VPS VIBRATIONS IN PHYSICAL SYSTEMS

DOI: 10.21008/j.0860-6897.2022.3.14
Vibrations in Physical Systems, 2022, 33(3), 2022314

Sound Transmission Loss Calculation for Metamaterial Plate Using Combined Analytical and Numerical Approach

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Abstract In recent years acoustic metamaterials are broadly investigated in many different fields of acoustics and one of them is noise and vibration mitigation. The solution with highest potential are locally resonant metamaterials (LRS), which by creation of band gap effect in flexural wave propagation in structure improve its Sound Transmission Loss (STL). Standard STL simulation procedures can be fully analytical or numerical. Analytical solution, when it comes to metamaterial modelling, is fast but it does not take into consideration metamaterial geometry. On the other hand numerical solution even when considering small part of periodic structure, is time consuming and can generate numerical errors related for example to the mesh. In this work combined analytical – numerical method is analysed as the alternative for STL calculation. This method can be a substitute for basic simulation procedures concerning vibro-acoustic metamaterials, since the simulations results are comparable and it is less time consuming method. Formulas and simulation procedure for the presented method are described and compared with analytical and numerical simulation results as well as with STL measurement results.

Keywords: locally resonant structures, metamaterials, sound transmission loss, simulations, diffuse field.

1. Introduction

Vibro-acoustic metamaterials are periodic structures which exhibit improved acoustic properties in sound and vibration insulation. An important aspect of metamaterials is their lightweight structure and compact dimensions, especially when it comes to improvement of acoustic properties in low frequency region [1]. The metamaterials with locally resonant structures (LRS) were firstly exploited only for vibration mitigation but in recent years, the dependencies between stop band behavior in vibrations and sound reduction of the structure are widely investigated [2-6].

In order to investigate the effectiveness of a metamaterial structure one can perform simulations. Wave propagation in solid body simulations both analytical as well as numerical (with 3D models) are possible and quite easy to perform. Considering the vibroacoustic metamaterials the most suitable are dispersion curves [5, 6] or Frequency Response Functions simulations [7]. In case of acoustic properties simulations of the LRS connected to a solid body the numerical simulations become a complex problem. The necessity of bounding acoustic and solid objects in simulation significantly increases the computational complexity of the model. For structures with small dimensions like metamaterials creating proper mesh and conducting the simulation can be difficult. In literature most of the presented simulation results are limited to analytical solution – locally resonant structure is assumed to be a mass-spring system [8]. Occasionally numerical simulations for 3D models for plane wave excitation are performed [9, 10]. Sometimes even no acoustic simulations are performed and the effectiveness of the solution is assumed based on for example dispersion curves calculation [5]. However, performing acoustic simulations for vibroacoustic simulations for 3D models of the metamaterial is crucial and it is necessary when it comes to sound transmission loss estimation or optimization of the geometry in order to improve the effectiveness of the solution itself.

The aim of this paper is to present novel method which allows for sound transmission loss simulation of a plain plate with locally resonant structures in diffuse field. The method combines numerical calculations together with analytical solution. By combining numerical with analytical solution, it is possible to calculate sound transmission loss as well as with analytical solution but with addition of taking into account real geometry of the 3D model of the structure and also significantly improve time consumption in comparison purely numerical solution.

The study is organized in three parts. Firstly, Section 2 presents the mathematical formulation of the considered plate structure with locally resonant metamaterials attached and is diffuse field sound transmission calculation. Section 3 shows the simulation procedure of the sound transmission loss in Comsol and Matlab. In Section 4 experimental validation is presented, simulation and measurement results are compared. Finally Section 5 summarizes and discusses results presented in this study.

