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Analysis of the Possibility of Shaping the Sound Diffusion Coefficient of Diffusers Based on Acoustic Metamaterials

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Abstract The article presents the analysis of the possibilities of using acoustic metamaterial to build sound diffusers. The diffusers composed of slots loaded with a curved quarter-wave resonator were investigated. Such solutions induce dispersion and the slow sound effect to increase the effective depth of the quarter-wave resonator. The sound diffusion coefficient was calculated with the use of numerical models of diffusers. Next, obtained results were compared with the Schroeder diffuser. In conclusion, it was shown, that acoustic metamaterials affect the frequency band of the diffuser and the effectiveness of the obtained sound diffusers depend on relative width of slits.

Keywords: metamaterial, sound diffusion, diffusion coefficient.

1. Introduction

The sound diffusers ensure the even distribution of acoustic energy both in space and in time. Diffusers are commonly used in room acoustics to first reflection control and to eliminate acoustic defects such as an echo, flutter echo or colouration of the sound as a result of mirror reflections from hard surfaces [1]. Diffusers are widely used in control rooms, recording studios, auditoriums, and concert halls. Contrary to sound-absorbing systems, diffusers enable the preservation of acoustic energy in the room.

The main function of a diffuser is to scatter the reflected wave in many different directions. Diffusers are specialized panels that are a system of slots acting as quarter-wave resonators. The variation in the depth of the slits causes local delays in the phases of waves reflected from the bottom [2]. Designing the sound scattering is therefore the designation of an appropriate sequence of random numbers and the corresponding slit depths. The most commonly used is the QRD sequence. The lower frequency limit for sound scattering depends on the resonant frequency f_0 of the quarter-wave resonator $f_0 = c_0/4L$, where L is the largest depth and c_0 is the sound frequency in the air. This means, that obtaining low-frequency diffusion requires the construction of large-thickness diffusers.

The use of acoustic metamaterials with spatial diffusers introduces strong dispersion in slits. Such structures provide properties that are not found naturally in materials, such as negative effective bulk modulus, negative mass density [3], or slow-sound [4]. It enables obtaining maximum phase delay at low frequencies for relatively thin structures. There are known numerical analyses of diffusing systems based on metamaterial structures composed of slots loaded with Helmholtz resonators [5].

The presented research is a continuation of the work presented in the article [6], where the author presented the possibilities of numerical modelling of sound diffusion of diffusers based on acoustic metamaterials composed of slits loaded with quarter-wave resonators. The purpose of this article is to analyze the possibilities of using acoustic metamaterials composed of a slit loaded with a curved quarter-wave resonator SL+QR in the construction of sound diffusers. Such a structure allows the dimensions of the cells can be minimized and the relative width of the slot outlet to the air increased. As a result, it will allow to obtain better sound diffusion properties. The article also presents the possibility of using QRD sequence [1] to design diffusers based on metamaterials.

The applied structure of metamaterials is characterized by strong dispersion and reduces the phase velocity of the wave propagating in the material. This phenomenon is typical for metamaterials and provides a long delay in the phase of the reflected wave at lower frequencies. Consequently, it allows sound dispersion at lower frequencies compared to the classic QRD diffuser. A computational model of the diffuser was developed using the finite element method (FEM) to calculate the diffusion. The dimensions of the cells were tuned to obtain a diffusion of sound over a wide frequency range. The results were compared with a QRD diffuser with the same cell dimensions.

2. Numerical model of sound diffuser

2.1. Calculations of the sound diffusion coefficient

The quality of sound diffusion by diffusers was assessed by the uniformity of angular distribution of sound pressure over the diffuser. For a panel with a finite width of $2b$, the far-field angular pressure distribution $p_s(\theta)$ can be obtained from the Fraunhofer formula [2]:

$$p_s(\theta) = \int_{-b}^{b} R(x) e^{jk_0 x \sin\theta} dx, \tag{1}$$

where θ – angular coordinate k_0 – wave number in the air, R – reflection coefficient. The reflection coefficient R can be determined by the Transfer Matrix Method (TMM) [7-9] or Finite Element Method (FEM). In the article sound pressure $p_s(\theta)$ was calculated using the FEM. The author indicates in his previous work [6] that this method allows to obtain more accurate results. Diffusion coefficient *d,* which informs about the quality of sound diffusion was calculated according to the formula:

$$d_\theta = \frac{\left(\sum |p_s(\theta)|\right)^2 - \sum |p_s(\theta)|^2}{(n-1)\sum |p_s(\theta)|^2} \tag{2}$$

where n – number of analyzed points in space above the diffuser.

