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The use of acoustic metamaterials to build a broadband sound diffuser

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Abstract In this article, the author analyses the potential for extending the frequency range of sound diffusion through the use of different structures of acoustics metamaterial cells. The author carried out numerical calculations on various diffusers that consist of slots with Helmholtz resonators connected either in parallel or in series. Through the use of numerical models, the author optimized these structures to achieve a high diffusion coefficient in a wide frequency range. The author compared these results with calculations for a sound diffuser made of slits. Ultimately, the author designed a sound diffuser that is capable of diffusing sound in a broad frequency range.

Keywords: metamaterial, sound diffusion, diffusion coefficient.

1. Introduction

Sound diffusers are commonly used in various settings, such as listening rooms, recording studios, and large auditoriums, to control the initial sound reflections. Their purpose is to evenly distribute acoustic energy in all directions, which helps to eliminate acoustic issues like echoes and flutter echoes, as well as reduce the intensity of the first reflections [1]. It is important to design diffusers based on their specific location within the room. For instance, when designing diffusers for concert halls, optimization techniques are employed to minimize sound absorption, eliminate specular reflections, and redirect sound reflections [2]. Additionally, the optimized sound reflection patterns of diffusers can be directly incorporated into the geometrical models of rooms [3].

The principle of operation for commonly used diffusers involves shaping the directivity characteristics of the reflected sound by introducing local changes in the phase delay of the reflected sound within slots of varying depths [4]. The frequency range over which a diffuser operates effectively depends on its ability to achieve variable phase delay in the sound reflected by the individual cells or slots that make up the diffuser. The maximum phase delays for slot-type diffusers, such as QRD and PRD, occur at quarter wavelength resonances. These resonances are described by equation:

$$l_s = \frac{(2n-1)c_0}{4f}, \tag{1}$$

where l_s is the depth of the slit, n = 1, 2, 3, … are odd multiple, c_0 is the sound and f is frequency.

In metamaterial structures, the phase delay is manipulated by altering the dimensions of the structure. The common structures used to construct the cells are slots loaded with quarter-wave resonators [5] or Helmholtz resonators [6]. These structures exhibit unique properties not found in natural materials, such as negative effective bulk modulus, negative mass density [7], or slow sound [8,9]. This allows to achieve lower resonant frequencies and sound diffusion at lower frequencies.

This research builds upon previous work of the author on sound diffusers using acoustic metamaterials. One study focused on the numerical modelling of sound diffusion using diffusers composed of slits with quarter-wave resonators [5]. Another study examined the use of a slit loaded with a curved quarter-wave resonator for sound diffusers [10].

This article analyses the potential for constructing broadband diffusers using acoustic materials. Two cell structures of metamaterials, slots with serially and parallelly connected Helmholtz resonators, were investigated. Additionally, a modification to the cell shape was introduced to achieve sound diffusion over a wider frequency range.

2. Parameters of the sound diffusers, cells and slots

The quality of diffusers characterizes the diffusion coefficient. This parameter is determined by the angular distribution of the sound pressure $p_s(\theta)$ reflected by structure in the space over the top surface of the diffuser. Each cell or slot is described by the reflection coefficient R, which contains information about the amplitude of the reflected wave and the phase delay. To obtain a high coefficient of sound diffusion, the phase delays introduced by individual cells should be differentiated, while the modulus should be close to 1 so that the structure does not absorb sound. To calculate the far-field sound pressure $p_s(\theta)$ reflected by diffuser or reference plate with a finite width of $2b$ for an angle θ, the Fraunhofer formula [1] can be used:

$$p_s(\theta) = \int_{-b}^{b} R(x)\mathrm{e}^{jk_0 x \sin\theta} dx, \tag{2}$$

where: θ – angular coordinate, k_0 – wave number in the air, R – reflection coefficient.

The values of the sound pressure $p_s(\theta)$ reflected by diffusor in the space over the diffuser were determined in numerical model with use of the Finite Elements Method (FEM).

