteen-storey frame with built-in viscoelastic dampers is investigated. The stiffness of storeys varies in the following sequence: $k_1 = k_2 = k_3 = 10000$ kN/m, $k_4 = k_5 = k_6 = k_7 = 8000$ kN/m, $k_8 = k_9 = k_{10} = k_{11} = 7000$ kN/m, $k_{12} = k_{13} = k_{14} = k_{15} = 5000$ kN/m. Mass is the same on each floor $m = 42000$ kg. There are nine dampers, on floors: 3, 4, 5, 8, 9, 10, 13, 14 and 15. Dampers are modelled by the Zener model with the following parameters: $k_0 = 300000$ N/m $c_1 = 30000$ Ns/m and $k_1 = 500000$ N/m. The first three natural frequencies for frame with dampers are as follows: $\omega_1 = 1.73$ rad/s, $\omega_2 = 3.97$ rad/s, $\omega_3 = 6.52$ rad/s. Damping in the structure is neglected. The analysis is carried out for two different values of *m*: 2 and 6. In both cases the obtained results are very close to the exact solution. Results are presented in Tables 1-3. These tables first present the comparison of eigenvalues obtained using the subspace iteration method with the exact solutions and values of $\bar{\varepsilon}_{3,i}$ (Table 1). Secondly, the tables show the comparison of eigenvalues, values of $\bar{\varepsilon}_{3,i}$ (Table 2) and comparison of two selected eigenvectors (Table 3). Calculations are carried out for assumed accuracies: $\varepsilon_1 = 10^{-4}$ and $\varepsilon_2 = 10^{-4}$ for eigenvalues and eigenvectors, respectively. The actual value of $\bar{\varepsilon}_{3,i}$ (resulting from condition (23) for the *i*-th mode) testifies to the correctness of the obtained solutions. It increases for successive eigenvalues, but still takes a very small value.

Table 1. Results of eigenvalues for *m* = 2, number of iterations: 4

mode	Exact solution	Subspace iteration method	$\bar{\varepsilon}_{3,i} = \left\| r_{1,i} \right\| / \left\| r_{2,i} \right\|$
1	-0.0021+1.4279i	-0.0021+1.4279i	$1.4 \cdot 10^{-6}$
2	-0.0173+3.9653i	-0.0173+3.9652i	$7.8 \cdot 10^{-5}$

Table 2. Results of eigenvalues for *m* = 6, number of iterations: 46

mode	Exact solution	Subspace iteration method	$\bar{\varepsilon}_{3,i} = \lVert r_{1,i} \rVert / \lVert r_{2,i} \rVert$
1	-0.0021+1.4279i	-0.0021+1.4279i	$7.0 \cdot 10^{-14}$
2	-0.0173+3.9653i	-0.0173+3.9653i	$2.7 \cdot 10^{-15}$
3	-0.0483+6.5188i	-0.0483+6.5188i	$9.5 \cdot 10^{-15}$
4	-0.0930+9.1636i	-0.0930+9.1636i	$6.5 \cdot 10^{-14}$
5	-0.1273+11.4500i	-0.1273+11.4500i	$3.1 \cdot 10^{-9}$
6	-0.1517+13.8297i	-0.1520+13.8298i	$2.8 \cdot 10^{-5}$

Table 3. Eigenvectors $\mathbf{q}_1$ and $\mathbf{q}_6$ for *m* = 6

Number of degrees of freedom	mode 1		mode 6	
	Exact solution	Subspace iteration method	Exact solution	Subspace iteration method
1	0.0775+0.0003i	0.0775+0.0003i	-0.8930-0.0340i	-0.8930-0.0339i
2	0.1544+0.0005i	0.1544+0.0005i	-1.0693-0.0250i	-1.0693-0.0249i
3	0.2276+0.0005i	0.2276+0.0005i	-0.4192+0.0053i	-0.4192+0.0054i
4	0.3163+0.0003i	0.3163+0.0003i	0.7798+0.0303i	0.7797+0.0303i
5	0.4016+0.0001i	0.4016+0.0001i	1.2421+0.0321i	1.2421+0.0320i
6	0.4859+0.0004i	0.4859+0.0004i	0.4872-0.0113i	0.4872-0.0114i
7	0.5650+0.0007i	0.5650+0.0007i	-0.7572-0.0541i	-0.7571-0.0541i
8	0.6450+0.0005i	0.6450+0.0005i	-1.2731-0.0069i	-1.2730-0.0068i
9	0.7174+0.0003i	0.7174+0.0003i	-0.4266+0.0324i	-0.4266+0.0325i
10	0.7814+0.0001i	0.7814+0.0001i	0.8759+0.0320i	0.8759+0.0320i
11	0.8386+0.0003i	0.8386+0.0003i	1.2679+0.0190i	1.2679+0.0189i
12	0.9044+0.0005i	0.9044+0.0005i	-0.2194-0.0744i	-0.2193-0.0745i
13	0.9518+0.0002i	0.9518+0.0002i	-1.2494+0.0028i	-1.2494+0.0029i
14	$0.9839+8.3 \cdot 10^{-5}$i	$0.9839+8.3 \cdot 10^{-5}$i	-0.4579+0.0338i	-0.4579+0.0338i
15	1.0+0.0i	1.0+0.0i	1.0+0.0i	1.0+0.0i
16	0.1549+0.0068i	0.1549+0.0068i	-0.8199+0.3103i	-0.8197+0.3105i
17	0.2283+0.0080i	0.2283+0.0080i	0.0561+0.6111i	0.0562+0.6118i
18	0.3169+0.0076i	0.3169+0.0076i	0.9670+0.2607i	0.9671+0.2605i
19	0.5655+0.0075i	0.5655+0.0075i	-0.9905-0.2912i	-0.9900-0.2918i
20	0.6455+0.0066i	0.6455+0.0066i	-0.9483+0.4296i	-0.9482+0.4303i
21	0.7186+0.0057i	0.7186+0.0057i	0.1035+0.6793i	0.1033+0.6800i
22	0.9048+0.0045i	0.9048+0.0045i	-0.6768-0.5546i	-0.6751-0.5556i
23	0.9521+0.0030i	0.9521+0.0030i	-0.9428+0.4086i	-0.9432+0.4094i
24	0.9840+0.0015i	0.9840+0.0015i	0.1519+0.7443i	0.1511+0.7448i

The results of an extensive calculation indicate that the number of eigenvalues that can be determined depends on the damping level. In the case of a higher damping for a larger number of *m*, the method is not convergent. However, for the examples under consideration, *m* represents at least 30% of all degrees of freedom associated with the structure, so in many cases this number will be sufficient for further dynamic analysis.

5. Conclusions

In this paper the subspace iteration method was used to solve eigenproblems formulated for structures with viscoelastic dampers. The classical Zener model was used to describe the dampers, which included internal variables. The numerical example showed that it is possible to determine only a specific number of eigenvalues and corresponding eigenvectors with good approximation. The subspace iteration method required the reduced quadratic Hermitian eigenproblem occurring in each iterative loop to be solved. It was solved using the state space model. An element of novelty was obtaining solutions using the subspace iteration method for systems whose description included internal variables.

The number of eigenvalues and corresponding eigenvectors that can be determined depends on the damping level.

The discussed example showed that the subspace iteration method can be used to analyse the presented structures. Work will continue for systems in which the damping elements are described using fractional derivatives. Studies on improving the convergence of this method are also planned.

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