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Sound power level estimation - choice of the prior distribution

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Abstract Bayesian inference is one of the methods used to determine the sound power level of sound sources. This method requires knowledge of two probability distributions. The first is the sampling density, while the second is the prior distribution. In this study, the effect of the prior distribution on the sound power level estimation results was investigated. For this purpose, three prior distributions were used: 1) a normal distribution, 2) a distribution determined using the kernel density estimator, 3) a uniform distribution. The sound power level results determined by the engineering method were used to illustrate the proposed solutions and carry out the analysis. The results of the experiment were compared with the results of the sound power level determined using the precision method in the hemi-anechoic room according to ISO 3745:2012. The statistical inference has been carried out based on results of non-parametric statistical tests at the significance level α = 0.05.

Keywords: sound power level, engineering method, Bayesian statistics, prior distribution.

1. Introduction

The sound power level (L_{WA}) is one of the basic parameters characterising a sound source. This parameter is commonly used in acoustics to model the distribution of A-weighted sound pressure level in the environment, to assess noise pollution in the working environment, and to compare machines and devices of a given type with each other. For the reasons outlined above, it is desirable to know the exact value of this parameter. The exact value of the sound power level can be determined from sound pressure measurements in anechoic and hemi-anechoic rooms using the precision method described in ISO 3745:2012 [1]. In industrial conditions engineering and survey methods in accordance with ISO 3744:2010 [2] and ISO 3746:2010 [3] are usually used. These methods are used with some deviations from the requirements contained therein. Typically, these deviations relate to the measurement environment and the number of measurement points.

Because of the difficulties presented in applying the engineering and survey methods in industrial conditions, the use of non-parametric statistical methods, namely bootstrap, and Bayesian inference were proposed for the determination of sound power level. The paper [4] analysed the influence of the input parameters of the bootstrap method on the accuracy of sound power level determination, while the paper [5] presented the possibility of using Bayesian inference to determine the sound power level of machines and devices.

This paper is focused on the determination of the sound power level of sources by the engineering method using Bayesian inference. In this study, the effect of the prior distribution on the accuracy of sound power level determination was analysed in depth by comparing the results obtained with the sound power level determined by the precision method in a hemi-anechoic room over a reflecting plane. For this purpose, measurement data recorded around the source under actual measurement conditions meeting the requirements of ISO 3744:2010 were used. A discussion and analysis of the results obtained were carried out and the statistical inference was made using non-parametric statistical tests at the significance level of α = 0.05.

2. Bayes inference

Consider an estimate of the random parameter θ from data $\mathbf{x}$. Then the associated conditional density $p(\theta|\mathbf{x})$ is called the posterior density because the estimate is conditioned "after the measurements" have been acquired. Estimators based on this posterior density are usually called Bayesian because they are constructed from Bayes' theorem since $p(\theta|\mathbf{x})$ is difficult to obtain directly. That is, Bayes' rule is defined as follows [6]:

$$p(\theta|\mathbf{x}) = \frac{p(\mathbf{x}|\theta)\,p(\theta)}{\int_{\Omega} p(\mathbf{x}|\theta)\,p(\theta)\,\mathrm{d}\theta}, \tag{1}$$

where $p(\theta)$ is called the prior density (before measurement), $p(\mathbf{x}|\theta)$ is called the sampling density or likelihood (more likely to be true), and Ω is called the parameters space, where $\theta \in \Omega \subset \mathbb{R}^d$.

This approach requires knowledge of two distributions. The first is the sampling density $p(\mathbf{x}|\theta)$. It was determined based on sample $\mathbf{x}$ of size n sampled from the investigated population. The second is the prior density $p(\theta)$.

The Bayesian approach distinguishes between two types of prior distributions. These are informative and non-informative distributions. Informative distributions are used when the data on which the estimation is based are "weak", i.e. there is a suspicion that they are subject to errors at the acquisition, processing and preliminary analysis stages. On the other hand, non-informative distributions are used when the data analysed are "strong", i.e. there is a confidence that they are not subject to any errors at earlier stages of processing. The phrase "they speak for itself" is used concerning such data.

In this experiment, three different probability density functions were used as the prior distributions:

1. A normal distribution (NORMAL) – informative distribution – is determined by the results obtained for the precision method as follows [7]

$$\hat{f}_{\mathrm{N}}(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(\frac{-(x-\mu)^2}{2\sigma^2}\right) \tag{2}$$

with mean μ = 71.2 dB(A) and standard deviation σ = 2.7 dB(A) presented in Figure 1a.