2. Mathematical formulation of STL for plate structure with LRS

The problem considered in this paper is a single wall of finite size mounted in infinite baffle and semi-infinite acoustic field on both sides [11]. First of all, the formulas for bare plate without resonant structures have to be analysed. The formulas presented hereafter are derived in [11] for forced airborne sound insulation. The solution of this particular problem can be described using differential equations and solving them requires obtaining accurate approximations for the particular problem. As presented in Fig. 1, it is assumed that the sample is placed in infinite baffle in $z = 0$. On the source side the acoustic field consists of incident pressure p_i oriented with θ and φ in its elevation and azimuth angles respectively, reflected pressure p_r as well as scattered pressure p_s. On the other side of the sample acoustic field will consist in this case only of transmitted pressure p_t, therefore total acoustic filed can be written as $p(x, y) = p_i + p_s + p_r - p_t$.

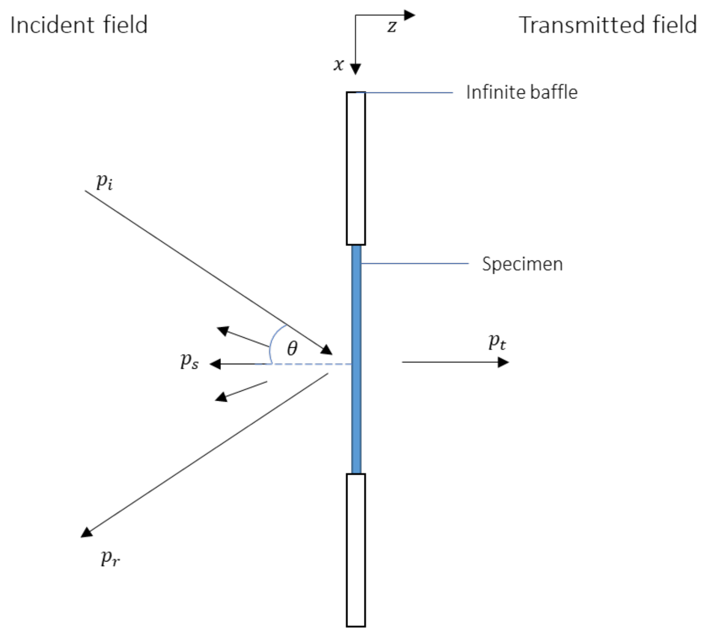

Figure 1. The model of finite sample in the infinite baffle with acoustic field on left and right side.

The transmission coefficies can be defined as

$$\tau(\theta, \varphi) = \frac{\Pi_t}{\Pi_i}, \tag{1}$$

where θ is the elevation angle of incident wave, φ – its azimuth angle, and Π_t and Π_i are transmitted and incident power respectively. This work is mainly focused on Sound Transmission Loss (STL) of the defined rectangular plate structure. The STL (sound reduction index) is defined as the variable R and is given as

$$R = 10\log_{10}\left(\frac{1}{\tau}\right). \tag{2}$$

Transmission coefficient τ can be calculated using Paris formula for diffuse filed transmission loss as follows [11]:

$$\tau_s = \frac{1}{\pi}\int_0^{2\pi}\int_0^{\frac{\pi}{2}} \frac{4\rho^2 c^2 \Re\{z_f\}}{\left|Z + 2\rho c z_f\right|^2} \sin\theta d\theta d\varphi\,, \tag{3}$$

where ρ is density of air, c – speed of sound in air, z_f is the radiation impedance and Z is wall impedance. Symbol $\Re\{z_f\}$ desribes real part of the radiation impedance. The time dependence is of the form $e^{i\omega t}$, where $\omega = 2\pi f$ is the angular frequency and t is the time. With Paris formula one can obtain sound insulation of structure under load. This equation is denoted in literature as fully theoretical solution and in most of the cases different approximations are used [11]. Wall impedance of the sample is denoted as Z and for thin structures is described with Kirchhoff plate equation as follows:

$$Z(\omega) = \frac{B'}{i\omega}\left(k_x^2 + k_y^2\right)^2 + i\omega m_p'', \tag{4}$$

