

# Natural vibrations of terrace floor boards made from modern eco-composite materials

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**Abstract** The objective of this study was to determine the natural frequency of a modern eco-composite made from a mixture of 100% recycled plastics and to compare obtained results with those reported for other materials, i.e. polyethylene and polypropylene. One type of structural support resembling a terrace floor board fixing was investigated. Moreover, the influence of board geometry (thickness, width and length) on the resonance phenomenon was investigated. The study was conducted by analytical and finite element methods in the Abaqus environment. It was found that increased board thickness led to a linear increase in natural frequency. Additionally, the tests showed that boards 20% and 30% longer than the reference dimension of 500 mm were characterized by a reduction in resonance frequencies by 30% and 49%, respectively. Results showed that board width had no effect on the natural frequencies of these recycled materials.

**Keywords:** floor board, natural vibration, bending, FEM, polyethylene, polypropylene, recycled plastics.

## 1. Introduction

Nowadays, one can observe an increasing use of spaces around buildings, with these spaces often playing an important function as they integrate people as well as provide a place that is friendly to relationship establishment and relaxation. One example of such spaces are terraces around private houses or public buildings. These small engineering structures offer many advantages, as they provide additional usable space and are relatively simple to design and assemble, as well as their construction is relatively inexpensive. An important aspect in the design of such structures is to investigate structural vibrations that may occur during their use. Structural vibrations usually range from 1 to 100 Hz and are transmitted through the structures or ground. Humans receiving these vibrations are particularly sensitive to accelerations in the frequency range of 4-8 Hz, not to mention the fact that humans themselves produce vibrations when walking, as specified in Table 1 [1].

**Table 1.** Walking frequencies generated by humans .

walking speed	walking frequency [Hz]
very slow	1.25
slow	1.6
moderate	1.85
fast	2.1
running	up to 4

When the walking frequency is equal to the natural frequency of a structure, resonance occurs. This phenomenon may also occur when a harmonic of the walking frequency becomes equal to the natural frequency. For example, walking at 2 Hz creates resonance in a floor with natural frequency at 2, 4, 6, 8 Hz [2]. When floors resonate with walking pace or other human activities, this can increase vibration amplitudes, causing discomfort and even affecting usability of the space. Excessive vibrations can lead to discomfort while walking or performing tasks that require stability [3]. Over time, repeated dynamic loads

can lead to fatigue failure and a reduction in the strength and durability of the floor system [4]. Solving these problems requires a comprehensive understanding of both the dynamic behavior of such structures and the factors influencing anthropogenic vibrations.

Bearing the above in mind, this study investigates the problem of vibrations in simply supported boards that can be used for creating a terrace floor. The terrace floor design is based on joists resting on the foundation or ground, with boards laid/fixed thereon. In a modal analysis, the behavior of a board resembles that of a beam. The problem of beam behavior has been investigated in numerous studies. Analytical calculations based on the Timoshenko theory and FEM were undertaken in [5]. An isotropic material was investigated using different support types and constant thickness of the system. A study [6] analyzed a simply supported beam with variable cross-section. The natural frequencies of this system as a function of the beam deflection line were calculated by analytical and numerical methods. Kloda et al. [7–11] conducted a series of studies devoted to simply supported isotropic beams, focusing on their linear and nonlinear dynamics. Analytical and numerical solutions were proposed, showing high agreement of the results. Carbon-epoxy composites were investigated in studies [12-13], where the natural frequencies were determined and compared with data reported in the available literature. Singh [14] conducted an analytical study of hybrid carbon and glass fiber-reinforced laminates. The impact of stacking sequences of laminate plies and laminate types on the natural frequencies of these laminates was examined.

Particularly interesting are natural materials which have a significant impact on the environment and surroundings due to their advantages. A study [15] investigated the insulating and sound-absorbing properties of various natural composites made of cork, wood fibers and different wood chip types. Their damping properties in low- and high-frequency ranges were tested, showing that these materials were suitable for fabricating structural components with specified soundproofing characteristics. An interesting study was conducted by Prasad et al. [16], who investigated hybrid beams made of jute and glass laminates. Natural frequencies for different boundary conditions were examined, with the results demonstrating that the presence of jute and glass fibers had a significant impact on the behavior of the beam.

