VIBRATIONS IN PHYSICAL SYSTEMS

DOI: 10.21008/j.0860-6897.2024.2.04
Vibrations in Physical Systems, 2024, 35(2), 2024204

The effect on the imaging quality of the frequency of the ultrasound wave transmitted from the ultrasound probe in the Doppler tomography method

Tomasz ŚWIETLIK

Wroclaw University of Science and Technology, 27 Wybrzeże Wyspiańskiego st

Corresponding author: Tomasz ŚWIETLIK, email: tomasz.swietlik@pwr.edu.pl

Abstract Doppler tomography (DT) is a method that allows the reconstruction of 2D or 3D images of the inside of the examined object. For this purpose, a two-transducer ultrasound probe is used. In this method, the Doppler phenomenon and the so-called Doppler signal are used to obtain an image. Therefore, the probe is one of the most important components of the measurement system of this method. It should be noted that Doppler tomography differs significantly from the well-known Doppler method of measuring blood flow in blood vessels. In the DT method, stationary cross-sectional images of the object under examination are obtained. In order to produce the Doppler effect in this case, the probe can move around or along the object being imaged. This paper will present a simulation of the effect of the frequency of ultrasonic wave generated by the ultrasound probe on the imaging quality for a single inclusion.

Keywords: Doppler Tomography, image reconstruction, Doppler signal.

1. Introduction

The Doppler effect has been used in medicine since the 1950s, when a Japanese scientist performed the first analysis of heart valve motion [1]. One of the most popular methods based on this phenomenon is the measurement of blood flow velocity in blood vessels using a continuous ultrasound wave [2].

Doppler Tomography (DT) otherwise known as Continuous Wave Ultrasonic Tomography (CWUT) uses the Doppler phenomenon in an unusual way. The method uses a two-transducer ultrasound probe that generates a continuous ultrasound quasi-flat wave. However, unlike classical methods, this probe moves around or along the object under examination. Due to the movement of the probe, according to the Doppler effect, a Doppler frequency will be added or subtracted to the frequency of the signals reflected from the inclusions located in the object. This fact makes it possible to image inclusions that scatter the ultrasound wave well, located, for example, inside the tissue. There are two ways of data acquisition in the DT method. In the first type of method, the probe moves along the test object. It is called linear geometry. The other type of DT data acquisition is circular geometry, in which the transducers move around the tissue section to be imaged [3]. This paper focuses mainly on Doppler tomography in circular geometry shown in Fig. 1.

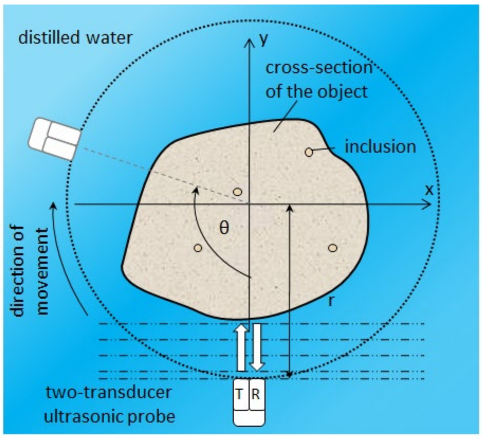

Figure 1. A method of measuring data for Doppler Tomography in circular geometry.

2. Principle of image reconstruction in DT method

Imaging simulation using Doppler tomography will allow the study of the effects of specific parameters on imaging quality, such as the frequency of the ultrasound wave transmitted by the transducer. In the case of real-world measurements, often many effects occur simultaneously, and it is very difficult to identify which one is the reason of changing a particular parameter. Therefore, in order to transparently and orderly examine changes in imaging quality, it is necessary to perform basic simulations.

In order to understand the results of the simulations more easily, it should be noted that the Doppler probe does not necessarily have to rotate around the examined object. Due to the nature of the Doppler effect, the same Doppler frequencies will be obtained when the test object moves around its axis, while the Doppler probe remains stationary. In medical research, such a situation is impossible to implement, while in non-destructive testing one can imagine setting the examined item (e.g., a metal cylinder) on a rotating table and scanning it with a stationary probe. However, the most important fact is that explaining the principles of image reconstruction is much simpler when the object rotates and the probe does not. Therefore, this paper is focused on this case.