Because the diffusion coefficient depends on the dimension of the diffuser, the normalized diffusion coefficient without the effect of scattering at the edge was calculated. In this case, the diffusion coefficient d_n was given by

$$d_n = \frac{d_\theta - d_p}{1 - d_p} \tag{3}$$

where: d_p – diffusion coefficient of the reference plane, d_θ - diffusion coefficient of the sound diffuser.

Other important parameter which was analysed is the phase shift φ introduced by of the metamaterial cells and the slits. This parameter informs about the delay of the reflected sound wave to the incident wave. The phase shift φ were determined as an argument of the complex sound reflection coefficient R:

$$R = \frac{p_r}{p_i} \tag{4}$$

where p_r – sound pressure on the top surface of cell, p_i – incident sound pressure on the top surface of cell.

2.2. Analysis of cells

In diffusers based on a phase change of sound reflection, the individual cells should introduce random phase shifts over a wide frequency range to obtain a high sound dispersion factor. Slits for the construction of QRD diffusers and a metamaterial composed of slits loaded with curved quarter-wave resonators (SL+QR)[10] were tested (see Fig. 1a, b).

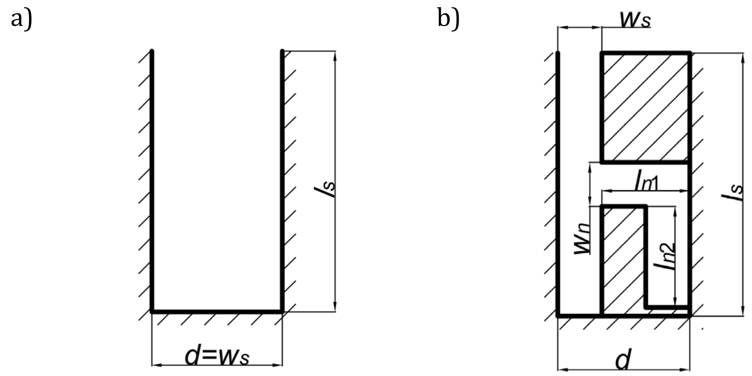

Figure 1. Models of the cells: a) slit, b) slit loaded of the curved quarter wavelength resonator (SL+QR).

The possibilities of shaping the phase shift by SL+QR cells were assessed based on a comparison with the phase shifts introduced by the slits. The dimensions of the cell and the slot with a width of d = 50 mm and a depth of h = 100 mm was adopted for the analysis. The influence on the change of the relative width of the slit w_s/d was also investigated.