A review of the literature has demonstrated that there are numerous works devoted to isotropic materials, while there are hardly any studies investigating polymeric materials. Therefore, this study investigates the natural frequencies of boards made from a modern eco-composite based on recycled polyolefins. In addition, results obtained for this eco-composite are compared with those reported for other modern materials such as LDPE, HDPE and PP. The impact of board geometry on the natural frequencies of this system are determined for one type of support corresponding to the real conditions of terrace floor design.

## 2. Material testing

Experiments involved determining the material constants of a modern composite material with a trade name of Ekoforme, developed by Ekoforme company. The material is a mixture of polyolefins, i.e. LDPE, HDPE and PP, which are 100% recycled materials. As part of the study, three-point bending tests were performed in compliance with ISO 178:2019 [17]. Samples for three-point bending tests were manufactured in accordance with ISO 178:2019 guidelines, with the following dimensions: total sample length 80 mm, support spacing 64 mm, sample width 10 mm, and sample thickness 4 mm. These proportions ensure the correct distribution of stresses and strains in the bending zone. Based on this standard, 5 specimens were prepared for testing. Their mechanical properties were determined on a Z010 Zwick Roell testing machine (Ulm, Germany). Prior to the actual measurement, the specimens were subjected to a preliminary force corresponding to a stress of 0.1 MPa, in order to ensure adequate contact between the specimen and the measuring instruments. The rate of the bending modulus test was 1 mm/min, while the main test rate was set at 5 mm/min. Stresses and strains of the specimens were measured during the experiments. Detailed results are given in Figure 1, showing the behavior of the specimens up to their failure. To better illustrate the deformation mechanism of the LDPE/HDPE/PP blend during the bending test, the force-strain diagram was divided into three characteristic zones: A, B, and C. Zone A corresponds to the elastic range of deformation, zone B represents the onset of plasticization and the development of microcracks, and zone C encompasses the stage of maximum load and material destruction. This division allows for a more comprehensive analysis of the material's behavior in real time and the identification of key phenomena affecting its strength. In addition, three zones are marked in this figure to illustrate the real-time behavior of the samples, as shown in Figure 2. For the tested LDPE/HDPE/PP blend samples, the average elongation at fracture was  $14.16 \pm 1.40\%$ . However average maximal force was  $92.10 \pm 6.05$  N. The averaged Young's modulus at bending was 963MPa. The next test involved determining the density of the polymeric mixture in compliance with ISO 1183-1 A [18]. The density of the specimens was determined

by hydrostatic (immersion) method using water as the immersion liquid. Measurements were conducted on a Radwag WPS 50SX analytical scales (Poznan, Poland), which allows precise determination of the weight of the specimen in air and under full immersion in water. All measurements were made at room temperature, and the density of distilled water was known and constant, which ensured high accuracy of the results. After several tests, the average density was 0.969 g/cm<sup>3</sup>. Moreover, the Poisson's ratio was determined in a tensile test, which was described in our previous paper [19].