Doppler tomography is not a method that relies on measuring the time of arrival of an ultrasound wave. It uses changes in frequency to reconstruct the image. Therefore, it is important to understand two key facts. Fig. 2a shows an example cross-section of a test object with three inclusions. Let's assume that only the inclusions reflect the ultrasonic wave. As they rotate around their axis, they will have their speed of motion. From the point of view of the Doppler effect and the DT method, the most important thing is the component of the speed of motion in the direction of propagation of the ultrasonic wave. For inclusions *a*, *b*, *c*, these velocity components are denoted as v_a, v_b, v_c. The DT method assumes that at each angle of rotation of the test object (or ultrasound probe) we record reflected signals (scattered in the direction of the ultrasound probe). Due to the Doppler effect, the frequency of the reflected signal is altered by the Doppler frequency. Let's assume that if for a given rotation angle the inclusion moves in the direction to the probe then this frequency takes a positive value, otherwise it is taken with a minus sign (Fig. 2a).

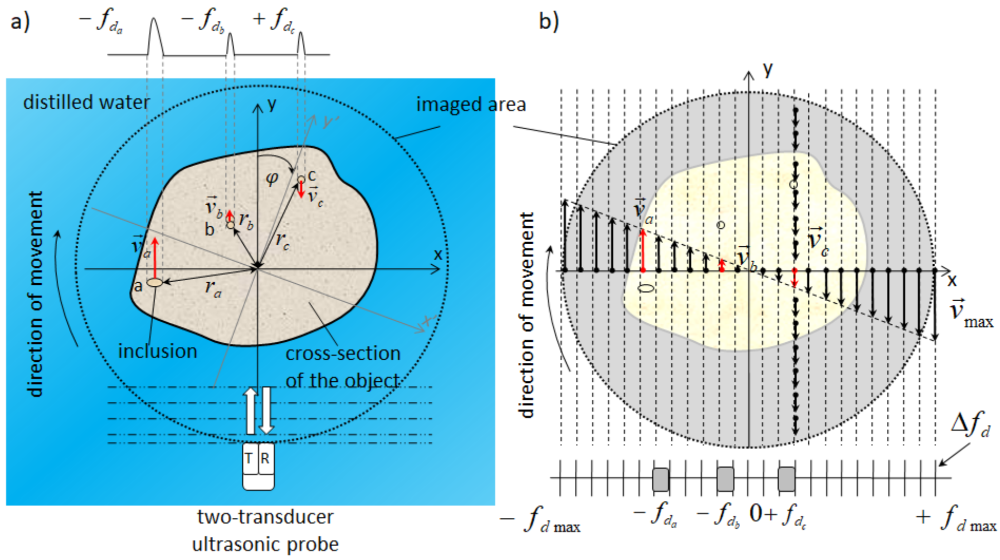

Figure 2. a) An example cross-section of an object with three inclusions at a given rotation angle. b) An illustration of the distribution of the movement velocity component *v* on the wave propagation direction and Doppler frequency f_d in the imaging area.

It is important to note that the velocity components v_a, v_b, v_c are related to the Doppler frequency by the formula:

$$f_d = 2 \cdot f_T \cdot \omega_{\text{turn}} \cdot r \cdot \frac{\cos(\varphi)}{c}, \tag{1}$$

where: $v = r \cdot \omega_{\text{turn}}$, ω_{turn} – angular velocity of rotation of the test object, r – distance of the inclusion from the center of rotation, c – speed of the ultrasonic wave in the test material, φ – angle of rotation of the test object, f_T – frequency of the signal transmitted from the ultrasonic probe.

Transforming Eq. (1), we get a direct formula for the velocity v:

$$v = \frac{f_d \cdot c}{2 \cdot f_T \cdot \cos(\varphi)}. \tag{2}$$

From Eq. (2), it is obvious that the Doppler frequency is directly proportional to the component of the inclusion velocity in the direction of wave propagation v. Figure 2b shows two facts (important features) about this velocity.

The first concerns the distribution of velocities on a line along the propagation of the ultrasonic wave (vertically). Inclusions placed on this line will have the same velocity v. Fig. 2b shows this with an example of the v_c component. Equation (2) also shows that Doppler frequencies at locations on the vertical line will have the same value.

