

# Conception of suspension system with variable stiffness for mobile robots

Sławomir DUDA <sup>©</sup>, Grzegorz GEMBALCZYK <sup>©</sup>, Zygmunt KOWALIK <sup>©</sup>, Tomasz MACHOCZEK <sup>©</sup>

Silesian University of Technology, Akademicka 2A, 44-100 Gliwice, Poland

Corresponding author: Grzegorz GEMBALCZYK, email: grzegorz.gembalczyk@polsl.pl

**Abstract** The paper presents the results of work on an innovative form of the suspension node. It is a concept of semi-active suspension that allows for control the stiffness system during operation of the device. This type of suspension can be used in robotic intralogistics systems, where mobile platforms must have a suspension that ensures adequate vibration damping, keeps the platform in a horizontal position, and at the same time provides adequate pressure to the ground of the drive wheels. A particularly interesting object with the potential possibility of using such a suspension are vehicles with omnidirectional wheels. The proposed design form is the first prototype version, which should primarily be used to validate computational models. The idea of the developed suspension element refers to the double torsion shafts.

**Keywords:** suspension, variable stiffness mechanism, bending, rotational compliance, omnidirectional wheels.

#### 1. Introduction

The most common method to counteract undesirable vibrations that occur in mechanical systems under a wide range of excitations, in order to achieve the desired dynamic behaviour of the system, is through design modifications. This is achieved by a formal modification of the parameters of these systems: their weight, rigidity, and damping [1].

Vehicle suspension is a system collected of springs, shock absorbers, and arms which connects a chassis to its wheels and allows relative motion between the two [2]. Suspensions are widely studied to improve ride comfort and road holding. According to the nature of the force-producing elements, suspension designs are classified into three main types [3]: passive suspension, semiactive suspension and active suspension.

Subject literature provides numerous studies on the determination of the optimum parameters of the suspension system for passive suspensions. The article [4] presents the results of research and development work on the passive suspension of an autonomous mobile robot with omnidirectional wheels. The work uses an optimization algorithm to select the parameters of the suspension system that meet the requirements of operating efficiency in a wide spectrum of kinematic inputs.

Active reduction of vibrations in mechanical systems is most often connected to the concepts of variable damping and variable rigidity. In order to ensure an effective vibration isolation solution, the modified parameter should be realised in real time mode [5]. The variation of damping will induce a change on the resonance magnitude through adjusting the dissipated vibration energy, and the variable stiffness will change the vibration transmissibility by changing the natural frequencies of the controlled system. In view of the a multitude of solutions on the vibration reduction, many researchers extensive research carried on with the concept of variable stiffness or damping.

One of the most commonly analysed methods to shape a dynamic response of vehicle suspensions in literature is controlling the vibrations by variable damping with the use of magnetorheological fluids (MRF). Magnetorheological fluid dampers are typical semi-active dampers with variable damping capability and have been widely used for vibration attenuation due to their reliability and fast response [6, 7].

The solutions that are less frequently presented in literature are based on the variable stiffness of the spring element [8]. In study [9], Youn and Hac developed a suspension of variable stiffness, using a semiactive air spring to change the stiffness of the suspension. The system offers the possibility to set three values of the stiffness of the spring. The conducted experiments verified the effectiveness of variable stiffness of the suspension in minimising the vibrations of the vehicle body. Another approach to actively selecting the stiffness of the suspension is modifying the geometry of the suspension. It may be done by modifying the position of the spring-damper set. The modification of the shock absorber mount allows to modify the stiffness of the suspension, which enables to adapt the suspension to the current road conditions in order to improve the comfort of travelling and driving safety. The effect of active control of variable suspension stiffness presented in the studies [10, 11] was obtained by the authors with the use of an electromechanical engine that was mounted on the body of the vehicle and powered a crank mechanism. The shock absorber was fitted articulated on the end of the crank. This allowed changing the position of the mounting of the shock absorber, which in turn allowed modifying the suspension stiffness.