2. Density estimated (KERNEL) – informative distribution – is determined by kernel density estimator (KDE) based on the results obtained for the precision method from equation [7, 8]

$$\hat{f}_{\mathrm{K}}(x) = \frac{1}{nh}\sum_{i=1}^{n} K\left(\frac{x-x_i}{h}\right), \tag{3}$$

where $K(\bullet)$ is a kernel function, h – smoothing parameter (bandwidth), x_i – elements of a sample, shown in Figure 1b. A normal kernel function was used in this study.

3. Continuous uniform distribution (UNIFORM) – non-informative distribution – is defined on the interval from a = 50 dB(A) to b = 100 dB(A) expressed as [7]

$$\hat{f}_{\mathrm{U}}(x) = \begin{cases} \dfrac{1}{b-a} & \text{dla} \quad a \le x \le b \\ 0 \quad \text{dla} & x < a \cup x > b \end{cases} \tag{4}$$

illustrated in Figure 1c.

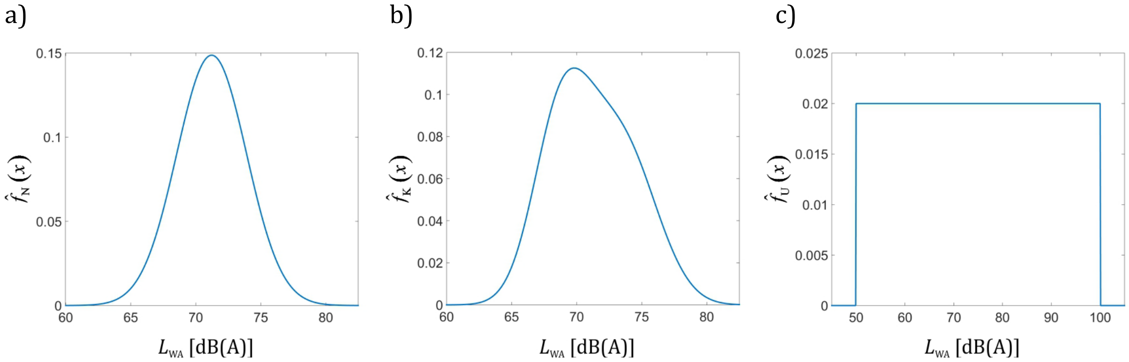

Figure 1. Probability density functions of the prior distributions: a) NORMAL, b) KERNEL, c) UNIFORM.

The Bayesian inference requires the use of numerical methods. In the hereby paper, the random walk Metropolis-Hastings sampling method (random walk M–H) [6] was used to generate samples θ_i^{BAY} from a posterior distribution. This sampling method is one of the main techniques that provide a means for drawing random samples from a posterior probability density function. The random walk Metropolis-Hastings method is a special case of the Metropolis-Hastings algorithm [9, 10].

The random walk M–H method is able to generate correlated Markov chain $\mathbf{x_{BAY}}$ from a continuous posterior distribution. The Bayesian estimate of the θ parameter is the expected value of posterior distribution, which was determined as the mean from the Markov chain after removed cycles burned [11]

$$\overline{\hat{\theta}}^{\mathrm{BAY}} = 10\lg\left[\frac{1}{N-S}\sum_{i=S+1}^{N} 10^{0.1\cdot\theta_i^{\mathrm{BAY}}}\right], \tag{5}$$

where θ_i^{BAY} – elements of Markov chain $\mathbf{x_{BAY}}$ from the posterior distribution, N – the size of generated Markov chain, S – number of burned cycles.

3. Research material

The study on the effect of the prior distribution on the results of the sound power level estimation using the engineering method in real-life situations was carried out with the use of data representing actual results. The sound power levels of the reference sound source B&K 4205 were used to illustrate the proposed solutions and carry out the analysis.