The second fact is that if a point (inclusion) moves away from the center of the imaging zone then its velocity v increases from 0 at the very center to some maximum value of v_{max}. Again, based on Eq. (2), note that the same fact is also valid for the frequency f_d.

Based on the above two features, the first step of image reconstruction is to determine the maximum Doppler frequency f_{dmax}. This can be done based on Eq. (1). The formula for calculating f_{dmax} is given as follows:

$$f_{d\max} = 2 \cdot f_T \cdot \omega_{\text{turn}} \cdot r_{\max} \cdot \frac{1}{c}, \tag{3}$$

where: r_{max} – radius of the imaging zone.

At this point it is possible to determine the range from $-f_{dmax}$ to $+f_{dmax}$. This is then divided into so-called Doppler bands of equal width Δf_d. This can be compared to dividing the entire imaging zone into vertical bands (ranges) shown by the dashed line in Fig. 2b. In order to reconstruct the image for each angle of rotation of the probe (or object under study) φ, we write the sum of the Doppler frequency amplitudes into a given Doppler band. If we perform this procedure for each angle φ, we obtain a matrix, which in tomographic methods is called a sinogram [3]. It is used for image reconstruction using the DT method. In practice, this means that for each rotation angle, a so-called Doppler signal must be recorded. This signal already contains only Doppler frequencies. It can be obtained, for example, by multiplying the signal transmitted and received by the probe and applying a low-pass filter with an upper frequency slightly above the f_{dmax} frequency. Then, a spectrum is calculated for the signals thus obtained. On the spectrum we determine the range [$-f_{dmax}$; $+f_{dmax}$] and divide it into Doppler bands of equal width Δf_d. Thus determined values of the amplitudes from the division of the spectrum in the Doppler bands for a given angle of rotation are entered in the corresponding line of the sinogram. When we fill in the rows for all angles of rotation, we obtain a matrix for image reconstruction.

The final step in image reconstruction is to apply one of the algorithms used in computed tomography to the calculated sinogram. In the present work, a fast algorithm called Filtered Back Projection (FBP) was used. It is worth noting that this algorithm allows reconstruction of the full image on the basis of half of the rotation data. A block diagram of image reconstruction in the Doppler tomography method is shown in Fig. 3.

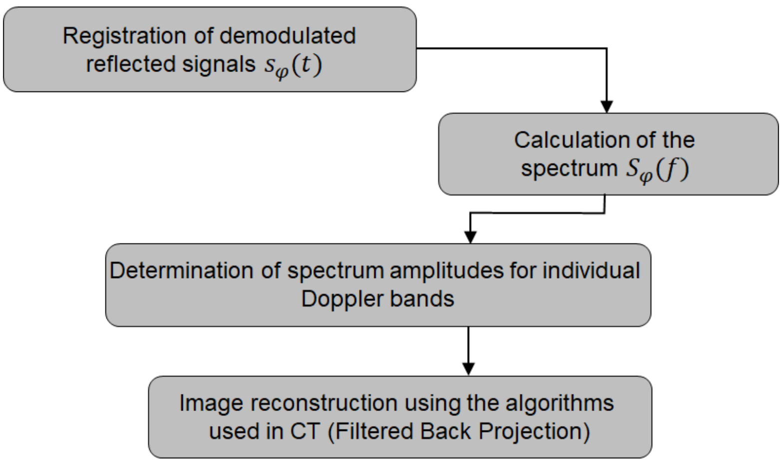

Figure 3. Block diagram of image reconstruction algorithm in Doppler tomography method.

3. Simulation and calculation of the Doppler signal

The Doppler signal is the most important signal in image reconstruction using the Doppler tomography method. As mentioned earlier, it contains only Doppler frequencies. In order to perform advanced simulations, it is required to determine the formula for the Doppler signal. This signal can be determined by multiplying the transmitted and received signal in the time domain. In the case of a single inclusion, in the frequency domain this will result in a sum and difference of the previously mentioned signals in addition to the fundamental frequency of the transmitted signal f_T and the received signal $f_R = f_T \pm f_d$. This means that we will get $\pm f_d$ in the spectrum, among other frequencies. Note that the sign (-) next to f_d carries information about the direction of the inclusion's movement, because from the perspective of physics, the frequency cannot be negative. After applying a low-pass filter with a cutoff frequency of just above f_{dmax}, a Doppler signal is obtained. This is how the Doppler signal is obtained in real measurements. For simulation purposes, however, a specific formula must be determined. For this purpose, we used formula (1) and the definition of the type of signal that is the Doppler signal obtained from rotating scatterers. This definition is cited in formula (4) [4], which describes a chirp-type signal:

$$s(t) = A \cdot \sin(\varepsilon(t))\,, \tag{4}$$

where: $\varepsilon(t)$ is the time function responsible for modulating the frequency of the signal, while A is the amplitude of the signal.

In addition, we determine the change in frequency of the signal using the formula:

$$f(t) = \frac{1}{2\pi}\frac{d\varepsilon(t)}{dt}. \tag{5}$$

Note that for the DT method, $f(t)$ is the change in Doppler frequency during rotation. It should be mentioned that the Doppler signal will be derived for a single point inclusion. Calculating the formula for a larger number of inclusions, or a given shape, comes down to summing the signals for all the points of which the simulated object is composed.

The derivation of the formula begins with the assumption that the signal is reconstructed from an inclusion placed on the rotating platform at a distance r_0 from the center of rotation and at an angle α_0 to the origin of the coordinate system. In addition, the platform rotates at a frequency of f_{turn}. Thus, from equation (1) follows the formula:

$$f(t) = \frac{2 \cdot f_T \cdot f_{\text{turn}} \cdot 2 \cdot \pi \cdot r_0 \cdot \cos(2 \cdot \pi \cdot f_{\text{turn}} \cdot t + \alpha_0)}{c}. \tag{6}$$

From equation (5) it was concluded that:

$$\frac{d\varepsilon(t)}{dt} = 2\pi \cdot f(t) \Rightarrow \varepsilon(t) = 2\pi \int_0^t f(\tau) d\tau. \tag{7}$$

Thus, substituting formula (6) into equation (7), we get:

$$\varepsilon(t) = 2\pi \int_0^t \frac{2 \cdot f_T \cdot f_{\text{turn}} \cdot 2 \cdot \pi \cdot r_0 \cdot \cos(2 \cdot \pi \cdot f_{\text{turn}} \cdot \tau + \alpha_0)}{c} d\tau =$$

$$= \frac{8\pi^2 f_T \cdot f_{\text{turn}} \cdot r_0}{c} \int_0^t \cos(2 \cdot \pi \cdot f_{\text{turn}} \cdot \tau + \alpha_0)\, d\tau =$$

$$= \frac{8\pi^2 f_T \cdot f_{\text{turn}} \cdot r_0}{c} \cdot \frac{1}{2\pi f_{\text{turn}}} \cdot \sin(2\pi f_{\text{turn}} \cdot \tau + \alpha_0) \Big|_0^t =$$

$$= \frac{4\pi \cdot f_T \cdot r_0}{c} \cdot (\sin(2\pi f_{\text{turn}} \cdot t + \alpha_0) - \sin(\alpha_0)\,.$$

Thus, the formula for the Doppler signal in the DT method has the form:

$$s(t) = A \cdot \sin(\frac{4\pi \cdot f_T \cdot r_0}{c} \cdot (\sin(2\pi f_{\text{turn}} \cdot t + \alpha_0) - \sin(\alpha_0)). \tag{8}$$

 DOI: 10.21008/j.0860-6897.2024.2.04

4. Effect of the frequency of the signal transmitted from the ultrasound probe (f_T) on the imaging quality of the DT method

Knowing how to calculate and simulate the Doppler signal and how to reconstruct the image using the DT method, let's look at the situation shown in Fig. 4. The simulated measurement system is shown there. It can be seen a rotating platform on which is placed a single very small inclusion that scatters the ultrasonic wave in each direction equally. The ultrasonic probe generating the f_T frequency wave is also marked there. The whole system is immersed in distilled water.