Next, the sound pressure $p_s(\theta)$ reflected by diffusor were used to calculate the diffusion coefficient of the sound diffuser d_θ according to the equation:

$$d_\theta = \frac{\left(\sum_{i=1}^{n} |p_s(\theta)|\right)^2 - \sum_{i=1}^{n} |p_s(\theta)|^2}{(n-1)\sum_{i=1}^{n} |p_s(\theta)|^2}, \tag{3}$$

where: n – the number of points distributed over diffuser.

Likewise, the diffusion coefficient of a flat reference plate d_p can be determined according to the equation (3), which involves the sound pressure $p_s(\theta)$ reflected by the reference plate. To eliminate the influence of diffusion at the edge of the diffuser, the diffusion coefficient d_θ of the diffuser is normalized to the diffusion coefficient d_p of a flat reference plate with identical dimensions as the diffuser:

$$d_n = \frac{d_\theta - d_p}{1 - d_p}, \tag{4}$$

where: d_n – normalized diffusion coefficient of the sound diffuser, d_θ - diffusion coefficient of the sound diffuser, d_p – diffusion coefficient of the reference plate.

The article also analyses the phase delay ϕ caused by the metamaterial cells and slots. This parameter indicates the delay between the incident wave and the reflected sound wave. The phase delay ϕ was calculated as the argument of the complex sound reflection coefficient R using equation:

$$R = \mathrm{Arg}\left(\frac{p_r}{p_i}\right) \tag{5}$$

where: p_r – sound pressure on the top surface of cell in Pa, p_i – incident sound pressure on the top surface of cell in Pa.

3. Numerical modelling of the sound diffusion

3.1. Models of the sound diffusers

Two different types of acoustic metamaterial cells were used to construct the diffusion panels. The first type, referred to as HR-Ds, consisted of slots with a Helmholtz resonator connected in series (Fig. 1b). The second type, HR-Dpv, involved slots with a Helmholtz resonator connected in parallel in a vertical arrangement (Fig. 1c). The cell with the resonator connected in parallel was modified from existing literature [6], with the resonator placed at the bottom of the slot instead of the middle, resulting in a lower resonant frequency and enabling sound diffusion at lower frequencies.

In the next step, numerical models of diffusers were made. It was assumed that all structures would consist of 11 slots or cells, denoted as N = 11. The structures were also assumed to be symmetrical. Each cell would have a width of d = 50 mm and a depth of h = 100 mm. Additionally, a constant distance of 5 mm was maintained between the cells, resulting in an effective width of 45 mm for each cell.

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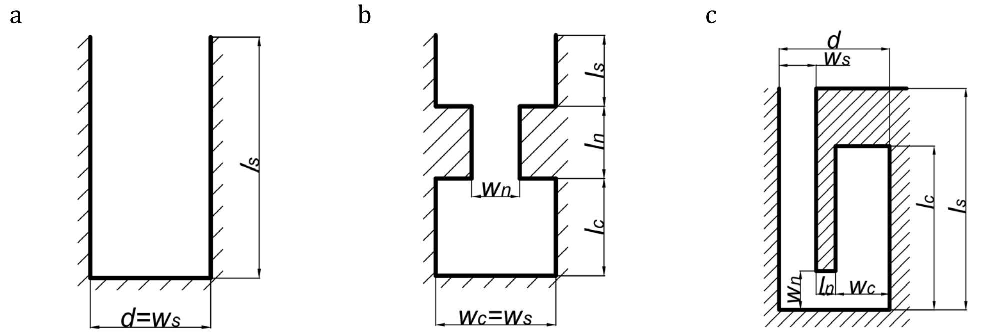


Figure 1. The cell models investigated in this study: a) a slot, b) a slot with a Helmholtz resonator connected in series, and c) a slot with a Helmholtz resonator connected in parallel in a vertical arrangement.

The QR-D diffuser was composed of quarter-wave resonators in the form of slots (Fig. 2a). The phase delay of the reflected wave in this type of structure is achieved by adjusting the depth of the individual slots. In the HR-Ds diffuser, which consists of Helmholtz resonators connected in series (Fig. 2b), the phase delay can be controlled by altering the total depth of the cell and adjusting the dimensions of the Helmholtz resonator. This involves modifying the width, length, and depth of the resonator's neck and volume. The cell geometry is characterized by four parameters. The HR-Dpv cell design is based on a slit loaded with a Helmholtz resonator in a vertical arrangement (Fig. 2c). This design allows for the adjustment of the phase delay of the reflected sound through variations in the dimensions of the resonator and the width of the main slot. For all cells a constant total cell depth h = 100mm was assumed.

