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Critical analysis of PN-EN 13979-1:2020-12: application limitations in fatigue assessment of railway wheels

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Abstract The paper focuses on PN-EN 13979-1:2020-12, which defines the technical requirements for steel solid monobloc wheels used in railway vehicles. The crucial role of the wheel-rail contact for the riding safety and comfort of passengers, as well as for the durability of rolling stock, was highlighted. Fatigue strength aspects of the wheels were analysed, including the amplitude and mean stresses which affect their long-term operation. Also discussed are factors that are not completely covered in the standard, such as varying operating conditions, thermal effects and material ageing, which can increase the fatigue wear process. The main purpose of the paper is to provide a comprehensive view of the importance of fatigue analysis and to identify areas that may require further research to improve the safety and operational efficiency of rolling stock.

Keywords: railway wheel, fatigue strength, PN-EN 13979-1, stress amplitude, railway rolling stock safety.

1. Introduction

Wheel-rail interaction is a crucial issue in the rail transportation system, as it directly affects riding safety, passenger comfort and the durability of rolling stock. The tiny contact area formed is responsible for the bearing of dynamic forces between the vehicle and the track, resulting, in particular, from track irregularities. The proper wheel-rail interaction ensures vehicle stability and guidance in curves and eliminates the risk of derailment. Also relevant to safety, the characteristics of this connection affect the mechanical loads on the running gear components and infrastructure through a direct relationship with the strength of these components. Improper load distribution can lead to an increased risk of failure, including fatigue cracks in wheels, which can directly affect operational safety. In terms of passenger comfort, the wheel-rail interaction should ensure a smooth ride, limiting vibration and noise in the railcars. Undesirable in the wheel-rail connection are highly dynamic phenomena that increase acceleration values, which negatively affect travel comfort. The interaction characteristics are also crucial to the durability of rolling stock, directly affecting the wear and tear of vehicle components such as wheels, axles and suspension components. It is desirable to reduce the need for frequent repairs, reduce the cost of rolling stock maintenance and increase the operational efficiency of the entire railway system. Wheel and rail condition management is one of the most important challenges in railroad infrastructure maintenance [1]. An important step to ensure the proper technical condition of rolling stock are the calculations of wheel strength in accordance with PN-EN 13979-1:2020-12, which specifies methods for evaluating the fatigue strength of wheels. In general, the guidelines of this standard should ensure the safe operation of newly designed wheels under various operating conditions, minimizing the risk of damage and failure. To complement the fatigue durability analysis presented in the article, a study was conducted on the impact of thermal stresses on stress distribution in railway wheels. This analysis includes an evaluation of temperature distribution results under typical operating conditions, such as intensive cyclic braking and downhill descent (e.g., on the Gotthard Pass). Considering these effects allows for a better understanding of potential differences in predicted stresses compared to results obtained solely by applying the normative requirements of PN-EN 13979-1:2020-12.

2. PN-EN 13979-1:2020-12 standard – requirements and application

The PN-EN 13979-1:2020-12 standard [2] is used primarily by wheel manufacturers, rolling stock design engineers, and certification bodies that assess the compliance of wheels with technical requirements. The application of the standard is aimed at ensuring the safe operation of wheels, minimizing the risk of failure through proper design and testing of wheel load capacities. The use of European standards facilitates the design of wheels that can be used in many European countries without the requirement of additional certification, which promotes the integration of the rail transportation sector [3]. With specific guidelines for materials and general design, these standards support the optimization of wheel design, which increases durability and reduces operating costs [1].

