

Using the MIMO Method to Evaluate the Modal Properties of the Elements of a Wheelset in an Active Experiment

Julia MILEWICZ , Daniel MOKRZAN , Tomasz NOWAKOWSKI , Grzegorz M. SZYMAŃSKI 

Poznan University of Technology, Institute of Transport, Poland

Corresponding author: Julia MILEWICZ, julia.milewicz@doctorate.put.poznan.pl

Abstract The study of the system composed of the inner disc and wheel rim of the 105Na type railway wheel, used in Polish Konstal streetcars, was aimed at determining the dynamic parameters of the object, such as the form and frequency of natural vibrations, and at evaluating the effectiveness of the method at given analysis settings. The experiment was conducted using triaxial piezoelectric transducers and a modal hammer with an aluminum head. A multiple-input, multiple-output (MIMO) testing approach was used because of the multiple excitation points and vibration measurements. A Fast Fourier Transform (FFT) of the measurements was performed in BK Connect software and the frequency response function (FRF) value waveforms were determined. The Rational Fraction Polynomial-Z method was used to extract modes from the frequency spectrum. In addition, the Complex Mode Indicator Function method was used, which resulted in the decomposition of the principal components of the FRF value matrix, allowing the identification of individual modes. The selection of the natural frequencies was performed on the basis of the obtained FRF and CMIF characteristics of the vibroacoustic response. Visualization of the form of the natural vibration was also performed. The result of the experiment was a set of comprehensive information on the modal properties of the studied object, which allowed to confirm the effectiveness of the selected method of analysis.

Keywords: modal analysis, FRF, light railway vehicle, wheelset.

1. Introduction

An objective indicator of technical condition can be the results of analysis of dynamic properties of objects, i.e. modal analysis. It is based on precise measurement of vibrations of the object, excited mechanically or electro-dynamically, and allows to determine the relationship between geometry and material and technical condition on the basis of the values of free vibration parameters and their form [1]. The objectives of using modal analysis include diagnosis of the object state, synthesis of control during active vibration reduction, verification and validation of numerical models and, consequently, modification of the structure. As a result of modal analysis, a modal model is obtained, i.e. a set of natural frequencies and their forms and damping coefficients. This model can be obtained theoretically, by solving a numerical structural model using a finite element mesh, or experimentally, by measuring the values of excitations and vibroacoustic responses. In the case of the experimental way, the results of the measurements are digitally processed by computational algorithms that estimate the model parameters. The vibration excitation can be realized by means of exciters or modal hammers - then we speak about impulse excitation. It is possible to measure the driving force thanks to built-in force sensors in the hammer, and the replaceable hammer tip allows to choose the appropriate stiffness (which changes the local impact conditions). In addition, tuning the hammer to the expected frequency range by using additional mass makes it easier to excite vibrations with specific parameters in the system.

Modal analysis is widely used in the diagnosis of railway vehicle components and subassemblies. Frequently used modal parameters in the frequency domain are the forms and frequencies of natural vibrations and damping coefficients corresponding to them. The analysis of the properties of the solid, its mass, stiffness and interaction with the environment allows to determine the resonance conditions and the influence of the created or propagating defects [2]. Experimental comparative studies of wheel structure types have shown that modal analysis for determining the frequency and form of natural vibration is an effective tool not only for determining the resonance conditions and damage diagnosis, but also for studying the vibroacoustic effects of the vehicle on its surroundings [3]. Moreover, modal analysis allows to build a model, which then can be tuned with the theoretical model, determined analytically. The numerical modal analysis of the streetcar monoblock wheel itself, was described in [4], and allowed the determination of

modes distinguishing the transverse, longitudinal and vertical excitations, which were related to the dynamics of the moving vehicle.

The aim of the described study was to experimentally determine the dynamic parameters of the system of elements of a railway vehicle wheelset using the MIMO (multiple input, multiple output) method, and ultimately to verify the effectiveness of the selected methodology and analysis settings.