In doctoral dissertation entitled "Shaping the dynamic characteristics of omnidirectional wheel-based mobile robot suspensions", author developed a passive suspension for omnidirectional wheels used in an AGV vehicle. The selection of the optimum parameters of the suspension system that ensure the minimisation of dynamic excitations originating from omnidirectional wheels [12-15] while driving was conducted for two speeds: 3 kph and 5 kph. These are the speeds at which the vehicle most often drives while performing its internal logistic tasks. The parameters of the suspension system, including the stiffness of springs, selected by the optimisation algorithm, are, however, a compromise between the most effective vibration insulation for each of these two kinematic excitations.

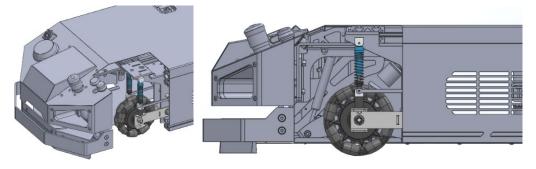


Figure 1. Model of the developed suspension node in an AGV vehicle.

The authors of this study present a concept of a mechanical system that enables the control of its stiffness, designated for the suspension of omni-directional wheels used in mobile robot solutions.

## 2. General assumptions of the suspension node design

The paper analyses the concept of using a suspension system with the spring element of a variable characteristics (Fig. 2).

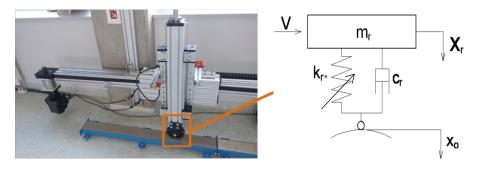


Figure 2. Target test site and model of a regulated stiffness suspension system.

The problems of selecting the optimum parameters of a suspension system for vehicles equipped with omni-directional wheels involve three main aspects that correspond to traditional suspension systems of vehicles moving on wheels, and are related to:

- minimising the transfer of vibrations caused by the shape of the envelope of an omnidirectional wheel;
- maximising the vehicle's stability while accelerating, braking and overcoming obstacles, i.e. minimising the oscillating tilt of the vehicle to the front and rear and minimising the phenomenon of driving outside the balance position;
- maximising the traction of the driving wheels during accelerating, braking, and turning.

Each of the discussed aspects is important for the functioning of the vehicle in the industrial environment, and failure to meet those conditions may result in damaging the vehicle, loss of adherence, falling out of the route or collision.

The most common operational speeds for the analysed AGV are the speeds of 3 km/h and 5 km/h. The frequency of excitations from omni-directional wheels (wheel model 125 mm Rotacaster, manufacturer's catalogue No. R2-1258-95/S10) assuming the perfect geometry of the wheel and driving at the speed of 3 km/h is  $f_1 = 34$  Hz and for the speed V = 5 km/h it is  $f_2 = 56$  Hz.

In order to meet the above assumptions, the optimum solution is to apply an active suspension system that would adapt its spring and damping characteristics to the current driving conditions.

#### 3. Concept of suspension node with modified stiffness

The test site presented in Figure 2 reflects the conditions of mounting the suspension node of the actual object presented in Figure 1. The static load of the weight of the body, both on the proposed test site and in the actual vehicle, is 10 kg. The authors considered using a regulated stiffness torsion spring as an intermediary element between the wheel and the vehicle body. The idea to use such solution was drawn from the work by Professor Murat Reis [16], who proposed this solution for applications in mobile robotics, in "series elastic actuators" drives. The structure of the regulated stiffness system was modified to adjust it to the current requirements and the previous deficiencies, which resulted from wedging of the beams while their active length was changed, were eliminated (by applying self-aligning bearings). Figure 3 presents the general concept of a regulated stiffness torsion spring.

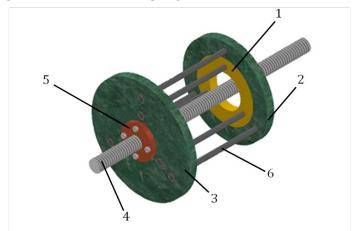


Figure 3. Simplified, conceptual model of a variable stiffness mechanism.

The specific markings refer to: number 1 refers to the part that is stiffly fixed to the vehicle frame. Number 2 marks the element that is stiffly connected to the wishbone of the analysed suspension node. Number 3 marks the control disc that is used to change the stiffness of the connection by changing the active length of beams (6). The control disc is connected to the frame and the wishbone by flexible beams marked by number 6. The position of the control disc and its distance from the suspension node is regulated by transferring the rotational movement of the trapezoidal bolt marked 4 to the linear movement carried out by the trapezoidal nut marked by number 5.