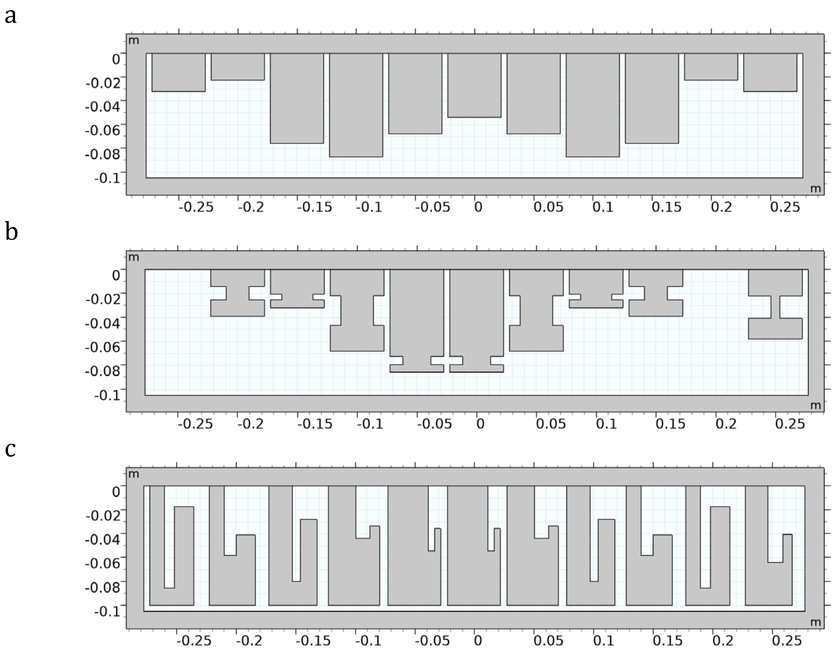


Figure 2. The cross-sectional configuration of sound diffusers:
a) QR-D, b) HR-Ds, c) HR-Dpv.

The diffuser models were created using COMSOL-Multiphysics. Two-dimensional models were constructed, featuring a circular main area with a 4-meter diameter, filled with air (Fig. 3a). Within the domain, a single period of the sound diffuser, consisting of N = 11 cells (Fig. 3b), was positioned in the centre.

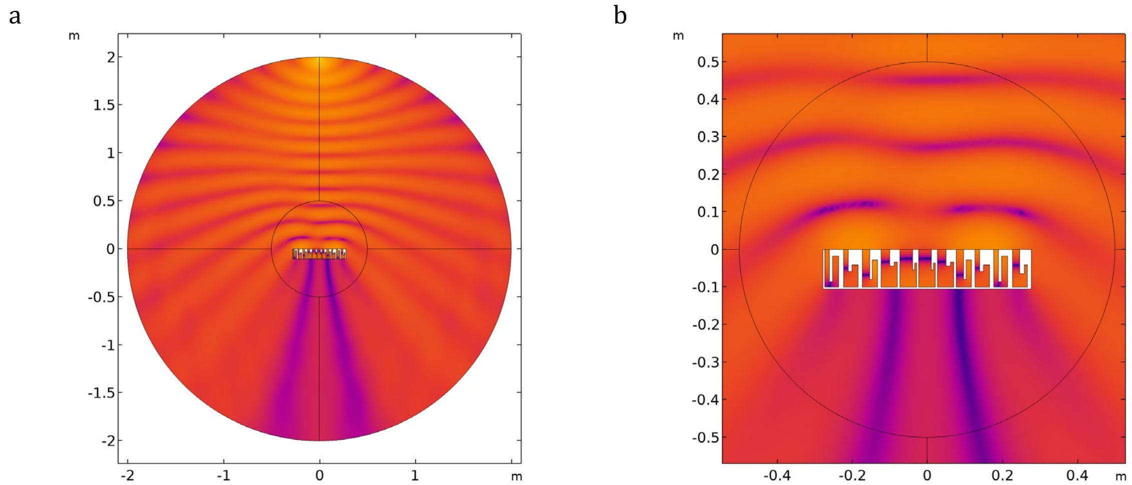


Figure 3. Model of the sound diffuser: a) diffuser placed in the main circular domain filled with air, b) cells with rigid boundary and filled with air including thermos-viscous losses.