The PN-EN 13979-1:2020-12 standard concerns design requirements and approval procedure for forged or rolled steel monobloc wheels that are used in railway vehicles. This is a European standard adopted at the national level. It was developed to unify standards for the design, testing and evaluation of the wheel strength, which are a key element affecting the safety and reliability of rail vehicles. The standard specifies technical requirements for wheels made of solid steel, such as certain shapes, dimensions and material quality. This includes the physical and mechanical parameters of manufacturing steel of the wheels to ensure strength for a wide spectrum of operating loads. An important aspect of the standard is the requirements related to mechanical behaviour, including fatigue strength. The evaluation criteria under cyclic loads that occur during operation are defined. These are dynamic loads resulting from wheel-rail interaction, such as lateral and vertical forces for cases of riding on straight track, curve or through switches and crossings. Standard 13979-1:2020 specifies a detailed procedure for fatigue calculations of wheels, whose compliance should ensure the carrying of cyclic fatigue loads during the total lifespan of the wheel. This process is crucial because fatigue damage can lead to cracks and wheel failures, raising serious safety hazards [4]. The standard describes the test procedures that wheels must pass to be accepted including bench strength tests as well as simulation analyses. This includes fatigue tests, fracture tests, and thermal load tests that may occur during braking.

3. Fatigue strength requirements and testing methodologies

Fatigue strength refers to the ability of the wheel material to withstand periodic loads without developing fatigue damage, such as cracks or deformations [1]. One of the quantities characterizing the load cycle is the stress range, which depends on the maximum and minimum stresses. The PN-EN 13979-1:2020-12 specifies methods for calculating the stress range, which, when compared to the criterion value, determines the fatigue strength of the wheel. The high stress amplitude can accelerate the fatigue process and increase the risk of damage. Accurate analysis can identify critical locations in the wheel structure that may be subjected to excessive stresses. However, the standard does not take into account the mean stress, which is an additional quantity that characterizes the fatigue cycle. Analysis of these stresses is important because, in some cases, despite their small amplitude, mean stresses can cause cumulative fatigue effects. A simplification of the assessment by neglecting the effect of mean stresses on fatigue strength can lead to an underestimation of the actual risk of cracking. The guidelines in the discussed standard only apply to the wheel web, without covering the hub or the rim assessment. As the contact loads are concentrated on the wheel thread and flange, which is a place subjected to rolling contact fatigue. The rim itself is subjected to varying loads from with-rail interaction and from dynamic effects associated with riding [5].

The standard provides guidance on how to interpret the results of simulations and endurance tests to predict the behaviour of a wheel under actual operating conditions. EN 13979-1:2020-12 recommends the use of both laboratory tests and computer simulations to assess fatigue strength. These simulations allow cyclic loading to be modelled under different operating conditions and the results then compared to the requirements of the standard. These tests make it possible to predict critical points where fatigue damage may occur, allowing design or material changes to be made to increase wheel life.

4. The simplified approach to fatigue analysis

PN-EN 13979-1:2020-12 specifies a number of requirements and guidelines for assessing the fatigue strength of railway wheels, but does not take into account all the factors that can affect fatigue phenomena.

Among the parameters that can have a significant influence on the fatigue process but are not fully considered in the standard are the varying operating conditions. The standard assumes some averaged loading conditions, while the actual wheel operating conditions can vary significantly depending on factors such as routes, train speed profile, varying weather conditions, and track conditions. These variables can result in higher or lower loads that have a direct impact on the fatigue wear process of the wheels. An example of this is an operation on routes with large differences in altitude, which causes more frequent braking and acceleration loads that increase fatigue wear.

Another aspect that is not considered in the standard is mean stresses. Although the standard includes an analysis of stress amplitude, it does not cover the influence of mean stresses, which can play a significant role in the fatigue process. Mean stresses can lead to the accumulation of fatigue damage, even if the amplitude meets criteria values. In practice, ignoring this factor can result in an underestimation of the risk of cracking, especially for higher mean stress values.

In the PN-EN 13979-1:2020-12, the fatigue analysis of railway wheels is based on the calculation of stress range, which is the difference between the maximum and minimum stress in all wheel material points during considered cyclic loading, while stress amplitude σ_a is expressed by the formula:

$$\sigma_{\mathrm{a}} = \frac{\sigma_{\max} - \sigma_{\min}}{2}, \tag{1}$$

where: σ_{max} – maximum cycle stress, σ_{min} – minimum cycle stress

In practice, however, consideration of stress amplitude alone does not fully determine the fatigue cycle. The mean stress σ_m expressed as:

$$\sigma_{\mathrm{m}} = \frac{\sigma_{\max} + \sigma_{\min}}{2}, \tag{2}$$

has an important influence on the fatigue process, as they affect the effective value of the stress acting on the material. Mean stresses cause the material to be subjected to additional continuous stresses, which can increase the risk of microcracks forming and propagating.