The essence of the functioning of the proposed system with regulated stiffness of the spring elements is connected to the change in the active length of the flexible beam (marked with 6 in Figure 3), which also translates in the change in the torsion angle at the same load (torque). Therefore, the key characteristic of the system is the function that relates the moment *M* loading disc 2 (connected to the wishbone) to the angle  $\theta$  of its rotation. This function may be determined by analysing the cooperation between individual elements of the mechanism.

Assuming that a single bar is deformed only by the bending moment induced by the force *P* acting at its end perpendicular to the axis of the undeformed bar, the  $\delta$  can be determined from the formula:

$$\delta = \frac{Pl^3}{3EI_x},\tag{1}$$

where: l – initial length of the deflected bar, E – Young modulus of the material from which the beam is made,  $I_x$  – the geometric moment of inertia of the beam cross-section.

The work of a single beam in the mechanism, e.g. the beam connecting discs marked 2 and 3 in Figure 3 is presented schematically in Figure 5.

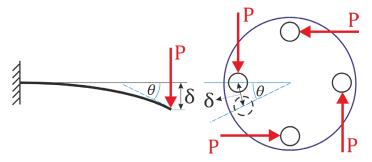


Figure 4. Schematic presentation of the operating principle of the stiffness regulation mechanism [1].

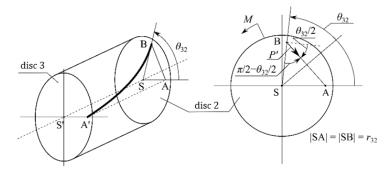


Figure 5. Schematic presentation of the deformation of a single beam in the mechanism.

The beam is mounted to the discs in such a way that it goes through the holes that are located at the same distances from the centres of the discs, i.e.  $r_{32} = |SA| = |S'A'|$ . Moment *M* is thus counteracted by the moment of reaction forces *P'* that emerge at the fixing points of the beams on disc 2. Therefore, for *n* beams, the formula is:

$$M = nP'r_{32}\cos\frac{\theta_{32}}{2},$$
 (2)

where  $\theta_{32}$  – the deflection angle of disc 2 to disc 3.

At the same time, from Figure 5 it may be noted that the deflection of the beam equals:

$$|AB| = 2r_{32}\cos\left(\frac{\pi}{2} - \frac{\theta_{32}}{2}\right) = 2r_{32}\sin\left(\frac{\theta_{32}}{2}\right).$$
(3)

The reaction forces P' are equal in value to the forces causing the deformation of the members. Thus, combining equations (1), (2) and (3) one obtains:

$$\theta_{32} = \arcsin\left(\frac{Ml^3}{3nEI_x r_{32}^2}\right). \tag{4}$$

The device constructed as shown in Figure 3. contains 2 pairs of discs, which are marked in the drawing as pairs: 3-2 and 3-1. They form a series connection between two spring systems, so they are loaded with a moment of the same value, and the angles by which the discs forming the pair rotate relative to each other (labelled  $\theta_{32}$  and  $\theta_{31}$  respectively) add up to the overall system deflection angle  $\theta$ . When  $r_{31}$  denotes the distance between the centre of discs 3 and 1, and the whole through which the beams connecting those discs pass, the following final formula is obtained:

$$\theta = \theta_{32} + \theta_{31} = \arcsin\left(\frac{Ml^3}{3nEI_x r_{32}^2}\right) + \arcsin\left(\frac{Ml^3}{3nEI_x r_{31}^2}\right).$$
(5)

Expanding formula (5) into a Taylor's series around the value of the moment M = 0, for small moments  $\Delta M$  and deflection angles  $\Delta \theta$  the following is obtained:

$$\Delta \theta \cong \underbrace{\frac{l^3}{3nEI_x} \left(\frac{1}{r_{32}^2} + \frac{1}{r_{31}^2}\right)}_{k} \Delta M.$$
(6)

Formulas (5) and (6) assume that the beams that connect two cooperating discs are the same, i.e. that they are constructed from identical material and have the same geometric moment of inertia of the cross-section. For this assumption, the number k in equation (6) may be interpreted as the susceptibility factor of the described system of various stiffness.