where B' denotes bending stiffness and m_p'' is mass per unit area of the plate. Eqation (4) is simplified form of wall impedance of the structure. The waves travelling through the structure are in the form $\exp[(-i(k_x x + k_y y)]$, where k_x and k_y are wave numbers and $k_x = k\sin\theta\cos\phi$ and $k_y = k\sin\theta\sin\phi$, k is the wavenumber in air and x and y are axis names. In case of the plate with locally resonant structures attached on one side, Eq. (4) must be rewritten in the following form to include the influence of resonant structures attached to the base plate [12]:

$$Z = \frac{B'}{i\omega}\left(k_x^2 + k_y^2\right)^2 + i\omega m_p'' - \frac{m_r''\omega s''(1+i\eta_s)}{i m_r''(\omega_0^2 - \omega^2) - \eta_s s''}, \tag{5}$$

where, s'' is the real part of spring constant of added resonant structures, m_r'' is the mass per unit area of added structures, η_s is the damping loss factor of added resonant structures and $\omega_0 = \sqrt{s''/m_r''}$ is natural resonant frequency of attached resonant systems.

Apart from wall impedance calculation of the sample, the final thing missing for transmission coefficient calculation is radiation impedance z_f, which is strictly connected with a shape of the sample. In most of the cases the rectangular shape of the structure will be considered. In this paper the rectangular shape is considered as well, because of the measurement setup in which the window for the sample is rectangular. In [11] the simplest form for z_f calculation is given as follows:

$$z_f = \frac{ik}{2\pi S}\int_0^a\int_0^b 4\cos(k\mu_x\kappa)\cos\left(k\mu_y\delta\right)\frac{e^{-ik\sqrt{\kappa^2+\delta^2}}}{\sqrt{\kappa^2+\delta^2}}(a-\kappa)(b-\delta)d\kappa d\delta. \tag{6}$$

where, a and b are dimensions of square plate in x and y directions respectively, $S = a \cdot b$ and denotes the area of the plate, $\mu_x = \sin\theta\cos\phi$, $\mu_y = \sin\theta\sin\phi$, where θ and φ are angles of the incident acoustic wave. The radiation impedance is not affected by locally resonant metamaterial which can be added to the base structure; it is the same in both cases.

As stated in [11, 13] azimuth angle φ has little effect on the radiation impedance and no effect on the wall impedance outcome. Therefore an approximation given as

$$\tau_s \approx 8\rho^2c^2\int_0^{\frac{\pi}{2}}\frac{\Re\{\overline{z_f}\}}{\left|Z + 2\rho c\overline{z_f}\right|^2}\sin\theta d\theta \tag{7}$$

can be used instead of Eq. (3) and $\overline{z_f}$ describes radiation impedance averaged over the azimuth angle φ.

3. Simulation procedure

3.1. General remarks

In pervious section the analytical sound insulation calculation is presented. This approach allows for more accurate sound insulation calculation than for example using mass law approach [4] especially for diffuse sound field or STL around coincidence region. The analytical solution can be used for sound transmission loss calculation also for locally resonant metamaterials. In analytical case the locally resonant metamaterial will be modelled as mass-spring-damper systems attached to the base plate in equal distances from each other. The basic dependencies can be studied using analytical simulations. The drawback of the analytical simulation is assuming theoretical model for locally resonant metamaterial. Considering real metastructures, the resonators have specific geometry which affects the global effectiveness. It would seem like the numerical solution is better idea for the problem.

One of the main reasons for exploiting resonant metamaterials for sound and vibration mitigation are small dimensions of the solution. Thus, for this reason the simulation model requires small dimensions of the mesh geometry which is highly computational demanding. For sound transmission loss simulation one need to create model with acoustic field and solid body interaction. Often, numerical simulations for metamaterials are simplified to a single unit cell of the structure (one resonant structure attached to a section of the base plate) as presented in Fig. 2. Simulations for single unit cell of the structure requires periodic boundary conditions and thus identical mesh on all outer boundaries of the model. Because of small dimensions of the metamaterial and the unit cell formation of proper mesh could be difficult, especially at the junction of acoustic field and solid body.