The formula describing the effect of mean stress on fatigue strength can be represented by the Goodman criterion [7], which take into account both the amplitude and the mean stress:

$$\frac{\sigma_{\mathrm{a}}}{\sigma_{\mathrm{f}}} + \frac{\sigma_{\mathrm{m}}}{\sigma_{\mathrm{t}}} \leq 1. \tag{3}$$

where: σ_a – stress amplitude, σ_f – material fatigue strength, σ_m – mean stress, σ_t – material tensile strength.

In the case of PN-EN 13979-1, the above correlations are not fully taken into account, which may lead to an underestimation of the risk of cracking. In practice, this may indicate that, although the stress amplitude does not exceed the permissible values with the acceptable values, mean stresses that are not taken into account may lead to earlier material failure, particularly in areas of the wheel where long-term continuous stresses are present.

In heavily loaded areas, such as the surface of the wheel rim, where there is contact with the rail, the mean stresses can lead to the more rapid appearance of microcracks [6]. This phenomenon can affect the actual durability of the wheel, irrespective of compliance with the standard based on stress range analysis. Therefore, for a better prediction of the fatigue strength of wheels, it is useful to take into account both the amplitude and the mean stresses using more advanced analytical approaches such as the fatigue criteria mentioned above.

The extension of the standard or the use of additional methods of analysis can therefore lead to a better reflection of the actual operating conditions of railway wheels, which contributes to the safety and sustainability of rolling stock.

The standard does not fully account for the influence of track irregularities and wheel-rail contact dynamics, which have a key impact on wheel stress distribution. Actual operating conditions, e.g. on tracks with corrugated or random defected rails, increase peak loads leading to an accumulation of damage. In addition, friction and wear on the wheel-rail interface, which depends on weather conditions, lubrication condition and contamination, can lead to changes in the dynamic loads acting on the wheel, affecting its service life.

The mechanical assessment part of the standard, do not impose the influence of thermal effects such as those resulting from heavy braking. Frequent braking, especially on high-speed or goods trains, leads to an increase in the temperature of the wheels, which can affect their material structure and induce thermal stresses. These stresses can combine with mechanical stresses, leading to accelerated fatigue wear. The total stress σ_{tot} in the wheel material can be expressed as the sum of the mechanical stress σ_{mec} and thermal stress σ_{term} [7]:

$$\sigma_{\mathrm{tot}} = \sigma_{\mathrm{mec}} + \sigma_{\mathrm{term}}. \tag{4}$$

Basic formula for calculating thermal stresses [7]:

$$\sigma_{term} = E \cdot \alpha \cdot \Delta T, \tag{5}$$

where: E – Young's modulus, α – thermal expansion coefficient, ΔT– temperature change.

The fatigue strength criterion, which takes into account the mean and thermal stresses can be described more comprehensively by a formula that integrates the thermal stress with the mechanical analysis, e.g. by modifying the Goodman formula:

$$\frac{\sigma_\mathrm{a}}{\sigma_\mathrm{f}} + \frac{\sigma_\mathrm{mec} + \sigma_\mathrm{term}}{\sigma_\mathrm{t}} \leq 1. \tag{6}$$

In the standard, the material properties of wheels as a result of long-term operation do not take into account microstructural changes, material ageing or micro-damage. These processes can reduce the fatigue capacities of the material and lead to a more rapid occurrence of damage, which in practice may require an earlier replacement of wheels. Although the standard defines test procedures and load ranges, these may not take into account accidental loads and extreme events, such as impacts or unusual dynamic loads that may occur as a result of running into an obstacle on the track or defects in the track layout. Such events can induce local overloads that accelerate fatigue processes.

Including these additional aspects in the fatigue strength assessment could improve the accuracy of wheel life predictions and better reflect actual wheel operating conditions. In practice, this would imply more complex simulation analyses and more advanced material testing, which could minimise the risk of unexpected failures and increase the safety and efficiency of rolling stock operations.