The sound power source B&K 4205 is a calibrated sound source whose output can be varied continuously between 40 and approximately 100 dB re 1 pW [12]. This is equivalent to a sound pressure level of 92 dB for wide-band noise, at a distance of 1 m from the B&K 4205, over a reflecting plane assuming a perfect hemispherical radiation pattern. The output can be wide-band pink noise in the frequency range from 100 Hz to 10 kHz, or octave-band filtered noise by using one of seven built-in octave band-pass filters [12]. The B&K 4205 consists of two separate units: the generator, containing all the controls, filters, battery pack, amplifiers and meter, and sound source HP 1001. The sound source HP 1001 contains two loudspeakers and their crossover units. A woofer loudspeaker is used for the 125 Hz, 250 Hz, 500 Hz and 1 kHz octave bands and a dome tweeter for the 2 kHz, 4 kHz and 8 kHz octave bands [12]. The sound power output of HP 1001 is controlled by an attenuator with a 40 dB range in steps of 10 dB and also by a continuously variable potentiometer.

The sound power level of this source has been determined using precision and engineering methods based on measurements of sound pressure. Measurements of sound pressure were made with the SVAN 959 sound level meter (SVANTEK, Poland) equipped with SV-type preamps and a 1/2 inch free-field 40AN microphone from G.R.A.S. The results of the background noise corrected A-weighted sound pressure levels recorded at each measurement point which have been used to determine the sound power level of the source using each method are presented in Figure 2.

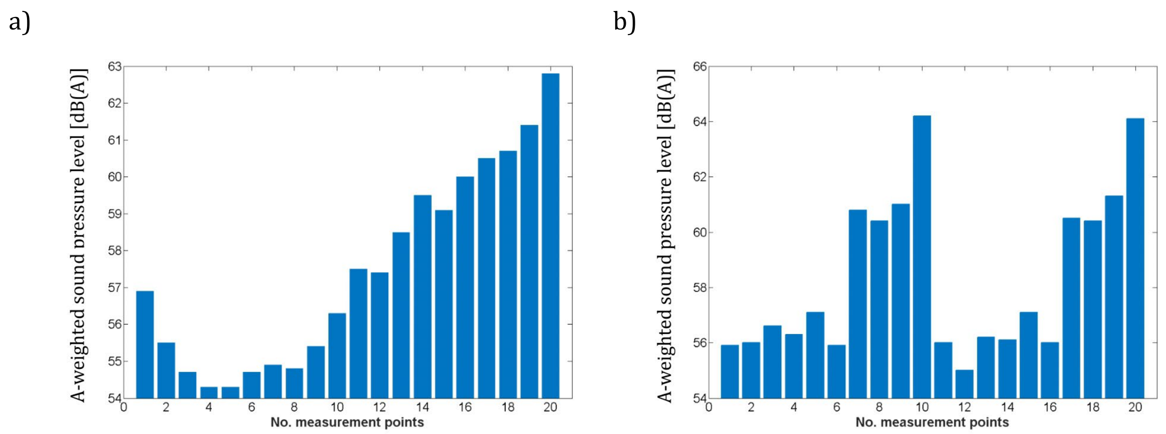

Figure 2. The A-weighted sound pressure level in measurement points recorded for:
a) precision method, b) engineering method.

The measurements for the precision method were carried out in the anechoic room. The M = 20 measurement points were located on a hemispherical measurement surface with a radius of r = 2 m in a hemi-free field according to Table E.1 in Annex E of ISO 3745:2012. On the grounds of the recorded A-weighted sound pressure levels shown in the Figure 2a, the sound power level for the precision method is L_{WApr} = (72.0±1.5) dB(A) and was used as a reference value for the analysis carried out in this study. The expanded uncertainty was determined in accordance with the methodology described in Section 10 and Annex I of ISO 3745:2012.

On the other hand, the measurements for the engineering method were made on the asphalt playing field. The M = 20 measurement points were located on a hemispherical measurement surface with a radius of r = 2 m in an essentially free field over a reflecting plane according to Table B.1 in Annex B of ISO 3744:2010. At 20 m from the measuring surface, there was no sound reflecting surfaces. Based on the A-weighted sound pressure levels (see Fig. 2b), the sound power level for the engineering method is L_{WAen} = (72.6±2.4) dB(A). The expanded uncertainty was determined in accordance with the methodology described in Section 9 and Annex H of ISO 3744:2010. These data constituted the examined population of size M = 20.

4. Experimental test

4.1. Course of the experiment

This experiment served to determine the effect of the prior distribution on the results of the sound power level estimation using the engineering method.