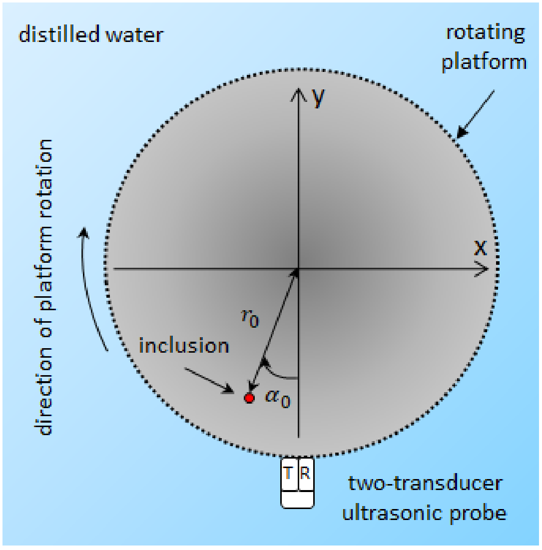

Figure 4. A setup for simulating the image of an inclusion placed on a rotating platform at a distance r_0 from the center of rotation and at an angle α_0.

In this paper, we simulated the image reconstruction of a single very small inclusion placed on a rotating platform at a distance of 2.5 cm from the center of rotation, at an angle of 0^0 to the ultrasound probe was simulated. The diameter of the imaged area was 15 cm (radius 7.5 cm). The rotation frequency of the platform f_{turn} = 2 turn/s, the velocity of the ultrasonic wave in distilled water c = 1482.38 m/s. In addition, it should be noted that the acquisition process given in the DT method can be simplified. In the second chapter it was mentioned that the Doppler signal should be reset at each rotation angle separately. It is much simpler to record this signal from a full rotation of the platform (or probe) and then divide such data into sections representing each angle of rotation. In the current simulation, the number of angles into which the Doppler signal is divided is equal to N_φ = 798 (full rotation). This gives a value of 400 per half turn.

In order to improve the quality of imaging, the so-called overlay algorithm, described in detail in the publication [5], was also implemented. Similar algorithms are applied in audio signal processing to improve frequency resolution. The most important parameter in this algorithm is the width of the time window for rotation angles. In this case, it is equal to 9^0, where 360^0 is the time of full rotation of the platform (or probe). Such a value gives an optimal improvement in the resolution of the inclusion image.

Figure 5 shows the result of the reconstruction of the inclusion image for a probe transmit frequency equal to f_T = 4 MHz. The maximum Doppler frequency calculated from equation (3) in this case is 5086 Hz. Figure 5 shows a clear image of the inclusion, which, remember, is very small in theory. On reconstruction, it has a resolution of about 2mm and a blur of about 5.5 mm. The concepts of "resolution" and "blur" of the image will be explained later in the paper. This means that at 4 MHz (with the simulation conditions mentioned above), the smallest inclusion image we can get is just about 2 mm. It should be noted that smaller inclusions can also be imaged, but their dimensions after reconstruction will still be no less than 2mm.

In order to better investigate this phenomenon, a series of simulations were carried out for f_T frequencies from 0.5 MHz to 7 MHz. Figure 5 shows a 3 cm by 3 cm slice of the image. Similar areas were cut out of the other reconstructed images and are shown in Fig. 6.

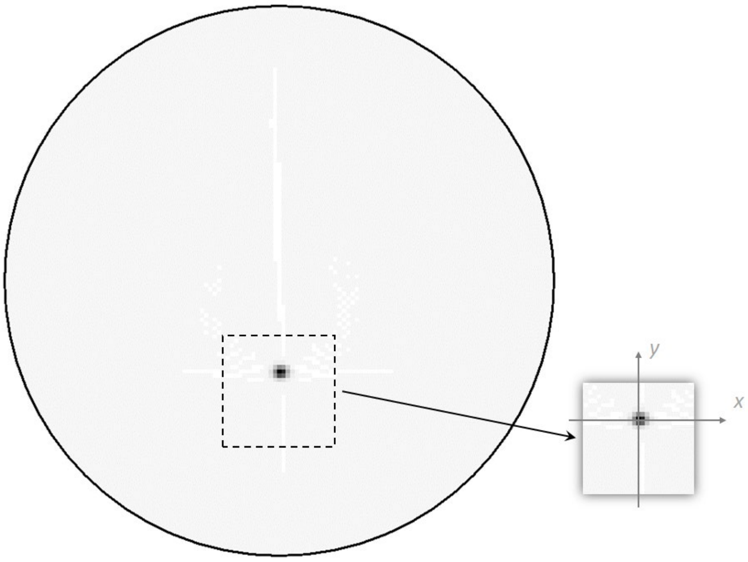

Figure 5. Simulated image reconstruction of a single very small inclusion placed on a rotating platform at a distance of 2.5cm from the center of rotation, at an angle of 0^0 to the ultrasound probe for a frequency f_T = 4 MHz.