The diffuser had a rigid boundary. Thermo-viscous losses were considered in narrow slots. To ensure free field conditions, the boundary condition of radiating a cylindrical wave was applied at the outer edge of the main domain. The far-field sound pressure $p_s(\theta)$ reflected by diffusor and the flat surface was calculated. The diffusion coefficients d_θ of the diffuser, reference plate d_p and normalized diffusion coefficient d_n were then calculated using formulas (3 and 4). Models were utilized then in the optimization process.

3.2. Optimization

The dimensions of the diffusers were optimized to maximize the sound diffusion coefficient in a broad range of frequencies. The Particle Swarm Optimization technique was used to optimize the dimensions of each individual cell in the diffusers. A fixed cell width of 50 mm and a total diffuser depth of 100 mm were assumed. The objective function for the optimization was defined as $F_0 = 1 - \overline{d_n}$, and the optimization process was carried out within the frequency range of 400-3150 Hz. The outcome of the optimization provided specific dimensions for the diffusers that ensured optimal sound diffusion in the assumed frequency range (see Table 1, 2, and 3).

Table 1. Optimized dimensions of the QR-D sound diffuser.

Parameter											
N	1	2	3	4	5	6	7	8	9	10	11
l_s [mm]	32.2	22.7	75.9	87.3	67.7	53.9	67.7	87.3	75.9	22.7	32.2
w_s [mm]						45.0					
d [mm]						50.0					
h [mm]						100.0					

Table 2. Optimized dimensions of the HR-Ds sound diffuser.

Parameter											
N	1	2	3	4	5	6	7	8	9	10	11
l_t [mm]	0.0	39.1	32.0	68.2	85.7	85.7	68.2	32.0	39.1	0.0	58.1
l_s [mm]	0.0	14.1	20.7	21.9	72.6	72.6	21.9	20.7	14.1	0.0	22.0
l_n [mm]	0.0	11.2	4.6	24.7	6.9	6.9	24.7	4.6	11.2	0.0	18.6
w_n [mm]	0.0	19.1	26.1	27.2	23.4	23.4	27.2	26.1	19.1	0.0	7.4
l_c [mm]	0.0	13.8	6.72	21.6	6.3	6.3	21.6	6.72	13.8	0.0	17.4
w_s [mm]						45.0					
d [mm]						50.0					
h [mm]						100.0					

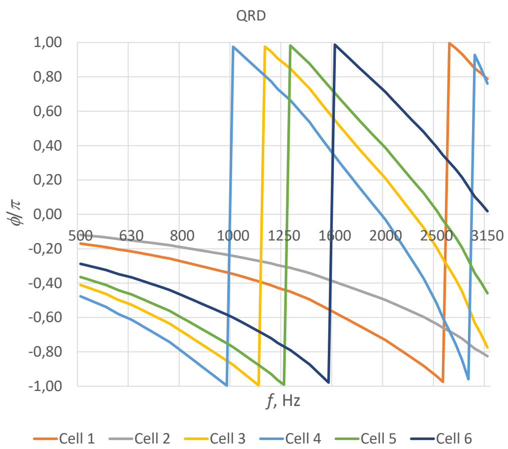
Table 3. Optimized dimensions of the HR-Dpv sound diffuser.