5. Simulation testing

In using Abaqus software order to test the assumptions of PN-EN 13979-1:2020-12, fatigue strength analysis of selected railroad wheels was carried out. The simulations were carried out to assess the compliance of the results with the requirements, particularly in the context of mechanical and thermal stresses, and to investigate the locations of potential fatigue limit violations.

For the purpose of the analysis using the Finite Element Method (FEM), a railway wheel was selected (Table 1). The numerical method enables the calculation of stress and deformation distributions in complex structures such as railway wheels. FEA allows both mechanical and thermal stress investigation in the whole structure, including stress concentration areas, such as near-drilled holes. Simulation studies of the static and fatigue strength of the wheel were carried out.

Table 1. Calculations input data.

Symbol	Description	Value	Unit
$2P$	Wheelset static vertical load	180	kN
-	material	ER8	-
d_new	Nominal wheel diameter	840	mm
d_worn	Minimum worn wheel diameter	790	mm
w^tol	Maximum interference between wheel hub and axle seat	0.287	mm

The loads were implemented as concentrated forces on the rolling surface of the wheel. In accordance with the requirements of PN-EN 13979-1:2020-12, the wheel was analysed in its maximum permissible wear condition. Prior to the application of loads, the wheel was preloaded, taking into account the maximum interference resulting from the fit between the wheel hub and axle seat. In order to represent the full spectrum of loads in the discrete model, the point of application of forces on the wheel rolling surface was varied with steps of 4.5° around the circumference, thus simulating the full rotation of the wheelset (Figure 1).

Table 2. Exceptional loads ($P = 90$ kN).

Symbol	Fz [kN]	F_{y1} [kN]	F_{y2} [kN]	F_{y3} [kN]
Case – curve	$90 + P$	0	$10 + \frac{2}{3}P$	0
	180.0	0	70.0	0

According to the standard, the case of static exceptional loads was considered, corresponding to specified cases (Figure 2) for loads summarised in Table 2.

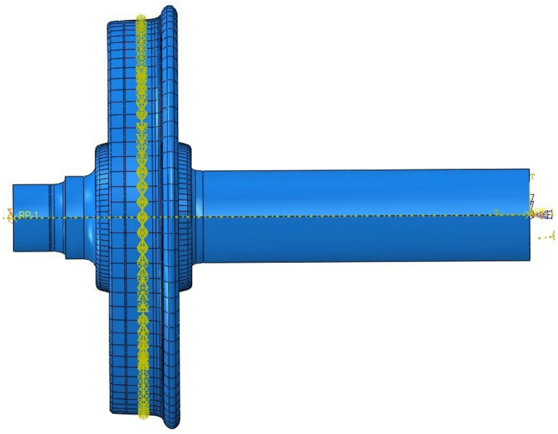

Figure 1. Loads and boundary conditions in a numerical model.

According to the requirements of the standard, in order to demonstrate sufficient fatigue strength of the monobloc wheel structure, it is necessary to carry out calculations of the stress fields for three load cases, which correspond to the situations of the vehicle passing through a straight section of track, curves and through switches and crossings. The scheme of the forces acting on the wheel in that case is shown in Figure 2. The load values used for the calculations for the leading wheelsets were taken from Table 3, as specified in the standard.

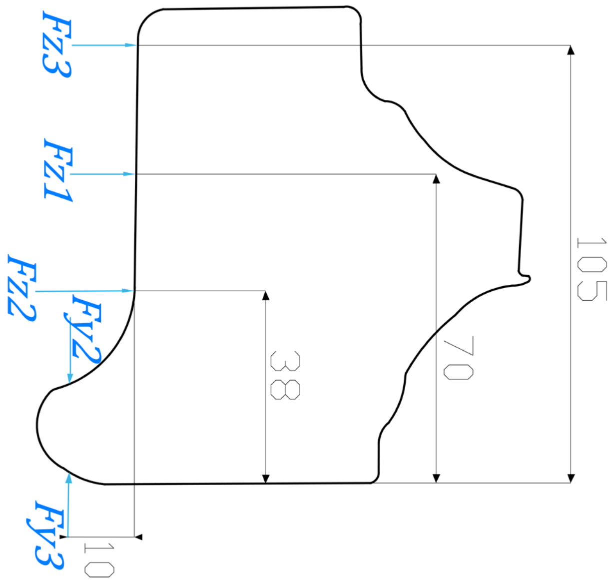

Figure 2. Scheme of the forces acting on the wheel [2].