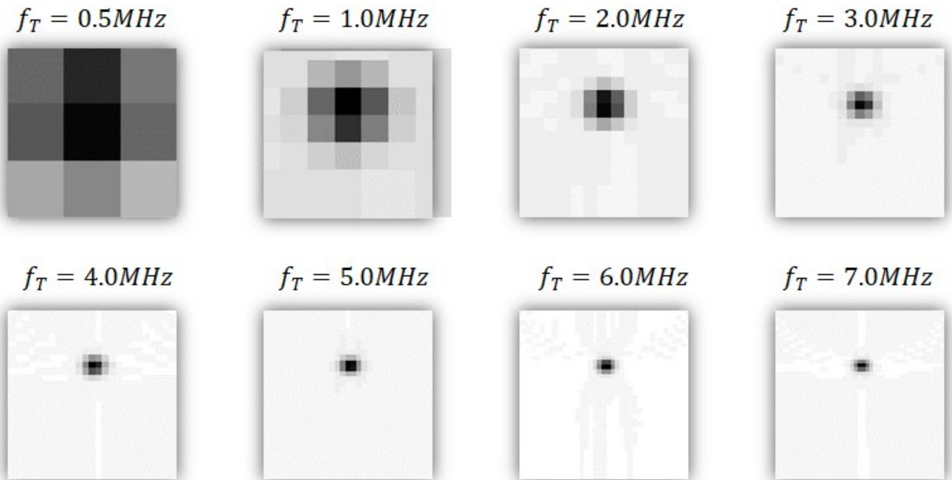

Figure 6. Simulated image reconstruction of a single very small inclusion placed on a rotating platform at a distance of 2.5 cm from the center of rotation, at an angle of 0^0 to the ultrasound probe for f_T frequencies of 0.5 MHz to 7 MHz.

From Fig. 6, it is clear that the higher the value of the frequency transmitted from the ultrasound probe, the better the resolution of the inclusion image. In other words, for higher f_T frequencies it is possible to imaging smaller and smaller inclusions. One reason may be the increase in the maximum Doppler frequency for consecutive f_T frequencies. In this case, for f_T frequencies from 0.5 MHz to 7 MHz, the value of $f_{d\max}$ takes the values 636 Hz, 1272 Hz, 2543 Hz, 3815 Hz, 5086 Hz, 6358 Hz, 7629 Hz, 8901 Hz, respectively. A higher $f_{d\max}$ frequency can increase the frequency resolution of the Doppler signal. This translates directly into improved image resolution. Let's take a closer look at this phenomenon. Figure 7 shows how to determine the previously mentioned image resolution and blur for two directions: along and across the image. Resolution is determined for a 3 dB drop (70.7%) in pixel values. Blur, on the other hand, is determined at 10% of the maximum pixel value. This is the level below which it is customary to assume that we are dealing with noise. With the parameters defined in this way, it is possible to perform a detailed analysis of the images in Fig. 6. Its results are shown in Fig. 8. There it is shown how the resolution and blur change along and across the image as the frequency of the ultrasonic wave is changed.