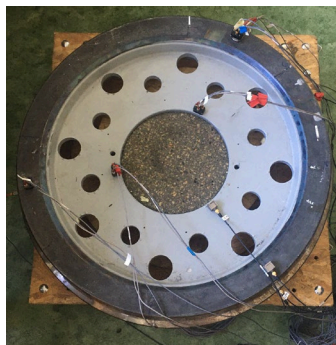
2. Research methodology

2.1. Research object

The selected object of research was a 105Na type railway wheel, used in Polish Konstal streetcars. The experimental modal analysis was performed on the system of the rim and inner disc (wheel rim). Both parts are made of carbon steel, and there is a press fit between them. Simplifying the structure of the test object by removing additional elements of the wheel, such as damping inserts, bolts, and outer discs, allowed the elimination of secondary vibrations that could result from the interaction of parts, which could interfere with the measurement of parameters for the whole object.

The BK Connect environment was used as the software for the experimental study. First, a geometric model had to be created to mark the locations of the applied excitations and the measurement points where the vibration transducers were located. Figure 1a shows the test object and Figure 1b shows the corresponding geometric model.

a)



b)

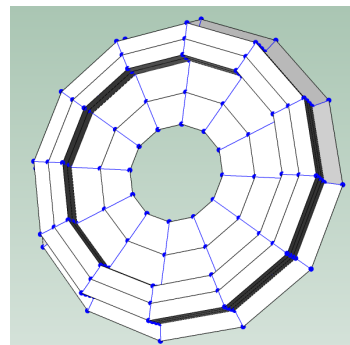


Figure 1. (a) Real test object and (b) geometric model in BK Connect software.

The model consisted of twelve segments, included the rim and flange, and reflected the proportions of the real object. However, it did not take into account the holes found in the real wheel, nor the complex geometry of the rolling surface, because it was intended only to locate the excitation and response points and define the analysis conditions, not to graphically represent the object.

2.2. Measurement station

It was assumed that the range of expected values of natural frequencies of the tested object is in the range of 1-3000 Hz. Ten triaxial piezoelectric transducers from Brüel & Kjær were used to measure the vibration response: four transducers type 4504 A (min. range of measured frequencies is 1-11000 Hz), three transducers type 4529 B (min. range of measured frequencies is 0.3-6000 Hz), two transducers type 4524 B (min. range of measured frequencies is 0.25-3000 Hz) and one single-axis transducer type 4507 B (min. range of measured frequencies is 0.1-6000 Hz). All transducers met the range and sensitivity requirements.

Three transducers were placed, every 120 degrees, on the disc near the hole (measurement points #4, 8, 9), three on the outer plane of the rim (measurement points #3, 7), three on the rim, including a uniaxial measuring vibrations in the tangential direction (measurement points #1, 5, 11), and 2 on the rolling surface (measurement points #2, 6). Figure 2b shows a visualization of the measurement points in BK Connect, against the model geometry. As a local coordinate system, the X-axis is assumed to be radial to the center of the wheel, the Y-axis is tangential and clockwise, and the Z-axis is vertical with an upward direction.

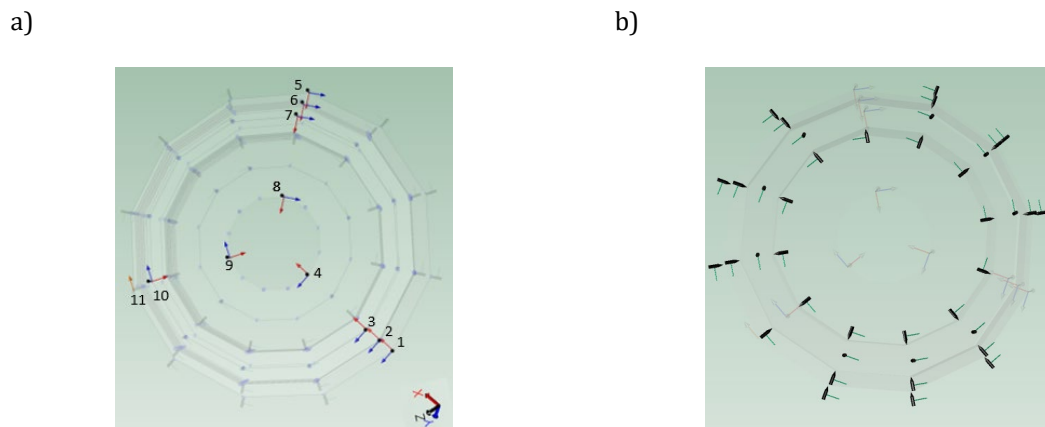


Figure 2. Location of measurement points (a) and location of excitation points, (b) against the geometric model.