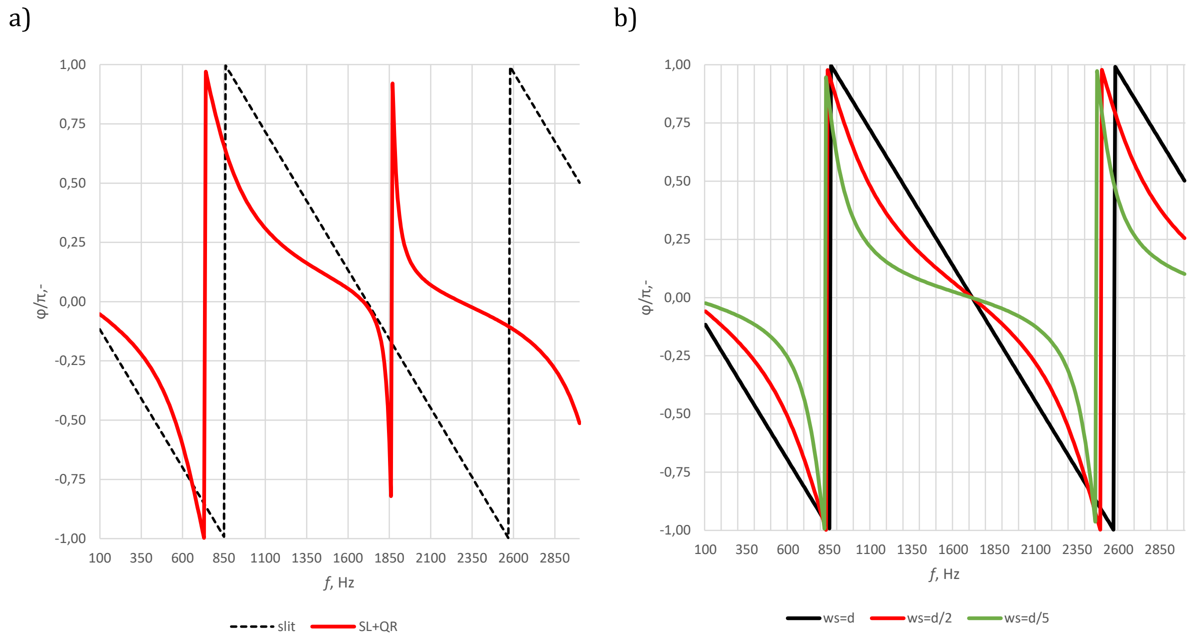

Figure 2. Comparison of phase shifts for SL + QR cells and the slit (a), and analysis of the influence of the relative slit width on the phase shift for w_s/d = 0.2, 0.5 and 1 (b).

The analysis of the reflection coefficient phase showed that the use of SL+QR type metamaterials enables the reduction of the frequency for which the reflection phase is reversed. For the slot, the phase reversal occurs at 860 Hz, for SL+QR at 740 Hz (Fig. 2a). It can be concluded from this that loading the slit with a resonator allows lowering the lower frequency for which it is possible to obtain a large phase shift. The use of such cells for the construction of a diffuser will therefore translate into better sound dispersion at low frequencies. In addition, the introduction of the curved QR resonator into the slot allows for changes in the reflection phase in narrower frequency ranges, which results from the greater number of natural frequencies of such a system. On the other hand, it should also be noted that for a metamaterial the range of large phase shifts is narrow, which is because the outlet of the slit is also narrow compared to the entire width of the cell. This is confirmed by the analysis (Fig. 2b) of the dependence of the phase shift for a slit with a constant depth (h = 100 mm) on the variable width of the outlet w_s/d. Thus, the narrowing of the slits similarly narrows the frequency range in which large phase shifts are obtained, which may affect the sound diffusion coefficient.

3. Numerical analysis of sound diffusers

To compare the sound diffusion coefficients of diffusers made of an acoustic metamaterial and a QRD diffuser, numerical FEM models of the entire diffuser were built.

Table 1. Dimensions of the sound diffuser SL+QR.

Parameter							
N	1	2	3	4	5	6	7
w_s [mm]				25.00			
w_n [mm]				16.70			
l_s [mm]	30.00	47.50	100.00	65.00	65.00	100.00	47.50
l_{n1} [mm]				25.00			
l_{n2} [mm]	1.00	10.25	38.00	19.50	19.50	38.00	10.25

The analyzed diffusers consisted of N = 7 cells with dimensions of d = 50 mm and l_s = 100 mm. The different phase shifts for individual cells were obtained by selecting the characteristic dimensions of the cells, which are presented in Table 1. For the diffuser based on SL+QR, the individual depths of the slots l_s and lengths of QR resonators l_{n2} were determined as for the QRD diffusers according to the quadratic residue sequence. The sequence number s_n for n-th slit is given by $s_n = n^2 \text{modulo} N$. On the other hand, the constant width of the slots $w_s = d/2$ and the resonator $w_n = d/3$ were assumed. The QRD and SL+QR sound diffuser cross-sections are shown in Fig. 3.

To compare the diffusion coefficient of the SL+QR diffuser with the QRD diffuser, its model was built based on the number N = 7, with the maximum depth of the slots l_s = 100 mm and the width of the slots w_s = 50 mm. The dimensions of the diffuser are given in Table 2.