Table 1. Material parameters considered in this study.

Quantity	Symbol	Unit	Value
Steel plate			
Density	ρ	kg/m³	7850
Thickness of plate	h	m	0.0005
Young modulus	E	GPa	210
Poisson ratio	ν	-	0.3
Structural loss factor		-	0.001
Length of unit cell	L	m	0.03
Polyamid PA12 (Nylon) – locally resonant structure			
Density	ρ	kg/m³	1100
Young modulus	E	GPa	1.8
Poisson ratio	ν	-	0.3
Structural loss factor		-	0.14

There is a way for simplification and time reduction needed for numerical simulation by taking advantage of using analytical solution. The analytical solution presented in Section 2 can be used for radiation impedance calculation as this parameter does not depend on the geometry of the metamaterial. This way acoustic field modelling can be omitted in the numerical model and thus the calculations can be simplified, without losing its precision.

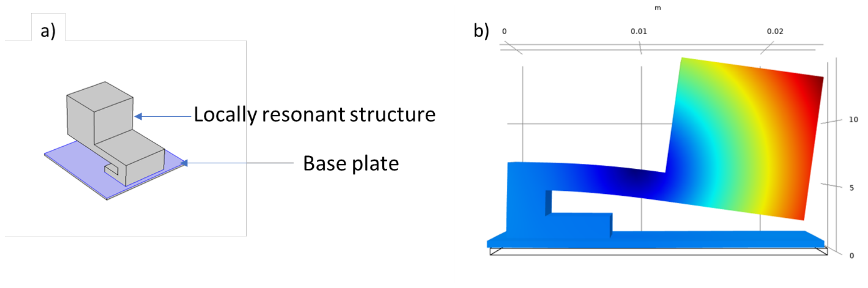

Figure 1. Comsol model of the unit cell with base plate and resonant structure (a). The excitation plane is marked with blue color. Mode shape of the first natural frequency of modeled structure (b).

3.2. Model description

Simulations were performed using COMSOL 5.6 software combined with Matlab Livelink. First of all a model of a single unit cell was created as presented in Fig. 2a. The resonant structure geometry is based on cantilever beam with mass at the free end and it is attached to the base plate. It was designed to has resonant frequency equal to 420 Hz and the mode shape of first natural frequency is presented in Fig. 2b. In this case base plate is steel plate 0.5 mm thick and the resonant structure is assumed to be made of PA12. Material properties used in simulations are presented in Tab. 1. The unit cell dimensions are adjusted to resonant structure dimensions; the length of the unit cell is set to 0.03 m. Model contains only one unit cell, so the periodic boundary conditions have to be implemented. The Floquet Boundary Conditions available in COMSOL assume spatial periodicity of the structure and allow for infinite structure simulation. The last thing implemented in Comsol software was boundary load applied to the structure. The plane acoustic wave incident on the structure was replaced by boundary load applied to the boundary marked in Fig. 2a on blue. By replacing pressure filed and plane wave excitation by boundary load the acoustic-solid interaction is not needed. It is beneficial for mesh quality, number of elements in mesh and therefore time needed for calculations.

3.3. Wall impedance simulation

Numerical simulations of solid body with boundary load excitation allow for the wall impedance of the structure calculation. Impedance of a structure can be calculated using formula

$$Z(\omega) = \frac{F(\omega)}{\hat{v}(\omega)}, \tag{8}$$

where $F(\omega)$ is applied force and $\hat{v}$ is vibration velocity amplitude. As stated in [12], the vibration velocity is assumed to be in the same form as incident pressure, so $v_p(x, y) = \hat{v}e^{-i(k_x x + k_y y)}$. Velocity amplitudes have to be calculated, extracted using Global Evaluation and averaged over the volume of the base plate.