Figure 2b shows the locations of the excitation points. A total of 44 force application locations were selected, of which: 9 each on the rim and rim surface parallel to the disc, 10 on the rolling surface, and 12 on the inner surface of the wheel disc rim. The choice of points depended on the location of the transducers (the excitations should not be too close to them) and the location of the additional holes that are in the real disc. At each of these points the vibrations were forced four times and the response values were then linearly averaged. Because of the multiple excitation and measurement points, this testing approach is referred to as MIMO (multiple-input, multiple-output)[5].

A Data Acquisition System (DAQ) consisting of three 12-channel modules from Brüel & Kjær type 3053-B-12 was used as the measurement apparatus, which allowed vibration measurements in the range of 0-25600 Hz. The synchronization and data transfer was made possible by PULSE Measurement System Switch UL-0265.

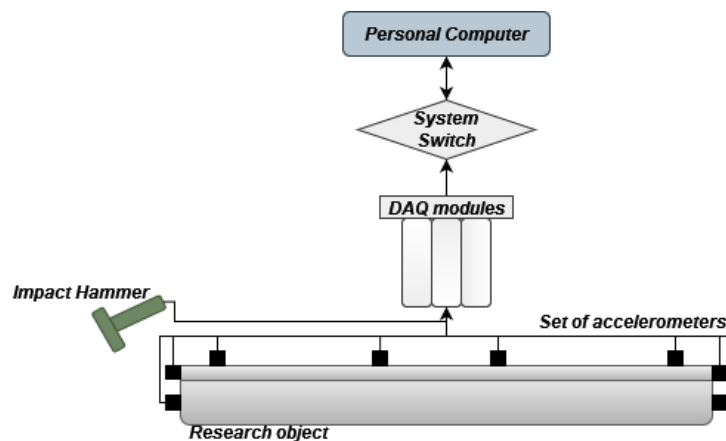


Figure 3. Scheme of the measurement path.

A Brüel & Kjær modal hammer, type 8206-002, with a sensitivity of 2.27 mV/N and a maximum driving force of 2200 N was used to perform controlled acoustic response excitations, a photograph of which can be found in Figure 4.8. An aluminum head was used, as this solution allowed vibration measurements to be made over the expected wide frequency range of the test.

2.3. Measurement and signal processing methods

By using the Fast Fourier Transform (FFT), it is possible to move from the time function, in which measurements are made, to the frequency function. Analysis in the time domain allows one to determine the values of the signal, while in the frequency domain the energy of the signal is considered, which is much more useful for describing structural dynamics phenomena. Thus, the signal can be analyzed from a different perspective, determining its components and dominant vibration frequencies and, consequently, determining the dynamic parameters of the object [6].

In the BK Connect program, FFT analysis of the measurements was performed and FRF characteristic waveforms were determined both for the vibrations measured by individual transducers and for the whole system. Table 1 shows selected values of the analysis settings.

Table 1. Selected parameters of BK Connect analysis settings.

Frequency range	3200 Hz
Frequency resolution	1 Hz
FFT lines	3200
Averaging domain	Line averaging
Excitation time window	10 s

In the case of two-channel analysis, i.e. when testing the vibration signal or acoustic pressure induced by a specific excitation, both the value of the excitation and the parameters of the obtained signal are recorded. The Frequency Response Function (FRF) [7] is then applied, which describes the ratio of the spectrum of the output signal, i.e. the vibroacoustic response to the excitation, to the spectrum of the input signal, i.e. the hammer force on the test object. The FRF is a mathematical, signal-independent descriptor of the system, which is defined by the formula:

$$H(\omega) = \frac{X(\omega)}{F(\omega)}, \quad (1)$$

where: $X(\omega)$ is the output signal in the frequency domain, and $F(\omega)$ – input signal in the frequency domain. Linearity and invariance of the model are assumed in such analyses [8]. The FRF matrix is understood as a frequency-domain representation of the linear structural dynamics, in which the linear spectra of the input signals are multiplied by the corresponding matrix elements to obtain the linear spectra of the output signals. This creates pairs of input and output values for successive degrees of freedom of the structure [9]. Depending on the nature of the noise and disturbances during the measurement, a specific estimator for FRF analysis is used to minimize the errors. For the case of modal hammer excitations, the H1 estimator is used for the noise in the output signal [10]. The value of the estimator is determined from the relation:

$$H_1(\omega) = \frac{G_{FX}(\omega)}{G_{FF}(\omega)}, \quad (2)$$

where $G_{FX}(\omega)$ is the cross-spectral density of the input (excitation) and output (response) signals in the frequency domain, a $G_{FF}(\omega)$ describes the spectral density of its own input signal (excitation) in the frequency domain

The Rational Fraction Polynomial-Z method, which is recommended for systems with many degrees of freedom and small disturbances [11], was used for mod estimation. It involves representing the function in terms of a measurable ratio of two polynomials such that the numerator and denominator values are of independent orders. This analytical form is numerically fitted to the FRF values obtained in the measurements by selecting appropriate coefficients and solving the roots of the polynomials [12], which allows for the extraction of the frequency and form of natural vibrations from the spectrum.

In addition to the standard use of the FRF function, the CMIF method - Complex Mode Indicator Function - was also used to analyze the results. It is based on the SVD (Singular Value Decomposition) decomposition of the normal matrix, formed from the FRF matrix [13], for each spectral line, to raise the determined components (singular values) to the second power. The SVD decomposition itself consists in decomposing the given matrix into the product of two orthogonal matrices and one diagonal matrix, in order to reduce its dimension and identify the repeating values. In this way, it is possible to identify individual modes, determine their magnitudes, forms and damping coefficients, as well as global modal parameters [14].

2.4. Results

The results of the experimental study are both FRF transition function plots, CMIF function plots, visualization of displacements in natural vibration forms, and correlation matrices between modes.

The frequency spectra showing the relationship between the values of the transition function on the ordinate axis, expressed in units of $(m/s^2)/N$, and the vibration frequency on the cut-off axis (in kHz) are presented as a "linear magnitude" visualization, which shows only the positive values of the spectrum on a linear scale for easy interpretation. The unit of $(m/s^2)/N$ relates to the ratio of the output signal (vibration acceleration, expressed in m/s^2) to the input signal (modal hammer force, expressed in N).

Figure 4 shows the FRF spectrum for the total measurements (data from all transducers measured in the three directions were automatically averaged by the BK Connect program). The amplitude has a different magnitude than the spectra for individual measurement points because the FRF values depend directly on the hammer force, which was not constant for all measurements (so when averaging the entire measurement, the amplitude maintained only proportions).

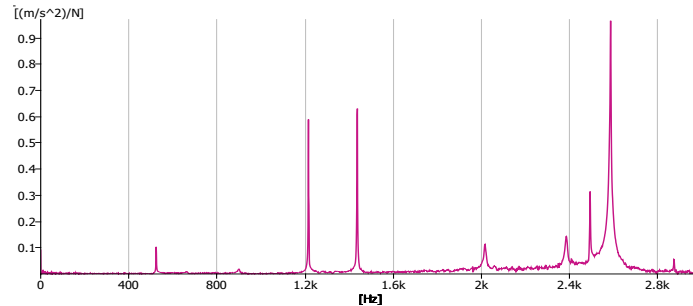


Figure 4. FRF characteristics averaged over data from all transducers and all excitation points.

Figure 5a shows the CMIF characteristics for the entire frequency range (and all excitation-response pairs), and Figure 5b shows the selected 400-600 Hz interval. The decibel scale is on the ordinate axis (the lack of unit is due to the nature of the CMIF function, while the notation dB/1 stands for the decibel scale with a reference value of 1 in the BK Connect program), and the frequency is on the cut-off axis.

The CMIF characteristics for the whole investigated range allows to evaluate the effectiveness of the vibration response measurement by given transducers and, consequently, to validate the choice of measurement point locations. It is possible to notice which signals indicating the natural frequency forcing were received by given transducers. The graph allows observation of the course of the function of selected transducers in comparison to the remaining ones, identification of the captured modes and also noticing the deviation of the results for some measurement points.