Parameter											
N	1	2	3	4	5	6	7	8	9	10	11
w_s [mm]	12.3	12.5	19.7	23.0	33.8	33.8	23.0	19.7	12.5	12.3	18.9
l_n [mm]	8.3	10.0	6.7	11.9	5.3	5.3	11.9	6.7	10.0	8.3	12.6
w_n [mm]	14.4	41.8	20.0	56.2	45.6	35.8	45.6	56.2	20.0	41.8	14.4
l_c [mm]	82.7	59.0	72.2	66.6	64.4	64.4	66.6	72.2	59.0	82.7	59.5
w_c [mm]	16.3	16.1	14.0	8.2	5.2	7.8	5.2	8.2	14.0	16.1	16.3
d [mm]						50.0					
$h=l_s$ [mm]						100.0					

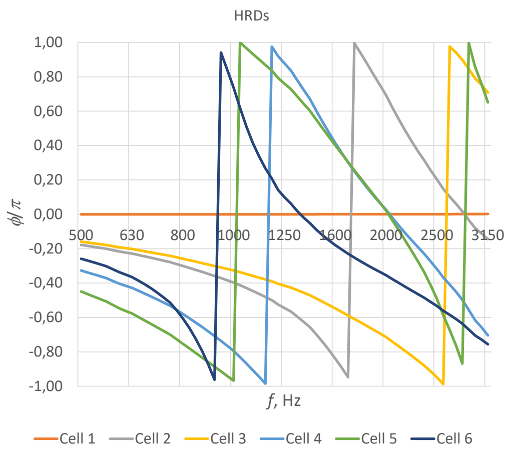
3.3. Analysis of results

For the optimal dimensions of the diffusers, numerical calculations of the sound pressure $p_s(\theta)$ reflected by the structures were carried out and then the normalized diffusion coefficient d_n and the phase delay ϕ for individual cells. The analysis of the phase delay ϕ is presented in the graphs (Figs. 4 a, b, c).

a



b



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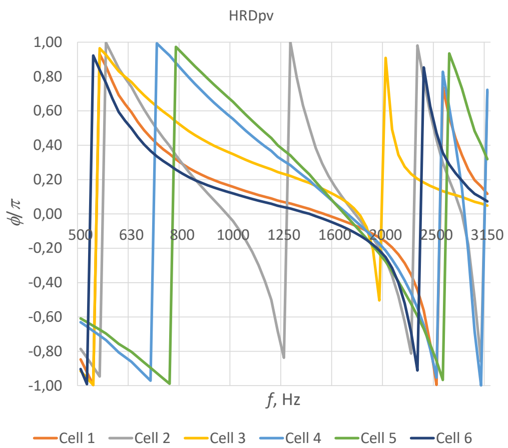

Figure 4. Comparison of phase delays introduced by cells and the slots in optimized diffusers: a) QR-D, b) HR-Ds, c) HR-Dpv.

To achieve a high sound diffusion coefficient, it is necessary to have diverse phase delays for the reflected sound from the cells. The best results are obtained when the phase delays for different frequencies form a random sequence [1]. The largest phase delays occur around resonant frequencies. For a diffuser using slots (quarter-wave resonators), the lowest operating frequency depends on the depth of the deepest slot (Figure 4a). Analysis shows that diverse phase delays for individual cells can be observed for frequencies above 1000 Hz. This structure achieves a high diffusion coefficient for a frequency range of 800-3150 Hz (Figure 5a).

The introduction of a Helmholtz resonator in series with the slot in a diffuser based on HR-Ds cells results in a lower resonant frequency and introduces large phase delays. Analysis of the phase delays indicates that frequencies above 900 Hz occurs a variety of delays (Figure 4b). This implies that the use of resonators expands the frequency range in which significant phase delays are present.