To this point the infinite structure is assumed. For wall impedance calculation outer dimensions are not important; the periodic infinite structure is sufficient for wall impedance calculations. Modelling finite structure would greatly increase model complexity.

3.4. Radiation impedance and transmission coefficient simulation

As the next step the radiation impedance of finite plate is calculated. Instead of modelling pressure field in numerical software, the analytical equations can be used. The calculations were performed in Matlab and the radiation impedance was calculated according to Eq. (6). The outer dimensions of the base plate were 450 × 550 mm to correspond the dimensions of the measurement setup considered in this paper. The numerical simulation result, wall impedance, was imported to Matlab using Comsol with Matlab Livelink.

The single simulation was performed for acoustic plane wave incident on the structure from selected elevation angle. The simulation was performed iteratively for different angles from 0° to 360° with 15° step. Using Eq. (7) the diffuse field transmission coefficient was calculated based on the simulations results obtained for all different angles.

4. Experimental validation

4.1. General remarks

In order to validate the simulation results the measurements in diffuse field were conducted. Measurements were performed in single reverberation chamber in AGH UST in Cracow. According to the normative procedure described in PN-EN ISO 10140-1:2021-10, airborne sound insulation measurements should be performed in two reverberation chambers with specimen between them, where sample should have at least few square meters of a surface. Locally resonant metamaterials presented in this paper have small dimensions as it is one of the main features of these structures. To prepare a measurement sample of several square meters surface it would require thousands of separate resonant elements attached to the base plate. Therefore measurements presented here, were performed in smaller, not normative measurements stand [14]. The stand allows for measurements comparable to normative ones from around 300 Hz; in lower regime the normal frequencies of the source room and the measured sample may have great impact on obtained results. Thus, for this reasons the designed resonator structures were tuned to 420 Hz.

4.2. Measurement procedure

The measurement stand is presented in Fig. 3. The reverberation chamber has 188 m^3 cubature. One corner of the reverberation chamber is adapted for sound insulation measurements; it has a window for mounting the 450 × 550 mm sample. Behind the sample there is separated space for omnidirectional sound source and a microphone (source room). The microphone is placed on a moving arm; it is moving during the measurement in order to average sound pressure in different places. On the other side of the sample there is receiver room with six microphones.

The sound reduction index R [dB] can be expressed as follows

$$R = L_1 - L_2 + 10\log_{10}\frac{S}{A}, \tag{9}$$

where L_1 and L_2 are sound pressure level in source and receiver room respectively, S is area of the sample in m^2 and A is absorption area of the receiving room and can be written as

$$A = \frac{0.163V}{T_{60}}. \tag{10}$$

T_{60} is the reverberation time and V is the volume of the receiving room. Because of small sound room volume as well as small sample dimensions the window correction must be considered in R calculations; the window where the sample is placed introduces differences in sound pressure levels in source and receiver rooms [14].

Measurements were performed using Labview and NI PXI 1082 in 1/24 octave frequency bands with pink noise signal on omnidirectional sound source and 7 microphones GRAS 46AQ. There were two measured samples:

- 0.5 mm thick steel plain plate,
- 0.5 mm thick steel plate with resonant structures attached.

The resonant structures were 3D printed using HP MJF 3D printer. In Figure 4a the plate with attached locally resonant metamaterial is presented. In Figure 4b sample in measurement window setup is presented. The edges of the sample plate were taped to the measurement window to prevent leakages.