The original sample size n simulates the number of measurement points based on which the sound power level is determined. In this experiment, the original sample size of n = 4 was used. This value is the minimum sample size that must be used to estimate a sound power level in an engineering method using Bayesian inference [5]. For this reason, 1000 samples of size n = 4 were drawn from the examined population of size M = 20. Each drawn sample was used to determine the sampling distribution $p(\mathbf{x}|\theta)$ using KDE, which was composited with each analysed prior distribution $p(\theta)$ according to Equation 1. In this way, a posterior distribution $p(\theta|\mathbf{x})$ was obtained for each sample drawn and each analysed prior distribution.

The reconstruction of probability distributions was performed based on the same number of Markov chain elements N for each sample of size n. The distributions were determined based on N = 110000 elements of which the first 10000 were treated as burned cycles S. Thus receiving 1000 posterior distributions with 100000 elements for each original sample and each prior distribution. Each distribution was used to determine the Bayesian estimate of the expected value of sound power level from Equation 5. As a result, 1000-element probability distributions of sound power levels were obtained for each prior distribution and further statistically analysed. These distributions are presented as histograms in Figure 3.

4.2. Results and discussion

The statistical analysis aims to test whether statistically significant differences exist between the datasets analysed. The datasets analysed are probability distributions obtained using different prior distributions. The basic statistical parameters of these distributions are shown in Table 1.

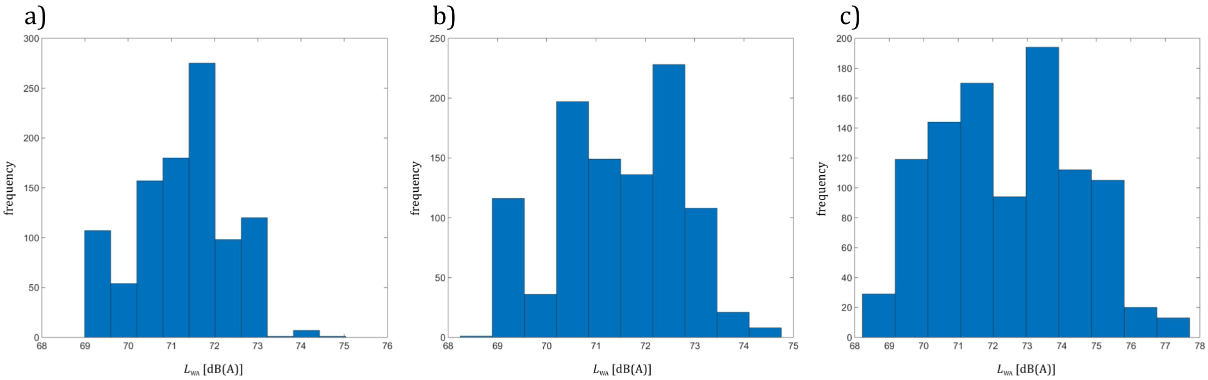

Figure 3. Probability distributions of sound power levels obtained using different prior distributions: a) NORMAL, b) KERNEL, c) UNIFORM.

As a first step, the normality of the obtained probability distributions was examined. Three normality tests were performed. For the Kolmogorov-Smirnov test, the probability value of p = 0 was obtained for all analysed distributions, while for the Lilliefors and Jarque-Bera tests, the value of this parameter was

p = 0.001 also for all analysed distributions. These values indicate that the analysed datasets do not come from a normal distribution. For this reason, non-parametric statistical tests were used.

Table 1. The basic statistical parameters of analysed distributions

Parameter	Unit	Prior distribution		
		NORMAL	KERNEL	UNIFORM
Mean	dB(A)	71.4	71.6	72.8
Standard deviation	dB(A)	1.1	1.2	2.0
Skewness	---	-0.26	-0.18	0.11
Kurtosis	---	-0.69	-0.46	-0.84

First, the Kruskal-Wallis test has been performed at the significance level α = 0.05 to check if there are statistically significant differences in estimated sound power level obtained for various prior distributions. The test gave the probability value of p = 0. This value is less than the assumed level of significance which proves the existence of statistically significant differences in at least one of the pairs compared.

In the next step, the Tukey-Kramer multiple comparison test at the significance level α = 0.05 was conducted to find out between which groups there are differences. The probability values obtained for this test are presented in Table 2. The results of the Tukey-Kramer test are also presented graphically in Figure 4. The graphs show the average value of ranks (symbol) together with the confidence level (horizontal line) for the sound power levels obtained using various prior distributions. Any two compared group averages are statistically different when their intervals are disjoint. Overlapping intervals mean that there are no statistically significant differences among the compared group averages. The results of post-hoc tests indicate that the sound power level obtained using various prior distributions are statistically different from each other, as p-values are lower than the adopted significance level α (see Tab. 2) and ranks intervals are disjoint (see Fig. 4).