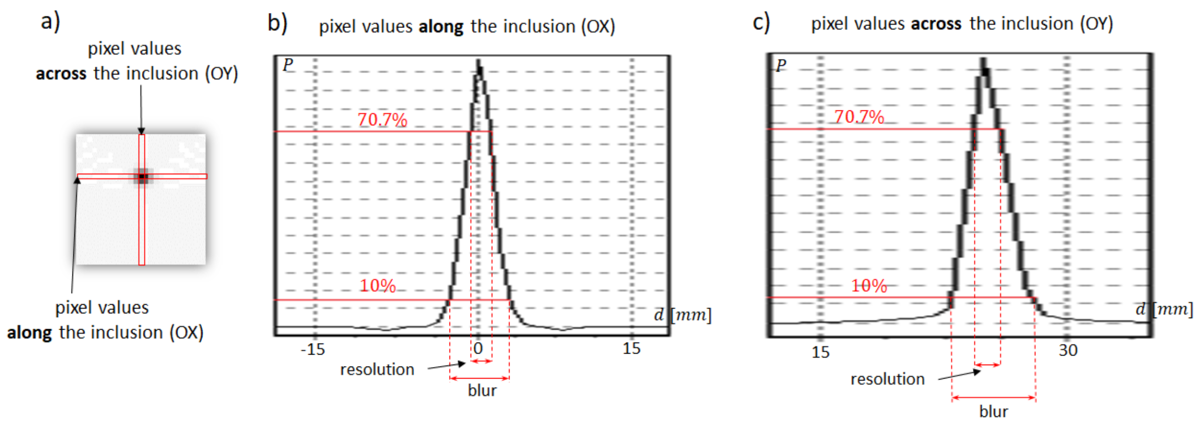

Figure 7. a) A section of the reconstruction of the inclusion image for the frequency f_T = 4 MHz with the pixels passing along (OX) and across the image (OY) marked. Pixel values b) along and c) across the inclusion image with marked resolution and blur.

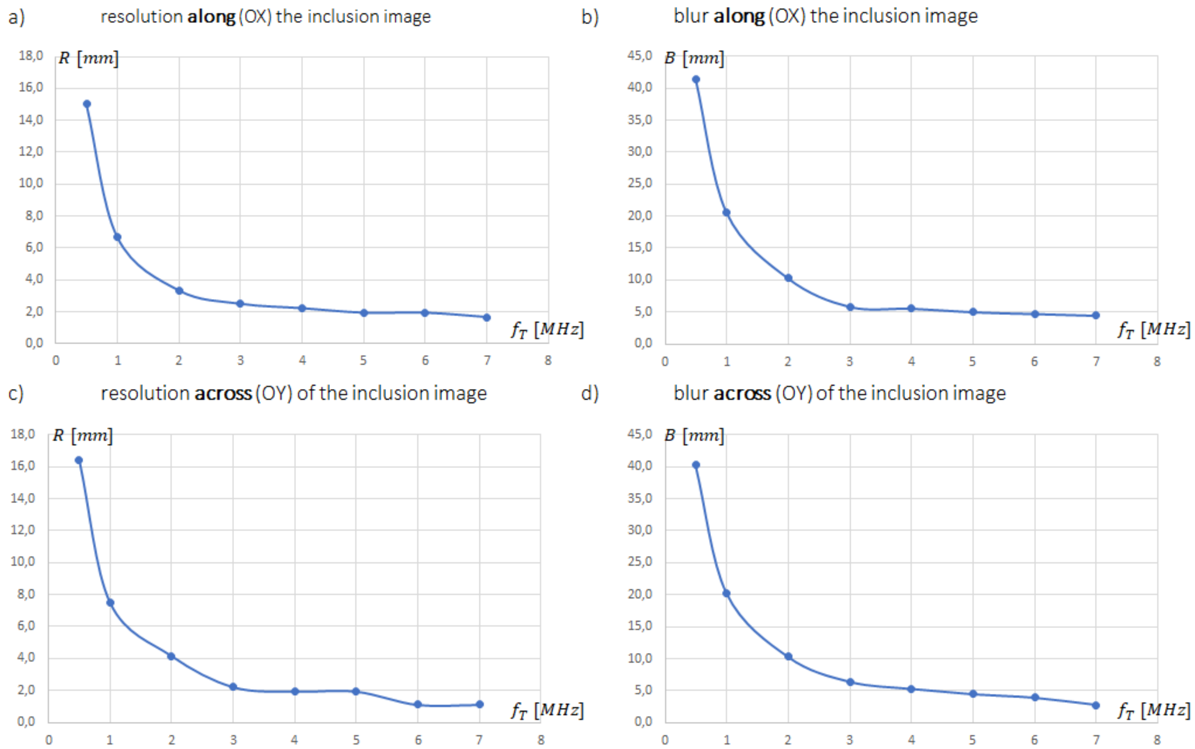

Figure 8. a) Resolution along the inclusion image for $f_T \in$ (0.5 MHz, 7 MHz). b) Blur along the inclusion image for $f_T \in$ (0.5 MHz, 7 MHz). c) Resolution across the inclusion image for $f_T \in$ (0.5 MHz, 7 MHz). c) Blur across the inclusion image for $f_T \in$ (0.5 MHz, 7 MHz).