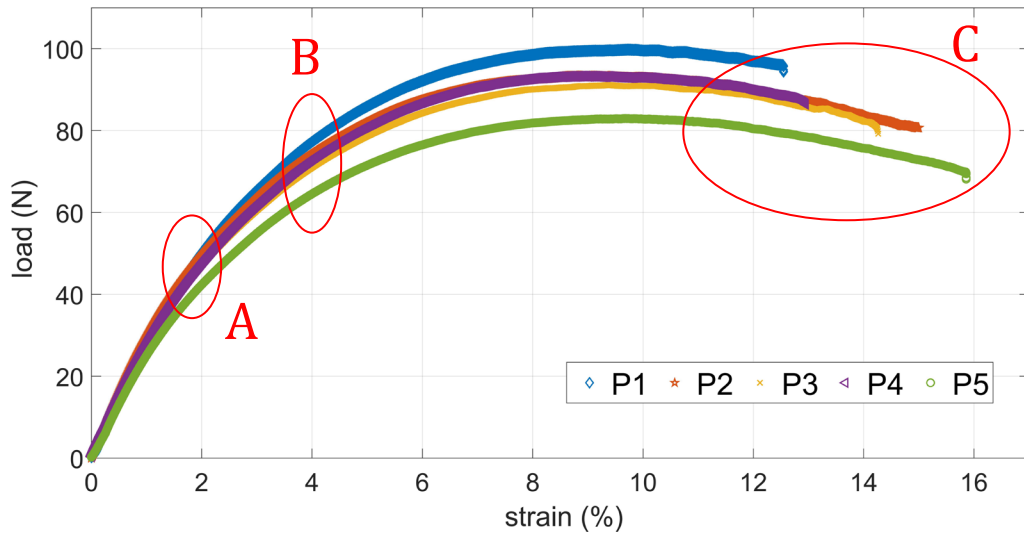


Figure 1. Load versus strain.



Figure 2. Sample during the experiment: a) zone A; b) zone B; c) zone C.

## 2. FEM modeling

### 2.1. Numerical model of a system

This section describes a numerical model of the system under study. The beam was modelled using continuum shell finite elements (SC8R) with reduced integration [20]. A mesh sensitivity study was performed on a representative specimen ( $L = 500$  mm,  $b = 100$  mm) to verify discretization convergence. The mesh density was increased until further refinement produced negligible changes (below 2%) in the computed natural frequencies. Finally for representative specimen 125 mesh elements obtained, which corresponds 436 nodes. Different beam lengths were investigated (Tab.2).

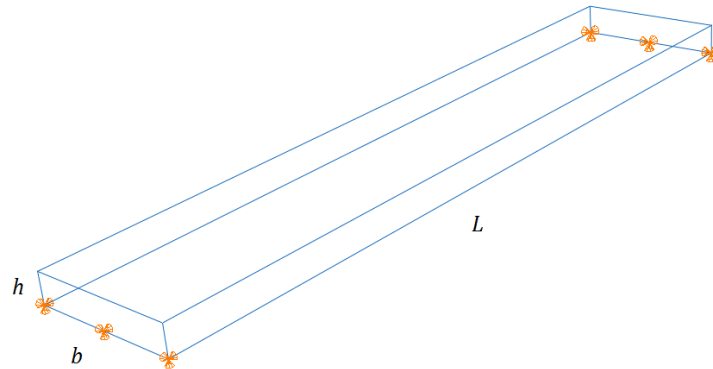
Table 2. Beam dimensions.

Length $L$ [mm]	Width $b$ [mm]	Thickness $h$ [mm]
500	100	20
600	130	30
700	160	40

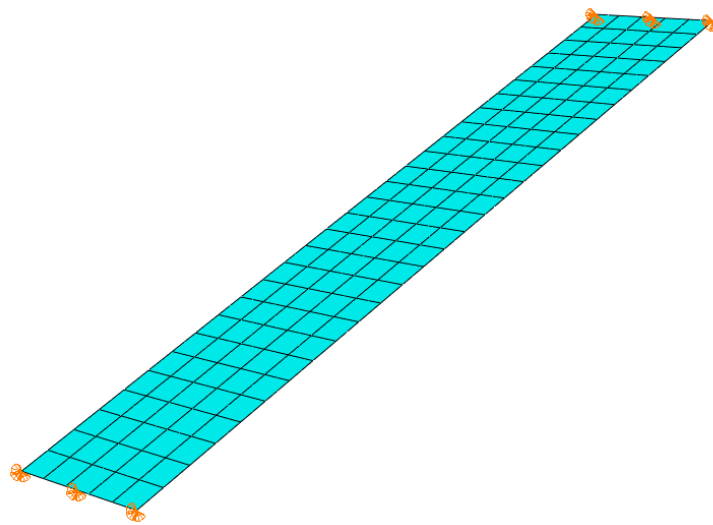
All analyzed structures were supported in the same way, i.e. they were simply supported at both ends. This support type reflects best the practical application of a terrace floor board which physically rests on

two beams that constrain the in-plane translational motion of this board while allowing it to rotate lengthwise. A schematic design illustrating the type of support is shown in Figure 3.