Table 3. Operating loads ($P = 90{,}0$ kN).

Symbol	Fz [kN]	F_{y1} [kN]	F_{y2} [kN]	F_{y3} [kN]
Straight track	1.25 P	0	0	0
	112.5	0	0	0
Curve	1.25 P	0	0.7 P	0
	112.5	0	63.0	0
Switches and crossings	1.25 P	0	0	0.42 P
	112.5	0	0	37.8

In case of exceptional loads, the criterion value for the assessment is the yield strength (R_e) of the steel. Where calculations are carried out in the linear-elastic domain, the reduced stresses according to the Huber-

Mises hypothesis shall not exceed this limit. Locally, it is permissible for the yield strength to be exceeded, but in such cases an additional elastic-plastic analysis is required. Its purpose is to demonstrate that the resulting deformations do not exceed 5% (The 5% limit is considered a practical and safe threshold for plastic deformations under exceptional loads, allowing the material to avoid permanent damage and ensuring operational safety.). For ER8 material, the yield strength is R_e = 540 MPa [6].

At each node of the numerical wheel model, for the given load cases, the stress state described by the six components of the stress tensor was determined. From which, the principal stresses and their directions were determined by rotating the tensor:

$$\sigma_{ij} = \begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{pmatrix}. \tag{7}$$

Of all the dynamic load configurations, the direction of the maximum principal stress $\sigma_1 = \sigma_{max}$ was determined. Then, for each load case, the minimum stress value σ_{min} in the direction of σ_{max}. was selected. The stress range at each node of the numerical model was calculated according to the relationship:

$$\Delta\sigma = \sigma_{\max} - \sigma_{\min}. \tag{8}$$

According to the standard, the stress range must not exceed a criterion value that depends on the wheel surface roughness (R_a). These values are summarised in Table 4 and the permissible stress range is assumed to be 360 MPa.

Table 4. Fatigue strength criteria [2].

Symbol	Description	Value			Unit
R_a	Maximum roughness	3.2	6.3	12.5	μm
$\Delta\sigma$	Stress range	360	360	290	MPa

Three cases of dynamic loads acting on the leading wheelsets were analysed, corresponding to straight tracks, curves, switches and crossings. In addition, the case of exceptional static loading occurring in curves was considered. The yield strength of R_e = 540 MPa was assumed, and an allowable fatigue stress range of $\Delta\sigma_{allowable}$ = 360 MPa.

In the diagram showing the fatigue stress distribution in function of radial coordinate $\Delta\sigma(r)$ it is possible to observe a significant exceeding of the fatigue limit on the outer surface of the wheel rim, which is due to the contact stresses that occur at the wheel-rail contact. This phenomenon is natural at points of elastic contact between two bodies, where a local stress state resembling triaxial hydrostatic compression is formed. This state does not directly cause fatigue according to the standard [2], but can lead to crack initiation and propagation as a result of varying rolling loads (rolling contact fatigue). Notably, in the analysed case, fatigue limit exceedances can also occur outside of contact zones.

Additionally, an analysis of thermal stresses and their impact on the fatigue durability of wheels was conducted. The numerical model reflected the actual mechanisms of heat generation (heating of the wheel's rolling surface during braking), conduction, and dissipation of thermal energy (radiation and convection at an ambient temperature of 300 K) as shown in Figures 4-5.

Thermal stress modeling was performed in accordance with Equation (5). The analysis included modeling the distribution of thermal stresses in various parts of the wheel, particularly in the rim, which is most susceptible to the combined effects of mechanical and thermal stresses. Figures 4 and 5 illustrate two key cases of temperature distribution in the wheel rim. The most unfavorable temperature distribution during cyclic braking, this demonstrates a significant temperature difference between the contact area with the rail and the inner layers of the rim material. This gradient leads to the generation of thermal stresses, which can combine with mechanical stresses. Temperature distribution during downhill descent (braking state), this distribution indicates more uniform, yet still significant, temperature changes due to prolonged thermal loading.