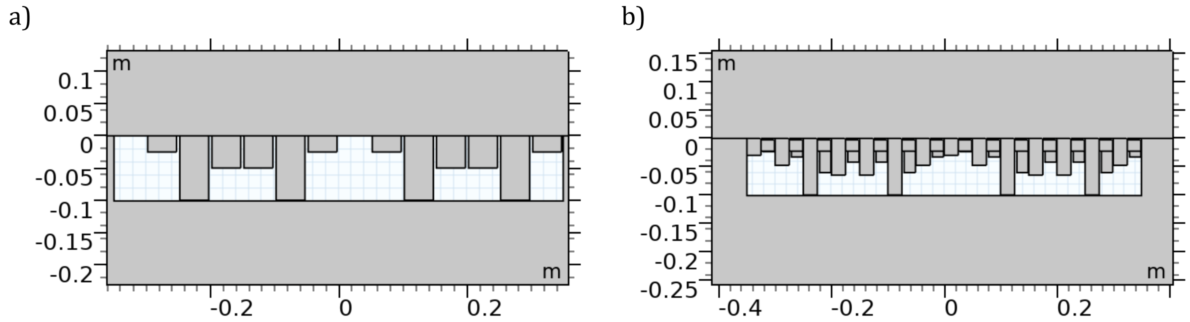

Figure 3. Sound diffuser cross-sections: a) QRD, b) SL+QR.

Table 2. Dimensions of the sound diffuser QRD.

Parameter							
N	1	2	3	4	5	6	7
w_s [mm]				50,00			
l_s [mm]	0.0	25.0	100.0	50.0	50.0	100.0	25.0

3.3. Numerical models of diffusers-FEM

The FEM model was developed in COMSOL Multiphysics. A 2D model was built, consisting of a circular main domain with a diameter of 4 m filled with air (Fig. 4a). In the middle of the domain were placed two periods of sound diffuser consisting of N = 7 cells (Fig. 4b). The diffuser boundary was rigid. In narrow slits, thermo-viscous losses were taken into account. At the outer edge of the main domain, the boundary condition of radiating a cylindrical wave was adopted to ensure free field conditions. The sound pressure of the reflected wave $p_s(\theta)$ was extrapolated to $r = \infty$ for the diffuser and the reflective surface, respectively. Then, according to formulas (2) and (3), the diffusion coefficients d_θ and d_p and d_n were calculated.

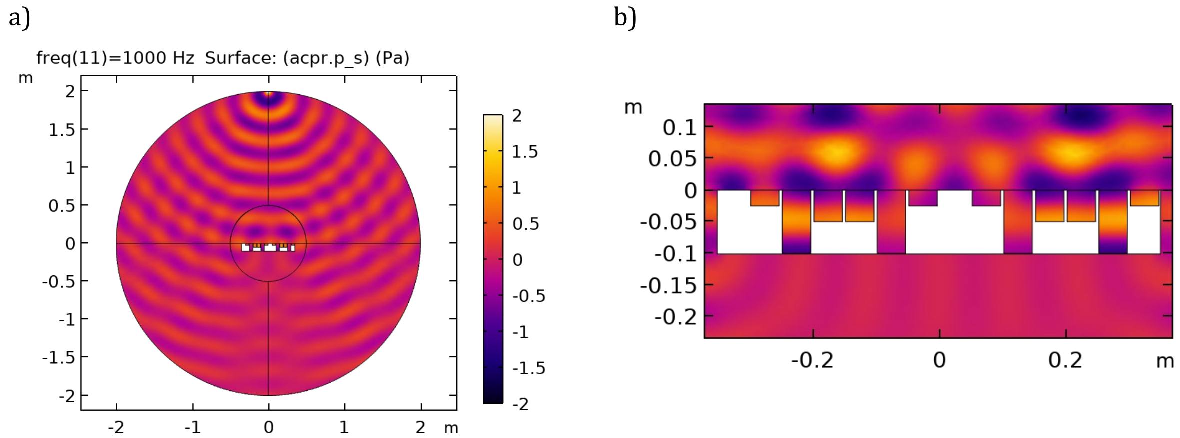

Figure 4. Model of the sound diffuser: a) slits placed in the main circular domain filled with air, b) slits with rigid boundary and filled with air including thermos-viscous losses.

3.3. Discussion

A comparison of the normalized sound diffusion coefficient d_n for QRD and SL+QR diffusers shows that metamaterials allow obtaining scattering at lower frequencies than QRD (Fig. 5d).