a)



b)



**Figure 3.** Schematic design of system support: a) analytical scheme b) numerical model.

## 2.2. Numerical simulations

The developed numerical model was used for numerical simulations in the Abaqus environment that is based on the finite element method. The scope of the modal analysis involved determining the first mode of natural vibration (Fig.4) and the corresponding natural frequency value. The analysis was made of the first mode only because from a practical point of view, this basic mode is the most important and dominant in real structures of this type. A higher natural frequency of the board implies its higher stiffness, which results in reduced deformability under external loads. Also, a high natural frequency implies a greater safety margin against resonance, reducing the likelihood of large deformations or damage. What is more, at higher frequencies, the board will have a higher dampening ratio, i.e. vibrations occurring on the structure will die out more quickly, which is a desired phenomenon for such structures.

The numerical results were validated via analytical calculations made with Equation 1. The natural frequency of a system with a given length strictly depends on the support type:

$$f = \frac{k^2}{2\pi} \sqrt{\frac{EJ}{A\rho L^4}}, \quad (1)$$

where  $f$  is the natural frequency,  $E$  is Young's modulus,  $J$  is the geometric moment of inertia,  $A$  is the cross-sectional area,  $\rho$  is the density,  $L$  is the length of the system,  $k$  is the coefficient dependent on the support type and vibration mode. For the case under analysis, it was a simple support, so the value of  $k$  was 3.142 [14, 21–22].

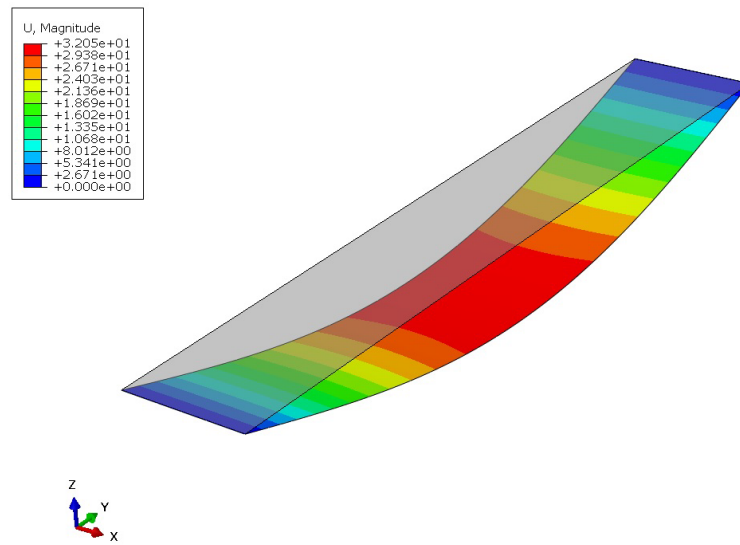


Figure 4. Analyzed mode of vibration.

### 3. Results and discussion

A detailed modal analysis was performed for three different spans between beam supports (beam lengths), as well as three different beam widths and thicknesses given in Table 1. Results obtained for the 100% recycled eco-composite were compared with those reported for other modern materials, i.e. polypropylene (PP), low-density polyethylene (LDPE) and high-density polyethylene (HDPE). Material data of all tested materials are given in Table 3.

Table 3. Material data.

Parameter	Eco-composite	PP [23]	HDPE [24]	LDPE [24]
Young's modulus [MPa]	963	1100	800	325
Poisson's ratio [-]	0.32	0.4	0.4	0.4
Density [kg/m <sup>3</sup> ]	969	900	870	870