### 4. Prototype of the device and determination of the stiffness characteristics

The prototype of the variable stiffness system was designed and then constructed at the Faculty of Mechanical Engineering of the Silesian University of Technology. The discs were created in 3D printing technology, and the beams were made from piano wire. The other elements, such as bearings, trapeze bolt and the matching nut, were selected from among standard elements.

Having constructed the prototype, the authors prepared a laboratory test stand to determine the stiffness characteristics of the mechanism depending on the active length of the spring elements, i.e. the distance between the fixed disc (No. 1 in Figure 3) and the regulation disc (No. 3 in Figure 3) and depending on the moment loading the disc connected to the wishbone (No. 2 in Figure 3). A schematic presentation of the test site is provided in Figure 6.

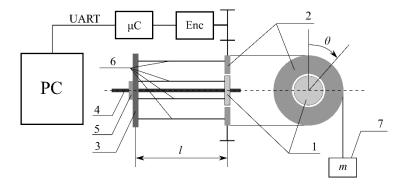


Figure 6. Target test stand model of a regulated stiffness suspension system.

The test site was designed so as to enable the measurement of the rotation angle  $\theta$  of disc 2 for the defined distance between discs 1 and 2 and regulation disc 3, depending on the torque loading the disc. The torque loading disc 2 is generated by a suspended weight of the mass of (mass *m* marked with No. 7 in Figure 6).

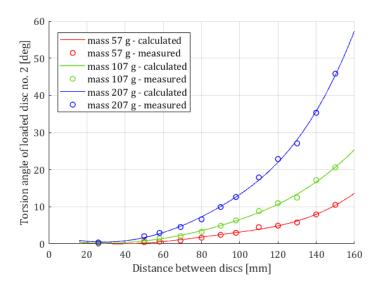
The torque arm from the weight of the suspended mass was 35 mm. Tests were conducted for 3 different weights of the mass of: 57 g, 107 g, and 207 g. The rotation angle  $\theta$  was measured with use of a two-channel incremental encoder, marked as "Enc." in Figure. 6. The encoder of a base resolution of 3600 imp/rotation was used. The rotational movement of disc 2 was transferred to the rotational movement of the encoder's disc with use of a belt transmission with a rubber V-belt. The transmission wheel on the side of disc 2 had the diameter of  $D_d = 68$  mm, and on the side of the encoder  $D_{enc} = 34$  mm. The impulses generated by the encoder were connected to digital outputs of the microcontroller (marked as " $\mu$ C" in Figure. 6) configured as external interrupt pins. Tests were conducted with the use of Teensy 4.1. microcontroller with ARM Cortex M7 processor, with a clock frequency of 600 MHz. The software written by the authors was used to read the number of impulses generated on both channels of the encoder by counting both the falling and rising edges of both signals in a procedure executed during an external interrupt, which increased the effective resolution of the encoder to 14400 imp/rev [17]. The number of impulses  $n_{imp}$  counted during the measurement was then sent to a PC via the UART interface and converted to the rotation angle  $\theta$  of disc 2 from the formula:

$$\theta = 360^{\circ} \frac{D_{\rm enc}}{D_d} \frac{n_{\rm imp}}{14000}.$$
 (7)

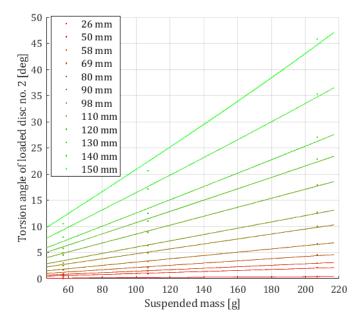
The resolution of the angle measurement conducted in this was  $1.25 \cdot 10^{-2}$  degrees.

The results of measurements of the deflection angle of disc 2, depending on the distance between discs *l*, are presented in Figure 7. Different colours were used to mark the series for 3 analysed values of weight suspended on the cord connected to disc 2. The curves with the designation "calculated" in the diagram were determined from equation (5), based on the data presented in Table 1.