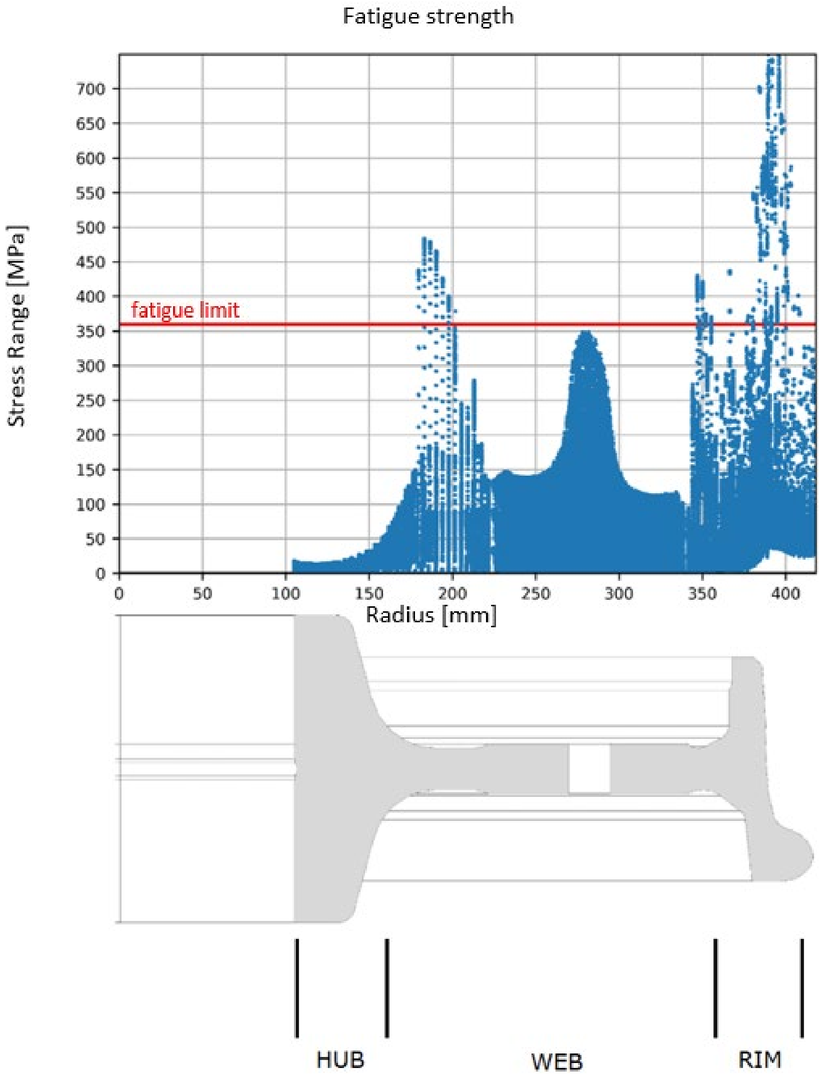

Figure 3. Stress range in function of radial coordinate Δσ(r).