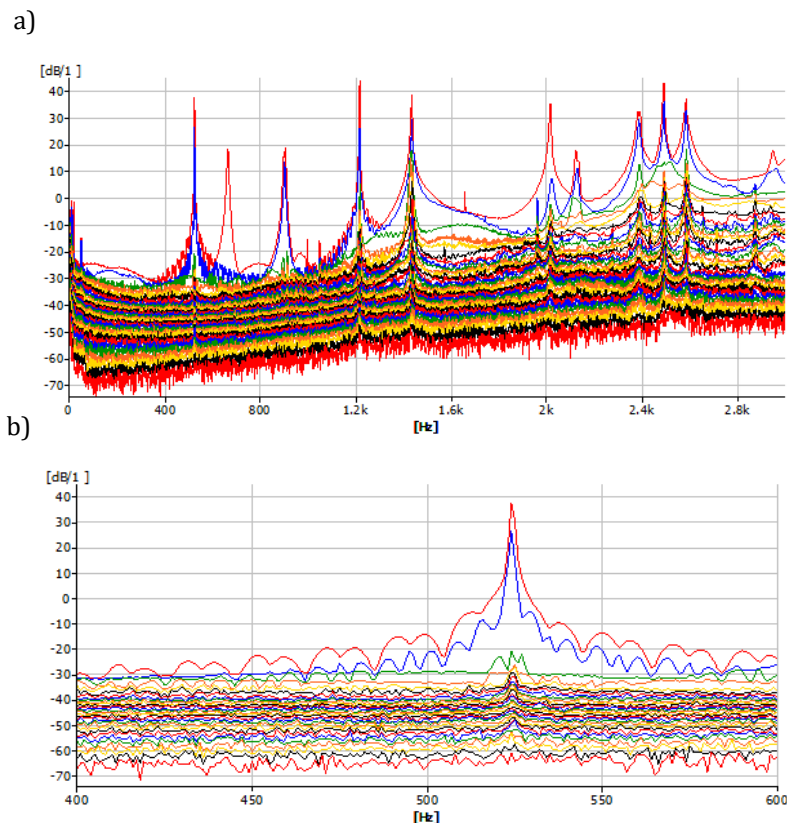


Figure 5. CMIF characteristics for: a) the entire frequency range tested b) the selected range 400-600 Hz.

Plotting the CMIF values in the selected range allows noticing the recurring amplitude increase for each transducer for the frequency of about 525 Hz and identifying the measurement points and vibration measurement directions characterized by the largest increase in amplitude (in this case red and blue courses correspond to measurement point No. 1, located at the periphery, and measurements in tangential and radial directions, respectively). This distinct extreme occurring at the frequency of 525 Hz clearly reflects the 2nd form of natural vibrations.

Based on the FRF and CMIF characteristics, a selection of 10 frequencies and natural vibration forms of the test object was made. In order to visualize the form of vibrations in BK Connect software, the distribution of resultant relative displacement amplitude was used (larger local displacements take increasingly lighter colors relative to the rest of the model).

Table 2. Forms and frequencies of natural vibrations of the system under test.

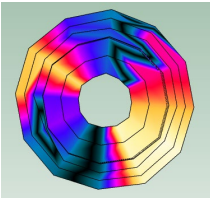
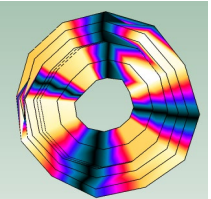
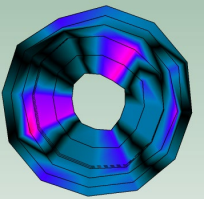
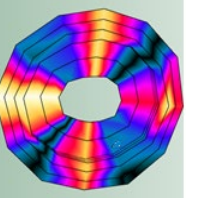
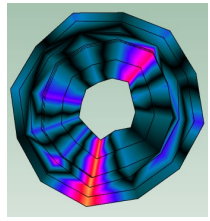
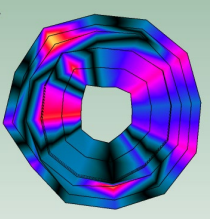
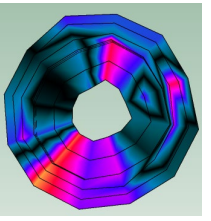
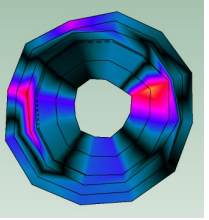
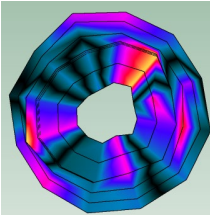
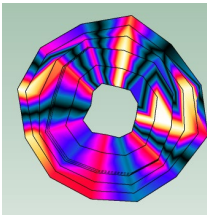
Number of mode	I	II	III	IV
Natural Frequency f [Hz]	11	524	904	1218
Form of natural vibration				
Number of mode	V	VI	VII	VIII
Natural Frequency f [Hz]	1433	2018	2387	2495
Form of natural vibration				
Number of mode	IX	X		
Natural Frequency f [Hz]	2586	2945		
Form of natural vibration				