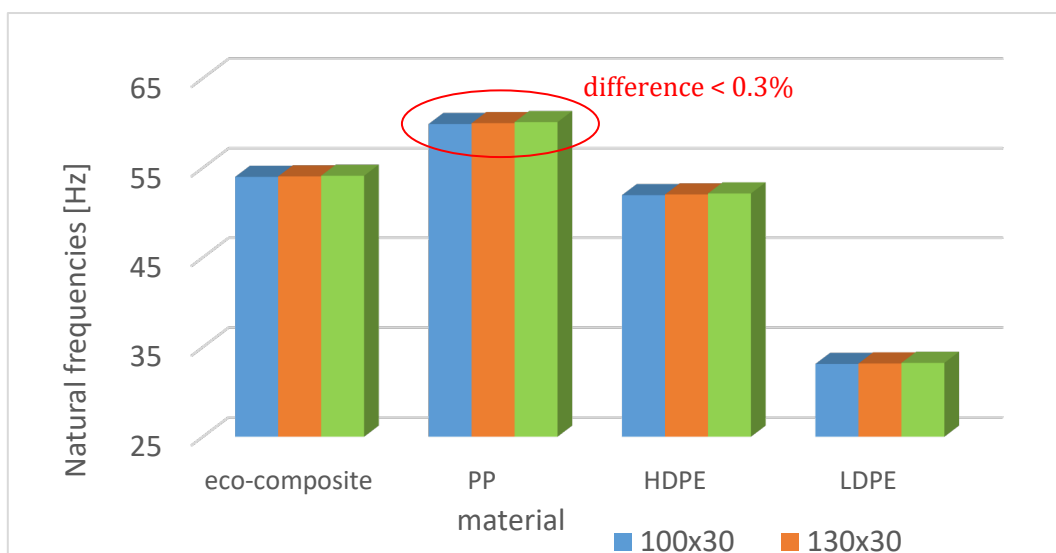


Figure 5. Natural frequencies versus board width, for a length of 500 mm.

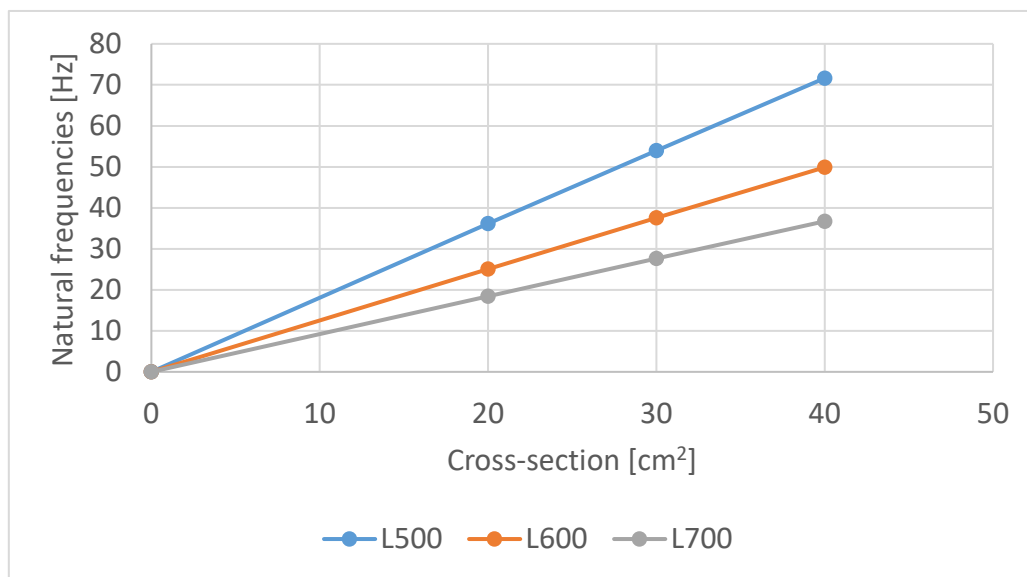
First of all, the impact of board width was determined. A 500-mm-long board was examined using all tested thicknesses and widths. The variations in board width had a practically invisible impact on the natural frequency. The same trend was obtained for all analyzed materials (Fig. 5). When the width was increased from 100 mm to 130 mm, the FEM-calculated maximum increase in the natural frequencies

amounted to 0.15% for PP and 0.09% for the eco-composite. Globally, a 60% increase in board width caused a less than 0.3% increase in the natural frequencies, which is a negligible error. It is worth noting that the analytical calculations showed no changes in the natural frequencies despite the applied board width variations, which results from the use of the simplified Euler-Bernoulli theory. Detailed results obtained for all analyzed materials are given in Table 4. The relative error of the results was below 1%, which confirms that the numerical model was designed correctly and that the numerical analysis was performed in a correct way.