Figure 8 clearly shows that higher frequency f_T exponentially improves both resolution and blur of the inclusion image. This is a true statement for both directions of examination of pixel values along and across the image. It should also be noted that above a frequency of f_T = 4 MHz the improvement is relatively small.

5. Conclusions

Doppler tomography is a relatively new method. More importantly, it is a not very well studied method. Therefore, it is important to learn how the parameters of this method affect the quality of imaging. The key role in DT is played by the ultrasound probe. Its most important parameter is the frequency at which the ultrasound wave is transmitted. By simulating the Doppler signal, the effect of this frequency on image reconstruction can be studied in detail.

For basic testing, a single very small inclusion is best suited to determine the smallest dimensions of the test object that can be reconstructed in the image. The results of such an experiment are shown in Fig. 6, where images were simulated for f_T frequencies up to 0.5 MHz to 7 MHz. It can be seen that the higher the frequency, the smaller the inclusion image, i.e. a better representation of a very small element. The results in Fig. 8 show more precisely what the relationship is between frequency and the resolution and blurring of the inclusion image. At f_T = 0.5 MHz, the resolution is 15.5 mm, while the blur is 40.5 mm. However, already for f_T = 4MHz the resolution is about 2mm, and the blur is about 5.5 mm. An improvement of more than 700% has been achieved. For f_T = 7 MHz, a discrepancy appears for values along and across the image. For resolution along (OX) R = 1.7 mm, while across (OY) R = 1.1 mm. This causes elongation of the image in the OX axis and is an undesirable phenomenon. Therefore, it can be concluded that the frequency f_T = 4 MHz is the optimal value.

The DT method requires further research and studies. They are still at an early stage, but Doppler Tomography has the potential of becoming method that allows scanning the interior of various types of objects under examination. One potential application of this method is the examination of women's breasts to detect cancerous lesions. One could imagine an ultrasound probe that moves in an enclosed tunnel around the breast being imaged. The patient could lie on a bed in which there would be a special opening with a vessel filled with distilled water. Inside the vessel would be a circular-shaped tunnel with an ultrasound probe. By moving this tunnel up and down (using a special mechanism), it would be possible to scan and reconstruct images for different cross-sections of the breast. This, in turn, would allow the reconstruction of 3D images of the breast. Similarly, a device could be constructed to reconstruct images of human limbs.

Another potential application of Doppler Tomography is non-destructive testing. In this case, the examined object can be placed on a rotating platform as shown in this paper. The stationary probe would record the signal for a given cross-section of the object. Then, thanks to the DT method, the image of a given cross-section of the object would be reconstructed. As was the case with the examination of women's breasts, the probe could move up or down, which would make it possible to reconstruct images of successive cross-sections of the object under examination. This would allow a 3D image to be reconstructed.

Additional information

The author declare: no competing financial interests and that all material taken from other sources (including their own published works) is clearly cited and that appropriate permits are obtained.

References

1. S. Satomura, S. Matsubara, M. Yoshioka; A new method of mechanical vibration measurement and its application; Memoirs Inst. Scient. Indust. Res. Osaka Univ., 1956, 13, 125–133
2. A. Nowicki; Ultrasonografia, wprowadzenie do obrazowania i metod dopplerowskich (in Polish); Wydawnictwo Instytutu Podstawowych Problemów Techniki, PAN, Warszawa, 2016
3. A. C. Kak, M. Slaney; Principles of computerized tomographic imaging; IEEE Press, 1988
4. L. Chaparro; Signals and systems using MATLAB. Second edition; Elsevier, 2015
5. T. Świetlik, K. J. Opieliński; Analysis for improvement of Doppler Tomography imaging of objects scattering continuous ultrasonic waves.; Archives of Acoustics, 2020, 45(2), 329–339; DOI: 10.24425/aoa.2020.133153