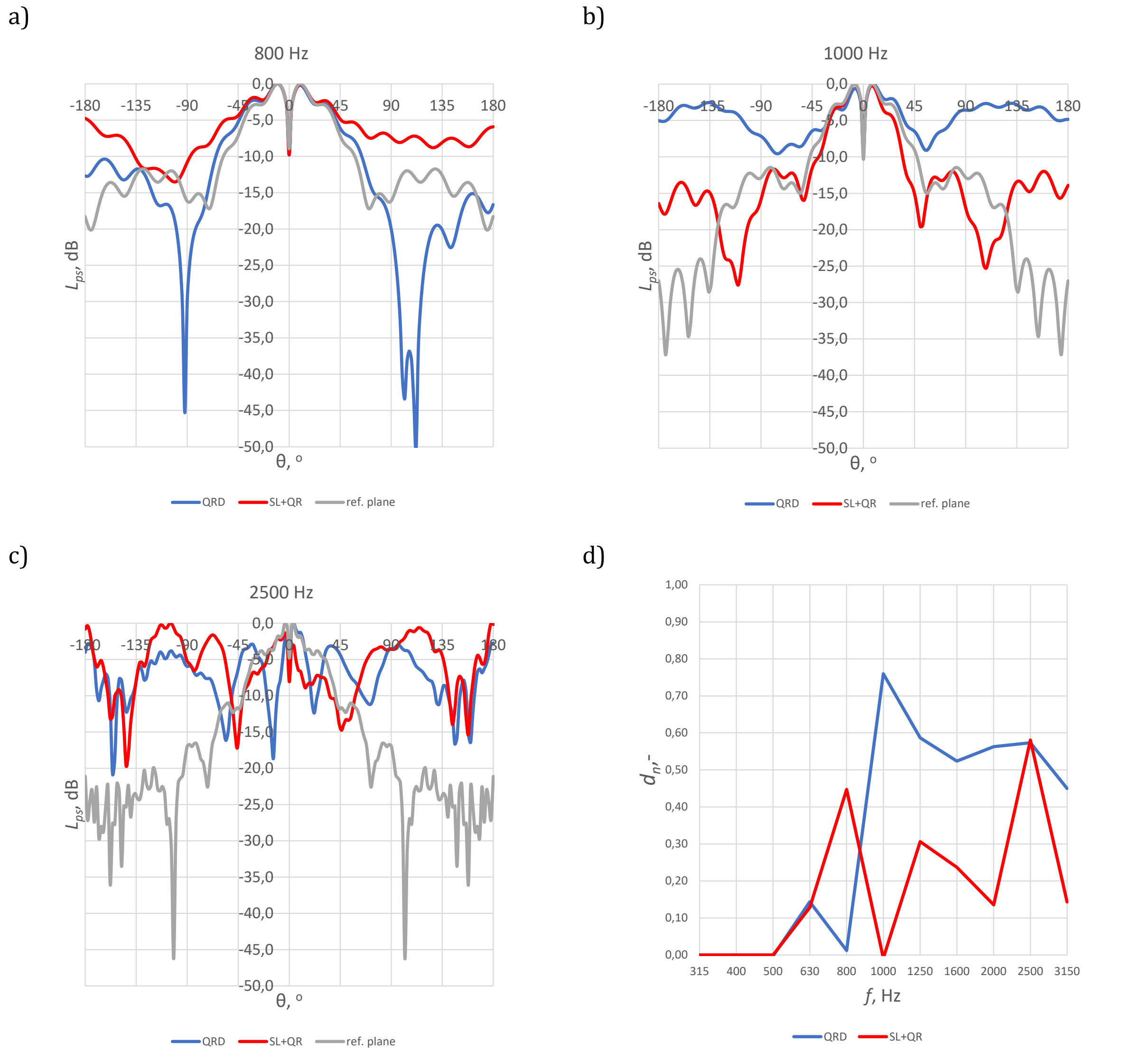

Figure 5. Reflected sound pressure level L_{ps} in frequencies: a) 800 Hz, b) 1000 Hz, c) 2500 Hz; d) comparison of the normalized diffusion coefficient.