**Table 4.** Natural frequencies obtained for different materials and methods [Hz].

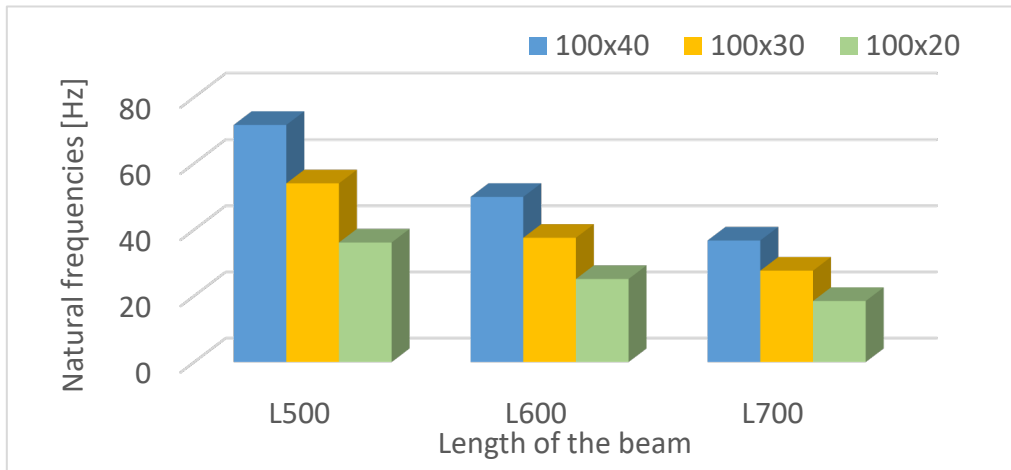
Length	Width	Thickness	Method	Eco-composite	PP	HDPE	LDPE
500	100	40	FEM	71.62	79.42	68.89	43.91
			Theory	72.38	80.27	69.63	44.38
		30	FEM	53.98	59.88	51.94	33.11
			Theory	54.29	60.20	52.22	33.28
		20	FEM	36.12	40.08	34.77	22.16
			Theory	36.19	40.14	34.81	22.19
	130	40	FEM	71.69	79.54	68.97	43.97
			Theory	72.38	80.27	69.63	44.38
		30	FEM	54.04	59.98	52.02	33.16
			Theory	54.29	60.20	52.22	33.28
		20	FEM	36.16	40.16	34.83	22.20
			Theory	36.19	40.14	34.81	22.19
160	40	FEM	71.78	79.68	69.11	44.05	
		Theory	72.38	80.27	69.63	44.38	
	30	FEM	54.11	60.10	52.13	33.23	
		Theory	54.29	60.20	52.22	33.28	
	20	FEM	36.22	40.24	34.91	22.25	
		Theory	36.19	40.14	34.81	22.19	

Considering the results, a board with a minimum width of 100 mm was selected for further analysis. In the next step, the effect of variable board thickness on the vibration damping in the structure was examined. Detailed analyses were performed for the recycled material, and results of these analyses are given in Figure 6. Linear dependences between thickness and natural frequency were obtained for the three tested beam lengths. It was found that an increase in beam thickness caused a linear increase in natural frequencies. The shortest beam was deflected to the horizon at an angle of 61.06°. However, when the board length was increased by 20%, the angle of beam deflection was 15% lower. For the longest board, the angle was 42.7°, this value being 30% lower than for the 500 mm long board.