Parameter	Symbol	Unit	Value
Young's modulus	Ε	GPa	206
Geometrical moment of inertia	$I_x$	$\rm mm^4$	0.7545
Distance between discs 1-3 and 2-3	l	mm	Variable during the experiment
Radius of the bar fixation between the discs 1 and 3	$r_{31}$	mm	18
Radius of the bar fixation between the discs 2 and 3	$r_{32}$	mm	30



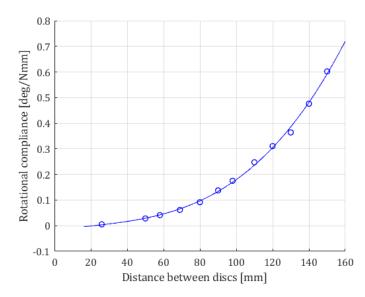
**Figure 7.** Characteristics of the torsion angle of the system, determined for different values of the torque generated by a suspended mass.



**Figure 8.** Characteristics of the torsion angle of the system as a function of the weights suspended on disc no. 2 of the analysed system, determined for different values of distance between the discs.

In Figure 8, the data collected during the tests are presented in a different way in order to highlight the relation between the torque affecting disc 2 and caused by the *m* and the deflection angle of this disc. Different colours represent data series for the analysed values of distance between the discs of the tested spring. The curves were drawn based on equation (5).

Figure 9 presents the data that refer to the elasticity factor of the analysed spring depending on the distance between the discs of the spring. Points mark the data calculated as directional coefficients of the line, selected for subsequent series of data presented in Figure 7. The drawn curve was determined based on equation (6) by calculating the coefficient denoted as k for different values of the distance between the discs in the system.



**Figure 9.** Characteristic of the rotational compliance of the analysed system as a function of the distance between the discs.

#### 5. Conclusions

The article presents a concept of a variable stiffness system that may be applied in suspension systems of small autonomous vehicles in order to adapt their frequency characteristic to excitations of a frequency that varies during the operation of the vehicle, especially when omni-directional wheels or Mecanum type wheels are applied. According to the presented concept, the stiffness of the system may be modified by changing the distance between discs that are connected with elastic beams.

The simplified theoretical analysis of the system assumes that during the operation, the beams are subject to deformations, whose only cause is the emerging bending moment. The analysis was the basis for the identification of the structural parameters of the system that influence its properties. These are: the diameter of the beams, the material from which the beams are made, and the distance between the centres of the discs and the holes in which the beams are embedded. The conducted analysis revealed that the elasticity of the system is proportional to the cube of the distance between the cooperating discs.

During the study, a prototype of the device was constructed and tested (Fig. 10).

The collected measurement data confirmed the results of theoretical analysis. This, in particular, confirms the correctness of the assumption that the beams in the device are first of all subjected to the load of the bending moment and that the formulated mathematical model can be used for further calculations. The measurement results also revealed that, for a specific distance between the discs, the deflection of the system is approximately proportional to the value of the loading torque, i.e. that the system behaves similarly to a classical spring.

Further work on a spring with variable stiffness should involve automating the stiffness adjustment system and developing a structure enabling its implementation on a test stand.

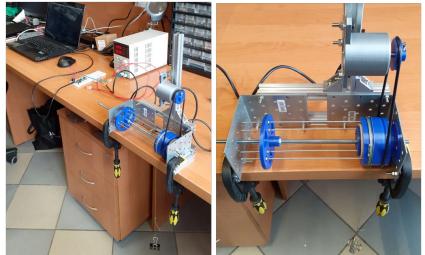


Figure 10. Prototype of variable stiffness system used in experiments.

# Acknowledgments

The research reported in this paper is the result of the PBL project entitled "Research on semi-active suspension systems with variable stiffness actuators". The PBL team consisted of: G. Gembalczyk (main supervisor), S. Duda, Z. Kowalik (auxiliary supervisors), P.L., O.K., A.K., M.S., K.J., M.W. (students). The PBL project was co-financed by the European Union from the European Social Fund in the framework of the project "Silesian University of Technology as a Center of Modern Education based on research and innovation" POWR.03.05.00-00-Z098/17.

## **Additional information**

The author(s) declare: no competing financial interests and that all material taken from other sources (including their own published works) is clearly cited and that appropriate permits are obtained.