Diffusion at lower frequencies is also confirmed by comparing the angular distributions of the reflected pressure levels L_{ps} by the diffuser SL+QR and QRD (Fig. 5a,b). For the SL+QR diffuser at 800 Hz, there are additional side lobes for large angles of reflection (Fig. 5a). Also, the analysis of the reflected sound pressure level distribution (Fig. 6a) for f = 800 Hz over the diffuser surface confirms that sound scattering takes place in this frequency band. Significantly greater changes in the pressure value on the diffuser surface can be observed than in the case of the QRD (Fig. 6b). On the other hand, for 1000 Hz, the distribution of acoustic pressure over the diffuser surface shows that the QRD diffuser achieved better diffusion(Fig. 6c,d). For the frequency f = 2500 Hz, the SL+QR diffuser also obtained high values of the diffusion coefficient (Fig. 5c,d), which confirms the possibility of using metamaterials to build broadband sound diffusers.

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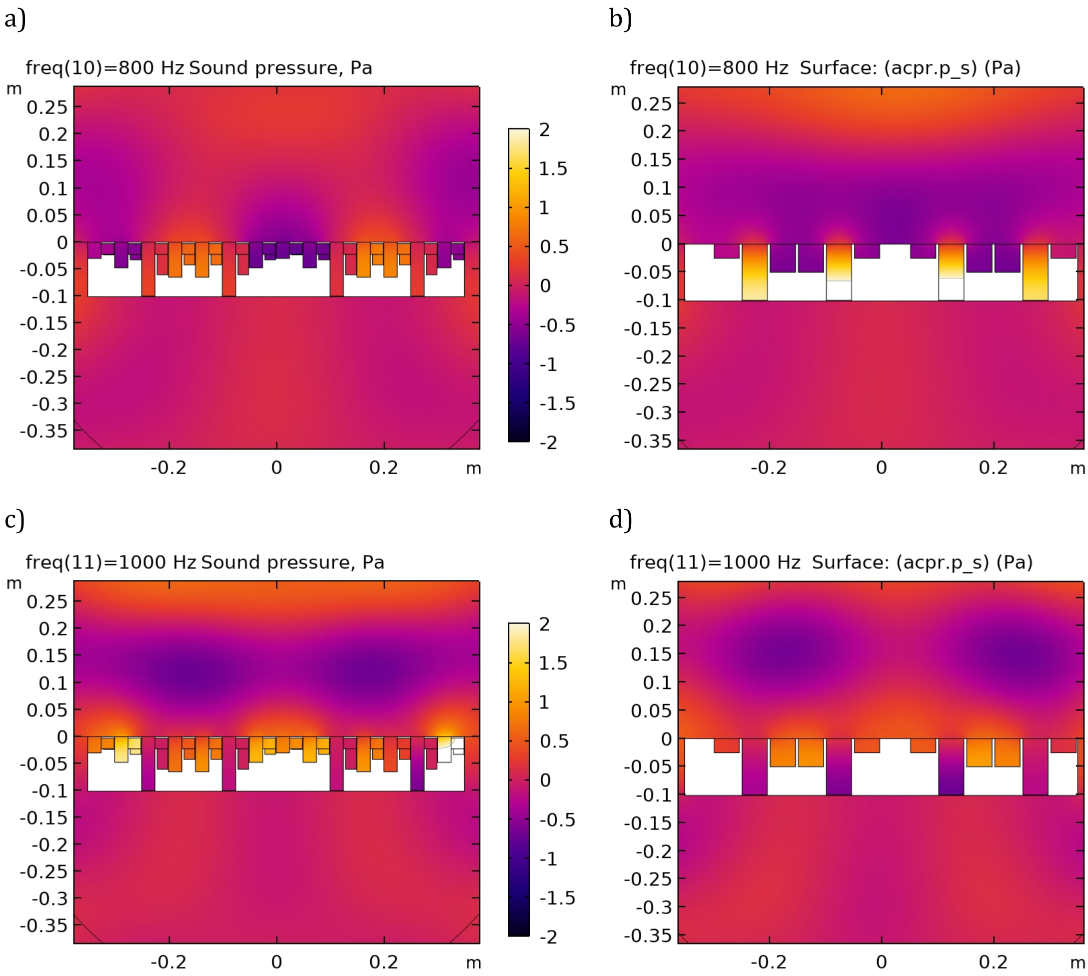

Figure 6. Comparison of the sound pressure level L_{ps} over surface of SL+QR diffuser in frequencies a) 800 Hz, c) 1000 Hz and QRD diffusers in frequencies b) 800 Hz, d) 1000Hz.