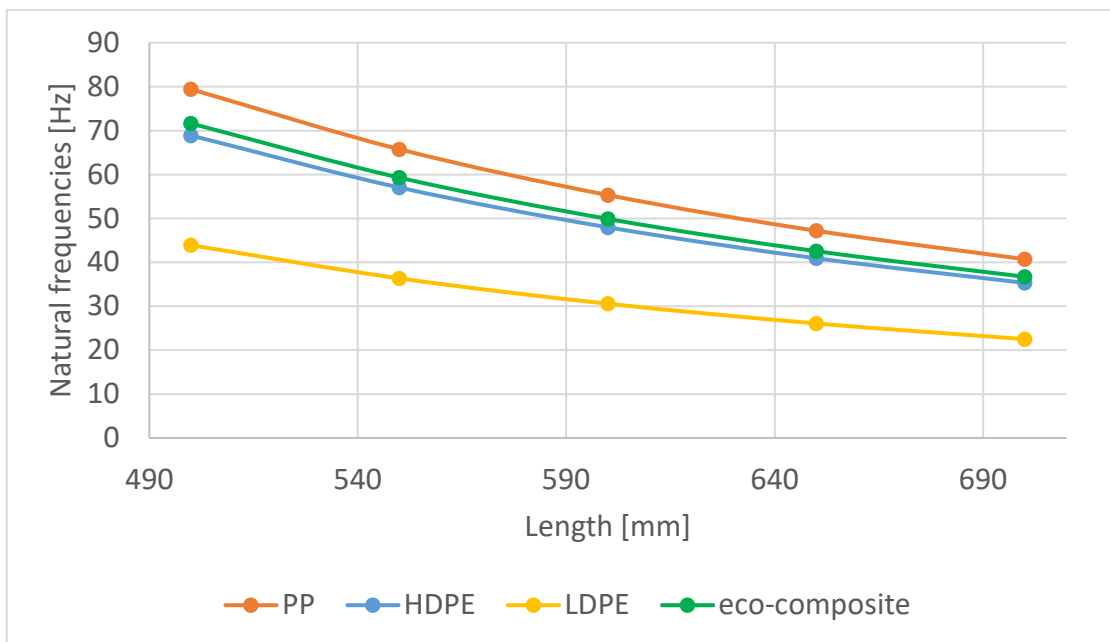
**Figure 6.** Natural frequencies versus cross-section of eco-composite.

After that, the impact of variable length on natural frequencies was determined for different cross-sections: 100 x 40; 100 x 30; 100 x 20. Results are plotted in Figure 7. It was found that boards 20% and 30% longer than L500 = 500 mm were characterized by a reduction in resonance frequencies of 30% and 49%, respectively. The same trends were obtained for all analyzed cross-sections. Moreover, the use of a 25% reduced cross-section (smaller board thickness) caused a 25% decrease in the natural frequency. Similar proportional relationships were observed for a beam with 50% reduced thickness.



**Figure 7.** Natural frequencies versus beam length, for eco-composite.

After that, the results obtained for the eco-composite were compared with those obtained for other modern materials in accordance with Table 3. A non-linear relationship was established between natural frequency and length, as shown in Figure 8 for a beam with the 100 x 40 cross-section. A comparison of the results obtained for the recycled material and polypropylene reveals an 11% difference between these results. On the other hand, the stiffness of the eco-composite was 4% and 39% higher than that of high-density and low-density polyethylene, respectively. Detailed values obtained for all materials, lengths and thicknesses are listed in Table 5.