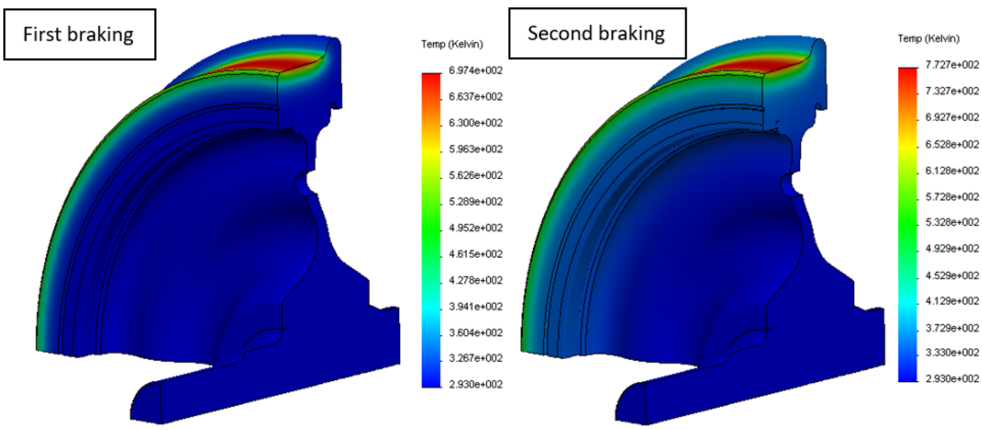

Figure 4. The most unfavorable temperature distributions during cyclic braking for a wheel with a rim in nominal condition.

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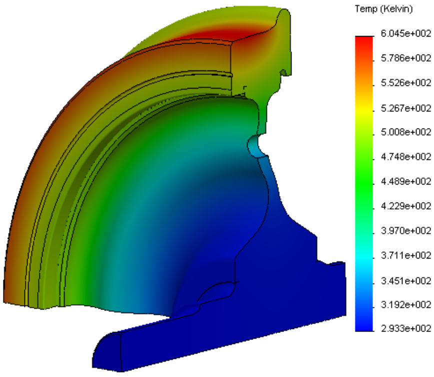

Figure 5. Temperature distribution during downhill descent in the braking state (Gotthard Pass, rim in nominal condition).

The simulation results (Fig 6-7) indicate a significant increase in total stresses in the wheel rim under intensive braking conditions. The highest values were observed at the contact surface with the rail, where thermal stresses further elevate the risk of initiating fatigue cracks. In the case of downhill descent, thermal stresses were more evenly distributed; however, their impact on fatigue durability remains significant.

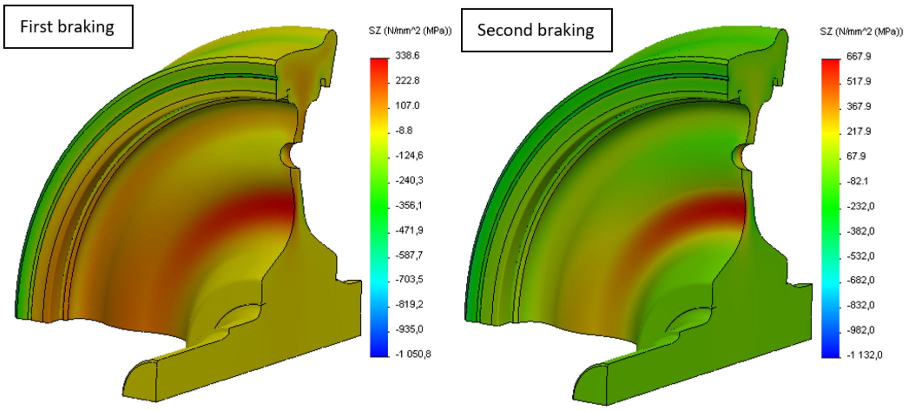

Figure 6. Stresses in the Z-direction during cyclic braking for a wheel with 25 mm radial wear.

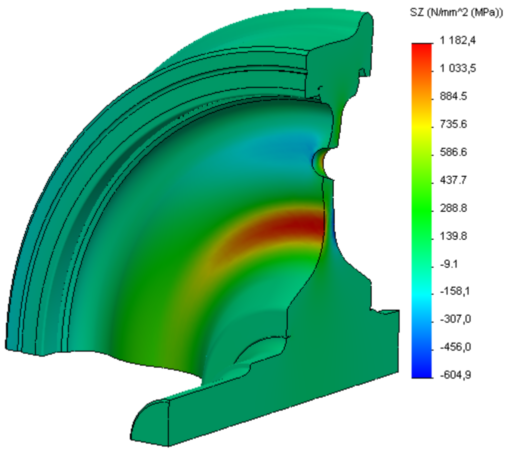

Figure 7. Stresses in the Z-direction during descent from a hill in the braking state (Gotthard Pass).