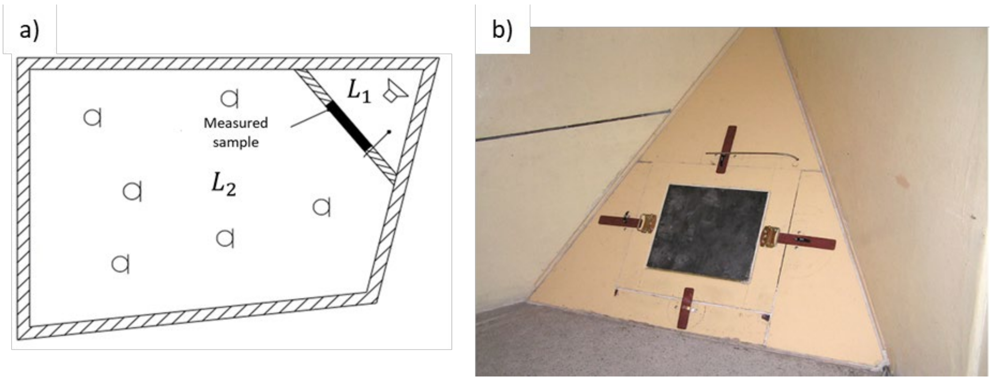

Figure 2. Scheme of the measurement setup with sound source, microphones and measured sample (a) and a photo of the measured sample in the reverberation chamber (b).

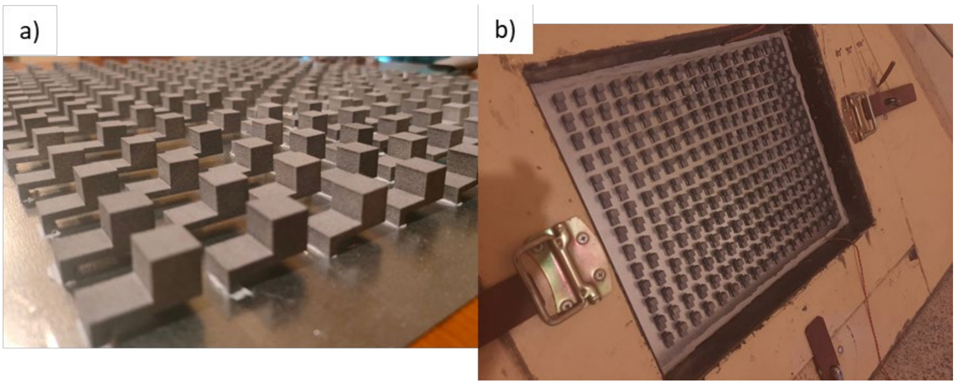

Figure 3. Photo of the steel plate with attached meatamaterial structures (a) and sample placed in meaurement window in the reverberation chamber (b).

4.3. Results

In Figures 5 and 6 the simulation and measurement results are presented. The Sound Reduction Index calculated from measurements results and given by Eq. (9) can be referenced to Sound Transmission Loss calculated in simulations and given by Eq. (2). In Figure 5 simulation results for plain steel plate are compared with measurement results for the same sample. In Figure 6 simulations and measurements for plate with attached metamaterials are presented. Since the metamaterial structure considered in this paper has low frequency effect of sound reduction index curve, the figures frequency band is narrowed down to 1000 Hz. The differences between simulation and measurement result for steel plain plate are not greater than 3 dB.

For the steel plate with attached resonant structure the differences are greater in resonance and antiresonance region. In measurement results the resonance frequency of the metamaterial is shifted to lower frequency region; there is around 20 Hz difference between simulation and measurement. This could be caused by too coarse mesh in numerical simulation and by the production method. The differences in amplitude in resonance and antiresonance region is caused by too low damping coefficient assumed in simulations. The value of this coefficient was predicted based on laboratory measurements of material parameters of PA12 [15] and the values will be validated in the future. Above resonance and antiresonance region the sound reduction index curve is shifted upwards because of the mass addition to the base structure caused by resonant structures. It can be said that combined analytical numerical approach for locally resonant metamaterial effectiveness simulation proved to be a good replacement for standard numerical simulations.