Table 3 shows the damping coefficients corresponding to the selected natural frequencies. Proportional damping results in real modes and oscillation in phase of the degrees of freedom determined in the study (the nodes of the modal model where the excitation occurs and the response is measured), while otherwise the forms of the natural vibration take on an imaginary character and the oscillations are out of phase [15].

Table 3. Damping coefficients corresponding to selected forms of natural vibration.

Number of mode	Frequency f [Hz]	Damping coefficient c [-]
I	11	1.966
II	525	0.002
III	904	0.312
IV	1218	0.017
V	1433	0.242
VI	2018	0.194
VII	2388	0.262
VIII	2495	0.077
IX	2586	0.093
X	2945	0.373

The highest damping occurs for the form with the lowest frequency, while for the other modes it does not exceed the value of 0.4, so it is relatively small.

3. Interpretation of results

In the studied range of 0-3000 Hz, 10 forms of natural vibration were determined from the FRF and CMIF characteristics with frequencies: 11, 525, 904, 1218, 1433, 2018, 2388, 2495, 2586 and 2945 Hz, with the highest averaged vibration acceleration amplitudes corresponding to the bands $f_{II}=525$ Hz, $f_{IV}=1218$ Hz, $f_V=1433$ Hz and $f_{IX}=2586$ Hz. The density of the natural frequency bands occurs in the second half of the FRF spectrum, particularly in the 2000-2600 Hz range. The FRF makes it possible to determine the dominant frequencies, while the CMIF characteristics represent which transducers best picked up the signals indicative of the modes, making it possible to accurately identify the locations of the measurement points most sensitive in determining the dynamic parameters of a given test object. The transducers located on the flange and rolling surface best measured the vibration response to the vertical forcing, which was likely related to the damping of the disc vibration by the support.

Some of the determined natural vibration forms take very large local relative displacements. Especially modes I, II, IV and X show the greatest susceptibility to impulsive forcing and involve the structure of the whole object. Modes V and VI are characterized by displacement in the area close to the disc bore edge, and mode VIII concerns mainly the rim.

Except for the form of natural vibration with the lowest frequency, the dimensionless damping coefficients reached low values.

4. Conclusion

The aim of this work was to determine the frequency and form of natural vibrations of the wheel rim-disc system, used in Polish Konstal 105Na streetcars, by means of experimental modal analysis using the MIMO assumption. In order to identify dynamic parameters of the research object, it was necessary to extract modes from the vibration spectrum using the CMIF method. It allowed the decomposition of FRF matrix and obtaining the information about the recurring data indicating the occurrence of free vibrations. The experiment also required making a geometric model in the BK Connect program and determining its degrees of freedom, as well as the excitation points (locations of modal hammer impacts) and response measurement points (locations of attachment of piezoelectric vibration transducers). After the analysis of FRF and CMIF characteristics of measurement results, 10 forms of natural vibrations were selected, which differ in amplitudes and areas of vibration displacement and damping coefficients.

Modal analysis allows to identify dynamic properties of the object in an effective and reliable way, both for the assessment of its technical condition, but also to determine the operating conditions. In the case of rail transport, the operating conditions influence the wear rate of the components, the comfort of the travel, and consequently also the traffic safety.

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Additional information

The author(s) declare: no competing financial interests and that all material taken from other sources (including their own published works) is clearly cited and that appropriate permits are obtained.

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