Thermal stresses, as calculated using a specific equation (5), have a significant impact on the overall stress, as described by another equation(4). The analysis in Figure 6 demonstrates that the inclusion of thermal stresses during cyclic braking can cause local exceedances of the fatigue limit on the rim's outer surface. For instance, during intensive braking, total stresses at critical locations can exceed the fatigue limit by a certain percentage, indicating a considerable risk of crack initiation.

While Figure 3 only presents mechanical stresses, the findings from Figures 6 and 7 highlight the critical role of thermal stresses in increasing the total stress levels. These results underscore the importance of considering thermal effects in fatigue analysis, as neglecting them, as per the current standard, could lead to an underestimation of fatigue risks under real-world operating conditions. Incorporating these effects would provide a more accurate and comprehensive assessment of wheel performance. It is important to note that comparing the amplitudes of mechanical and thermal stresses is not directly meaningful due to the different number of cycles involved for each type of loading. Mechanical loads typically occur over a significantly higher number of cycles compared to thermal loads, which are associated with specific braking events. Therefore, a direct comparison without considering the loading frequency and duration may lead to misleading conclusions regarding their respective contributions to fatigue damage.

6. Conclusions

The simulation study, conducted using the finite element method (FEA), analyzed wheels in their maximum wear state, considering operating parameters such as the maximum interference for a given fit and the varying dynamic forces acting on the wheel's rolling surface. The results of these simulations provided detailed data on the stress distribution within the wheel and identified areas where the stress exceeded the limit values specified by the standard.

The findings from this research suggest that the current standard's focus on stress amplitude does not account for the influence of mean stresses, which could be an important factor in fatigue cracking [7]. Although the standard addresses stress amplitude, it overlooks how mean stresses, related to static loading, can cause fatigue cracks even when the stress amplitude is within the specified limits. This observation points to a potential direction for future research, where including mean stresses in fatigue strength analyses may improve predictions.

Furthermore, the simulations highlighted the lack of consideration in the standard for thermal effects, particularly under conditions of heavy braking, such as those experienced by high-speed or goods trains. The results indicated that thermal stresses, arising from significant temperature increases due to intense braking, combine with mechanical stresses and contribute to increased fatigue wear of the wheels. This observation also suggests an area for future investigation, where incorporating thermal effects into fatigue analyses could improve the accuracy of wheel life predictions.

Incorporating thermal stresses into the fatigue durability analysis reveals significant differences in results compared to an assessment based solely on the normative requirements of PN-EN 13979-1:2020-12. The obtained results indicate that a normative analysis, which excludes thermal effects, may underestimate the fatigue risk under real operating conditions.

The analysis demonstrates that thermal stresses significantly influence the total stress state of railway wheels, particularly under intensive braking conditions. During cyclic braking, as shown in Figure 6, the combined stresses exceed the fatigue limit in specific regions of the wheel rim, indicating a clear risk of crack initiation. In contrast, during prolonged thermal loading, such as downhill braking (Figure 7), total stresses remain below the fatigue limit but approach it closely, signaling a potential cumulative effect on long-term fatigue durability. These results highlight a critical oversight in PN-EN 13979-1:2020-12, which does not account for thermal effects, leading to an underestimation of fatigue risks under real-world operating conditions. Addressing this gap would enhance the predictive accuracy of fatigue assessments and improve the safety and reliability of railway systems.

In light of these findings, the study advocates for a more comprehensive approach to fatigue strength analysis that extends beyond the current simplification of considering stress range alone. Future research could focus on incorporating a full description of stress cycles, including mean stresses and thermal effects, to provide a more accurate assessment of wheel condition. This would not only enhance the predictability of wheel life but also contribute to the overall safety and reliability of the railway system.

While standards play a crucial role in guiding the design and analysis of railway wheels, the results from these simulation studies suggest that integrating additional factors such as mean stresses and thermal effects could significantly improve the accuracy of predictions. However, this remains a potential avenue for future research, and further investigation is needed to explore these factors more thoroughly. Such efforts could improve safety, reduce the risk of wheel failure, and lower the cost of maintaining rolling stock.

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Additional information

The authors declare: no competing financial interests and that all material taken from other sources (including their own published works) is clearly cited and that appropriate permits are obtained.

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