a b

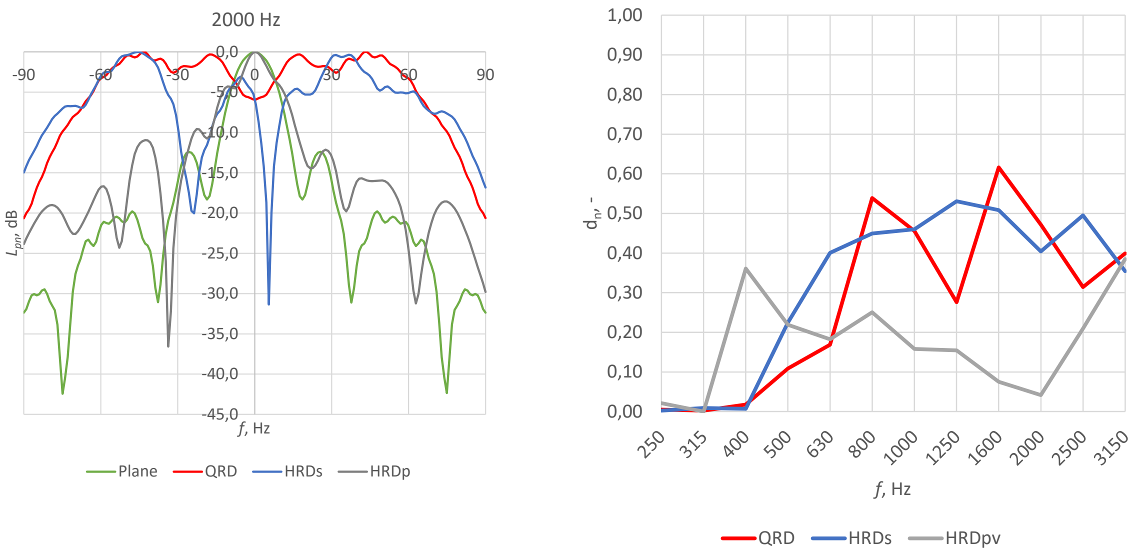

Figure 5. a) Reflected sound pressure level L_{pn} in frequency 2000 Hz.
b) Comparison of the normalized diffusion coefficient.

The diffuser made from cells with parallel Helmholtz resonators connected (HR-Dpv) showed a wide range of phase shifts at 500 Hz. However, between 1600-2000 Hz, each cell produced similar phase delays (Figure 4c), resulting in a lack of sound diffusion (Figure 5b). The distribution of reflected sound pressure level $L_{pn}(\theta)$ was like a flat surface (Figure 5a). Above 2000 Hz, there was a range of diverse phase changes, leading to uneven sound diffusion characteristics (Figure 5b). The analysis of the sound diffusion coefficient indicated that the diffusers with QR-D and HR-Ds achieved the highest coefficients (Figure 5b). The frequency range in which sound diffusion occurred aligned with the range where phase shifts were differentiated. The diffuser constructed with cells containing parallel Helmholtz resonators connected HR-Dpv yielded the lowest diffusion coefficient values (Figure 5b). This can be attributed to the fixed depth of the main channel. Additionally, changing the dimensions of the Helmholtz resonator in a vertical cell arrangement did not result in a change in phase shift across a wide frequency range.

3.4. Modification of the HR-Dpv cell

To achieve a broader frequency range for phase delay adjustment, the cell's shape has been modified. The main slot's depth was decreased to 50 mm, while the cell's width was increased to 100 mm (as shown in Figure 6a).

a b

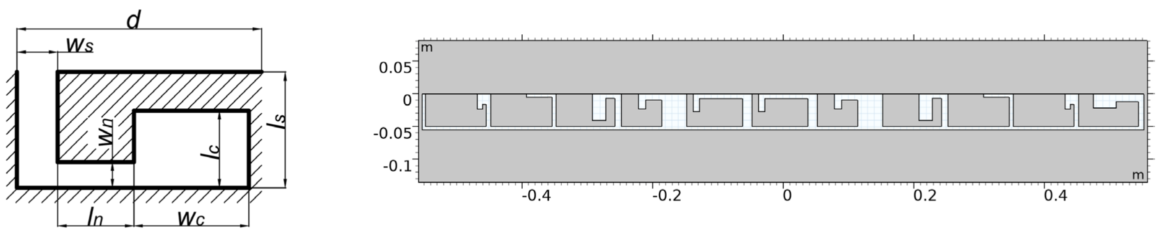

Figure 6. a) The modified cell model HR-Dph utilized for constructing diffusers.
b) The cross-sectional configuration of the optimized sound diffusers HR-Dph.

The modified diffusion structure consists of cells arranged horizontally, with a period that is double the width of the previous ones. The depth of the diffuser has also been halved (as shown in Figure 6b). The optimization process was repeated using the same cost function. The optimization was focused on the frequency range of 315-2500 Hz.