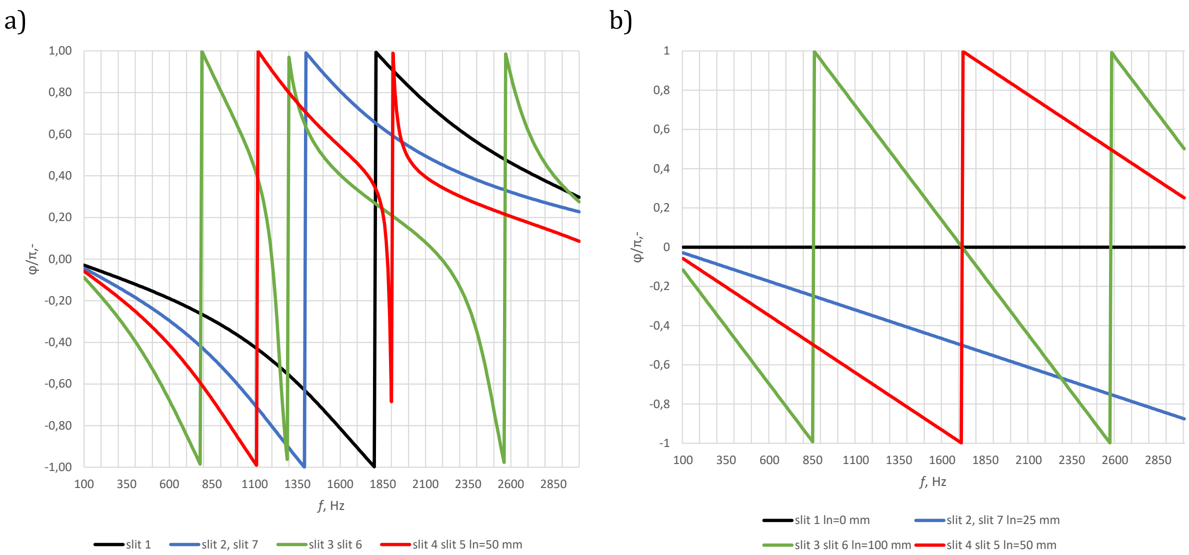

Figure 7. Phase shifts for individual diffuser cells: a) SL+QR, b) QRD.

A significant limitation of the metamaterials used is that in the operating band of the QRD diffuser f > 1000 Hz, the SL+QR diffuser obtained lower d_n values. This may be the result of improperly selected phase shifts of individual cells. Also, the analysis of the reflection phase of QRD and SL+QR diffusers confirms that QRD allows obtaining large phase shifts in a wide frequency range (fig. 7b), while for SL+QR the phase shifts change much faster (fig. 7a), which is the result of the fact that such a system has many more natural frequencies dependent on the dimensions of the cell.

The relative slit width in w_s/d also has a significant impact on the values of the coefficient. For a QRD diffuser w_s/d = 1, therefore, it obtains a large differentiation of the reflection phase on the surface. On the other hand, the surface of the diffuser made of metamaterial is largely a flat surface, which in effect reduces sound diffusion.

4. Conclusions

The article presents an analysis of the possibilities of using metamaterials to build sound diffusers. It has been confirmed that the resonators connected to the slit introduce dispersion, which enables to obtain phase delays of the reflected sound for lower frequencies than for QRD diffusers. This allows for the design of sound diffusers for lower frequencies.

The metamaterial cells, which had wider slit outlets to air, obtain a large differentiation of the phase of reflection coefficient and higher values of the sound diffusion coefficient. On the other hand, the cell with a narrow slit is largely a flat surface, which no phase delay introduces and in effect reduces sound diffusion. It indices that when designing diffusers based on metamaterials, the relative width of the slot outlet to the air should be maximized.

It has also been shown that it is possible to use a QRD sequence in the design of the sound diffuser based on metamaterials. According to this sequence the depth of the slots and the length of the QR resonators were determined. Assuming that the slit and resonator widths are constant, this method allows for simpler design of sound diffusers compared to the time-consuming design using optimization.

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