# References

- 1. D.J. Inman; Vibration with control; John Wiley & Sons, 2017
- 2. R.N. Jazar; Vehicle Dynamics, 2nd ed.; Springer, 2013
- 3. C. Alexandru, P. Alexandru; A comparative analysis between the vehicles' passive and active suspensions; Int. J. Mech., 2011, 5(4), 371-378
- S. Duda, G. Gembalczyk, W. Klein, O. Dudek; Tuning of characteristics of dynamic driving suspensions in an autonomous robot with omnidirectional wheels; J. Theor. Appl. Mech., 2022, 60(2), 293-302; DOI: 10.15632/jtam-pl/149415
- 5. P. Martynowicz, S. Kciuk, A. Mężyk; Rotary shock-absorber with magnetorheological valves; Adv. Mat. Res., 2013, 628, 505-511; DOI: 10.4028/www.scientific.net/AMR.628.505
- 6. E. Świtoński, A. Mężyk, S. Duda, S. Kciuk; Prototype magnetorheological fluid damper for active vibration control system; J. Achiev. Mater. Manuf. Eng., 2007, 21(1), 55-62
- S. Sun, J. Yang, W. Li, H. Deng, H. Du, G. Alici; Development of a novel variable stiffness and damping magnetorheological fluid damper; Smart Mater. Struct., 2016, 24(8), 085021; DOI: 10.1088/0964-1726/24/8/085021
- 8. I. Youn, A. Hać; Semi-active suspensions with adaptive capability; J. Sound Vib., 1995, 180(3), 475-492; DOI: 10.1006/jsvi.1995.0091
- 9. R. Tavares, J.V. Molina, M. Al Sakka, M. Dhaens, M. Ruderman; Modeling of an active torsion bar automotive suspension for ride comfort and energy analysis in standard road profiles; IFAC-PapersOnLine, 2019, 52(15), 181-186; DOI: 10.1016/j.ifacol.2019.11.671
- 10. C. Arana, S.A. Evangelou, D. Dini; Series active variable geometry suspension for road vehicles; IEEE/ASME Trans. Mechatron., 2015, 20(1), 361-372; DOI: 10.1109/tmech.2014.2324013

- M. Yu, C. Arana, S.A. Evangelou, D. Dini; Quarter-car experimental study for series active variable geometry suspension; IEEE Trans. Control Syst. Technol., 2019, 27(2), 743-759; DOI: 10.1109/tcst.2017.2772912
- S. Duda, O. Dudek, G. Gembalczyk, T. Machoczek; Developing a test site for testing the suspension of vehicles with omnidirectional wheels; Vib. Phys. Syst., 2020, 31(3), 2020304; DOI: 10.21008/j.0860-6897.2020.3.04
- 13. N. Thongpance, P. Chotikunnan; Design and Construction of Electric Wheelchair with Mecanum Wheel; J. Robot. Control (JRC), 2023, 4(1), 71-82; DOI: 10.18196/jrc.v4i1.17095
- 14. E. Rubies, J. Palacín, E. Clotet; Overview of the Trajectories of an Omnidirectional Mobile Robot based on a Single Motion Command; 17th Iberian Conference on Information Systems and Technologies (CISTI), Madrid, Spain, 2022, 1-4; DOI: 10.23919/CISTI54924.2022.9820053
- J. Palacín, E. Rubies, E. Clotet, D. Martínez; Evaluation of the Path-Tracking Accuracy of a Three-Wheeled Omnidirectional Mobile Robot Designed as a Personal Assistant; Sensors, 2021, 21(21), 7216; DOI: 10.3390/s21217216
- 16. M. Reis, N. Ebrahimi, A. Jafari; Elastic Actuator Design Based on Bending of Cylindrical Beam for Robotic Applications; Actuators, 2020, 9(3), 80; DOI: 10.3390/act9030080
- W. Burlikowski, P. Kielan, Z. Kowalik; Synchronous Reluctance Machine Drive Control with Fast Prototyping Card Implementation; Arch. Electr. Eng., 2020, 69, 757–769; DOI: 10.24425/aee.2020.134627

© **2024 by the Authors.** Licensee Poznan University of Technology (Poznan, Poland). This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (http://creativecommons.org/licenses/by/4.0/).