The use of modified cells allowed for a decrease in the resonant frequency to 600 Hz. This was achieved by reducing the size of the main slot, which acted as an additional vibrating mass for the resonator. It is important to note that the horizontal arrangement of the cell provided phase delays over a wider frequency range (Figure 7). Phase delay differentiation can be achieved for frequencies up to approximately 2500 Hz. This can be achieved by increasing the size of the resonator's neck and volume in the horizontal configuration, thereby potentially lowering the second resonant frequency. However, a disadvantage of this solution is that when the main slit is narrow, there is a very limited frequency range in which the phase delays are significant (Figure 7, specifically cell 2, 3, and 5). This can result in a decrease in the sound diffusion coefficient.

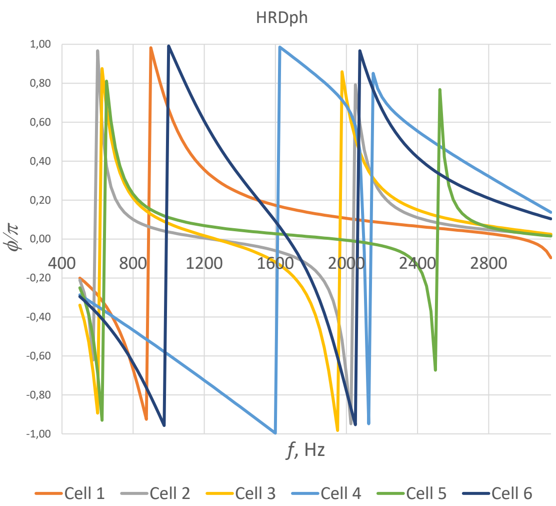

Figure 7. Comparison of phase shifts introduced by cells and in optimized diffuser HR-Dph.

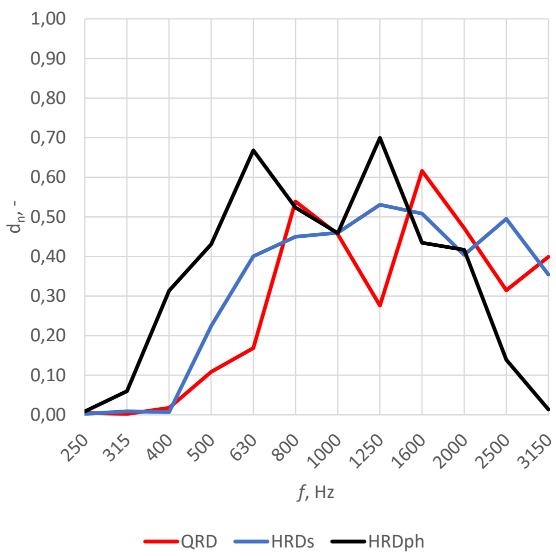

Figure 8. Comparison of the normalized diffusion coefficient of the optimized sound diffusers.

The diffuser with horizontally arranged HR-Dph cells successfully diffuses sound in a broad frequency range of around 500-2000 Hz (Figure 8). This cell design allows for a thinner diffuser compared to the QR-D diffuser and provides diffusion at lower frequencies. Conversely, the diffuser using HR-Ds cells also achieves a wide diffusion range of 630-3150 Hz. However, this range occurs at higher frequencies compared to HR-Dph cells and the diffuser is also twice as thick.

4. Conclusions

The article examines the potential application of acoustic metamaterials to the construction of broadband sound diffusers. The study examined two different types of cells used in the construction of the diffuser and compared their effectiveness to slot-type diffusers. The presented study showed that using resonators in the cells for the construction of sound diffusers enables the adjustment of phase shifts in the lower frequency range compared to slots. When resonators are connected in series, sound diffusion can be achieved across a broader frequency range, but the dimensions of the diffusers are similar to slotted diffusers. On the other hand, using cells with resonators connected in parallel in a horizontal configuration allows for sound diffusion at extended low frequencies and reduces the thickness of the diffuser.

Additional information

The author declare: no competing financial interests and that all material taken from other sources (including their own published works) is clearly cited and that appropriate permits are obtained.

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