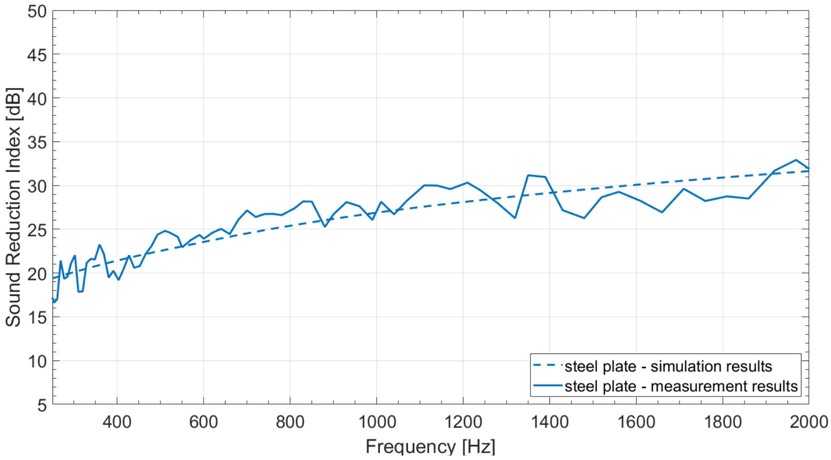

Figure 4. Simulation (dashed line) and measurement (solid line) results comparison for steel plain plate.

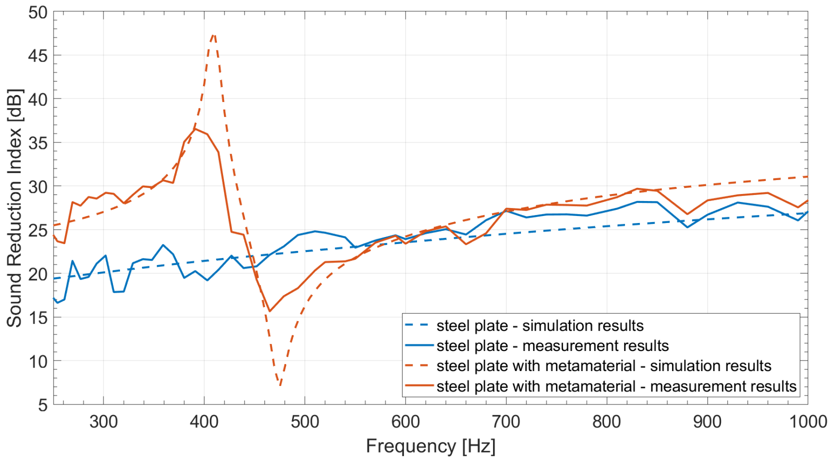

Figure 5. Comparison of the simulation and measurement results for steel plain plate (blue lines) and the same steel plate with metamaterial (red lines).

The combined numerical – analytical simulation procedure allowed for significant time reduction. Computational time needed for standard numerical simulation of unit cell was approximately 40 min for one angle of incidence, while for combined analytical numerical approach it was approx. 1 min in the same case. The difference in time is caused by omitting the acoustic domain in the second model. It made it possible to reduce the number of mesh elements as well as allowed for faster analytical calculation of radiation impedance. Depending on the number of incidence angles used for the calculation the overall computational time will vary.

5. Conclusions

This paper has presented the potential of using combined analytical and numerical approach for sound transmission loss of locally resonant metamaterials simulation. This kind of simulations allows for reducing the time needed for sound transmission loss calculation even for acoustic diffuse field. It has been proven that the simulations correspond well with measurement results. Presented simulation technique will allows for carrying out optimization of metamaterial parameters, what was impossible witch standard simulation methods. Combined simulation approach will be beneficial not only for metamaterial simulation but also for sound transmission loss of normal structures like multilayer and sandwich structures. In the future the simulation method will be improved and validated on more broadband measurements also for metamaterials matched to coincidence problem. The important site of future work would be realization of optimization of 3D geometrical models in order to search for the optimal possible solution for selected problems.

Additional information

The author(s) declare: no competing financial interests and that all material taken from other sources (including their own published works) is clearly cited and that appropriate permits are obtained.

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