Table 2. Probability values of the Tukey-Kramer test.

p-value			
Prior distribution	NORMAL	KERNEL	UNIFORM
NORMAL		0.0028	$9.56 \cdot 10^{-10}$
KERNEL	0.0028		$9.56 \cdot 10^{-10}$
UNIFORM	$9.56 \cdot 10^{-10}$	$9.56 \cdot 10^{-10}$	

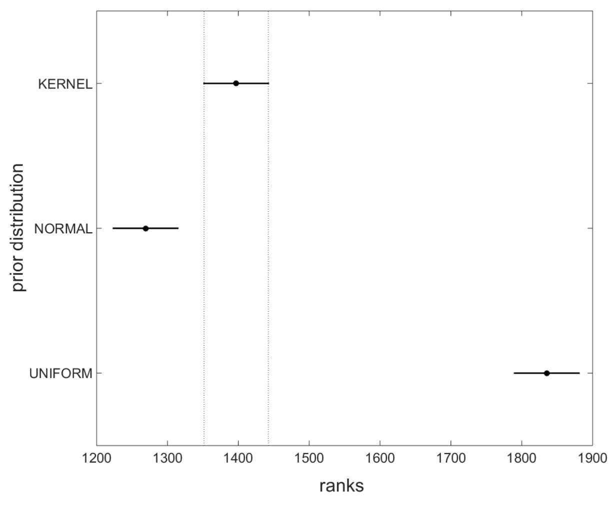

Figure 4. Results of the Tukey-Kramer test.

The bias of Bayesian sound power level estimates obtained using different prior distributions was also analysed in relation to the sound power level determined using the precision method. The mean of bias was defined as

$$\overline{b}_{L_{\mathrm{WA}}} = \frac{1}{B}\sum_{b=1}^{B}\left(L_{\mathrm{WA},b} - L_{\mathrm{WApr}}\right), \tag{6}$$

where $L_{\mathrm{WA},b}$ – Bayesian estimate of sound power level obtained using different prior distributions, L_{WApr} – sound power level determined using the precision method, B – the size of sound power level probability distributions obtained using different prior distributions, B = 1000.

For the sound power level distribution obtained using the NORMAL prior distribution, the mean value of the bias was $\overline{b}_{L_{\mathrm{WA,\,NORMAL}}} = -0.7$ dB(A), using the KERNEL prior distribution $\overline{b}_{L_{\mathrm{WA,\,KERNEL}}} = -0.6$ dB(A), while for the UNIFORM prior distribution the value of this parameter is equal to $\overline{b}_{L_{\mathrm{WA,\,UNIFORM}}} = 0.4$. This means that using NORMAL and KERNEL prior distributions the Bayesian sound power level estimates are underestimated by an average of 0.7 dB(A) and 0.6 dB(A) respectively while using UNIFORM prior distribution they are overestimated by an average of 0.4 dB(A).

5. Conclusions

This paper analysed the effect of the prior distribution on the accuracy of the engineering method for determining sound power levels using Bayesian inference. To this end, a simulation experiment was carried out to generate Bayesian probability distributions of sound power levels using three different prior distributions. Two informative distributions (NORMAL and KERNEL) and one non-informative distribution (UNIFORM) were used.

The statistical analysis was carried out based on the Kruskal-Wallis test. Next, multiple comparison procedures were used for pairwise comparisons between the means using the non-parametric Tukey-Kramer test at the significance level α = 0.05. Statistical analysis showed that the sound power level obtained using different prior distributions are statistically different at the assumed level of significance. The least bias on the expected value of the sound power level was obtained using the non-informative UNIFORM prior distribution. Based on the results presented, it can be recommended to use the uniform distribution as a prior distribution for estimating the expected value of the sound power level with the engineering method using Bayesian inference with "strong" data.

The numerical experiment results presented in this paper refer only to one sound source. The effect of the presented prior distributions on the sound power level estimation results of other sources may differ from that presented. Therefore, the prior distribution used to determine the sound power level of any other source must be adapted to the characteristics of that source, as the proposed methodology can be used to determine the prior distribution for other types of noise sources.

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Additional information

The author declares no competing financial interests and that all material taken from other sources (including their own published works) is clearly cited and that appropriate permits are obtained.

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