**Figure 8.** Natural frequencies versus length, for a beam cross-section of 100 x 40.

**Table 5.** Natural frequencies for all tested cases [Hz].

L [mm]	b [mm]	h [mm]	Method	Eco-composite	PP	HDPE	LDPE
500	100	40	FEM	71.62	79.42	68.89	43.91
			Theory	72.38	80.27	69.63	44.38
			Error (%)	1.05	1.06	1.06	1.05
		30	FEM	53.98	59.88	51.94	33.11
			Theory	54.29	60.20	52.22	33.28
			Error (%)	0.57	0.54	0.53	0.52
		20	FEM	36.12	40.08	34.77	22.16
			Theory	36.19	40.14	34.81	22.19
			Error (%)	0.20	0.14	0.12	0.13
600	100	40	FEM	49.89	55.32	47.98	30.58
			Theory	50.27	55.74	48.35	30.82
			Error (%)	0.75	0.76	0.77	0.77
		30	FEM	37.54	41.64	36.12	23.02
			Theory	37.70	41.81	36.26	23.11
			Error (%)	0.42	0.40	0.39	0.40
		20	FEM	25.09	27.84	24.15	15.39
			Theory	25.13	27.87	24.18	15.41
			Error (%)	0.17	0.11	0.11	0.12
700	100	40	FEM	36.72	40.72	35.32	22.51
			Theory	36.93	40.95	35.52	22.64
			Error (%)	0.57	0.57	0.57	0.58
		30	FEM	27.61	30.62	26.56	16.93
			Theory	27.70	30.72	26.64	16.98
			Error (%)	0.32	0.31	0.31	0.30
		20	FEM	18.44	20.46	17.74	11.31
			Theory	18.46	20.48	17.76	11.32
			Error (%)	0.13	0.08	0.12	0.10

#### 4. Conclusions

This study addressed important issues concerning the dynamics of floor boards made of modern and practical materials. Structural elements of this type can be used for creating a terrace floor, the design of which requires higher resistance to resonance. An important factor in the design of such structures is to examine structural vibrations that may occur during their use. These structures are primarily designed for humans, who themselves produce vibrations up to 4 Hz when walking across a floor. To avoid the occurrence of undesired resonance, it is important that the designed structure has a high natural frequency. The high frequency level results in a higher damping ratio, which – in turn – contributes to higher comfort when using the terrace floor.

The study was performed on the specimens of an eco-composite made from a mixture of 100% recycled materials. The main components of the mixture were: high-density polyethylene, low-density polyethylene and polypropylene. Although all the components are thermoplastics, their combination creates a heterogeneous material with properties different from the properties of the individual components. The components differ in chemical structure and physical properties, which results in the creation of a multiphase material with new functional features, so this material is called an eco-composite. As part of the conducted modal analysis, the first mode of natural vibration and the corresponding natural frequency were determined. The analysis was performed by the finite element method in the Abaqus environment. The numerical results were compared with analytical findings, where the first natural frequency value was determined. Detailed analyses were conducted for different cases of board geometry, with length varying in the range of 50-70 cm, thickness ranging 2-4 cm and width ranging 10-16 cm. In addition, the results for all analyzed cases were compared with results obtained for other modern thermoplastics such as PP, HDPE and LDPE. First of all, the effect of board width on the system's behavior was investigated. It was found that width in that cases had no impact on the natural frequencies of the system. After that, a relationship was established between board thickness and structural vibration damping. It was found that an increased board thickness caused a linear increase in the natural frequency. The results showed that when the cross-section was reduced by 25% (board thickness was smaller), the natural frequency of the system decreased

by 25%. The system behaved similarly when the cross-section was reduced by 50%, i.e. its resonance frequencies decreased by 50%. In the next step, the impact of length on natural frequencies was determined. A length of 50 cm was taken as the reference dimension. When the dimension was increased by 20%, the board exhibited natural frequencies that were 30% lower, whereas a 40% increase in board length led to a 49% reduction in its natural frequencies. The same trend was observed for all cross-sections under analysis. The results also showed that the stiffness of the recycled eco-composite was 4% and 39% higher than that of other commercially available new materials, i.e. HDPE and LDPE, respectively.

According to market data the price of recycled polyolefin granules is on average 30–50% lower than that of virgin polymers. Additionally, the processing of recycle requires a lower melting temperature (approximately 160–180°C), which results in reduced energy consumption and lower CO<sub>2</sub> emissions. Combined with the possibility of reusing production waste, this significantly lowers the overall manufacturing cost of components made from the “Ekoforme” material. Therefore, the developed eco-composite represents a cost-effective and environmentally justified alternative to conventional materials used in decking and façade construction, combining mechanical efficiency with a low environmental impact.

This study is the first stage of research investigating the natural frequencies of boards made from modern eco-composite materials. Further works will involve experimental verification of the numerical results obtained in this study.

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### Additional information

The authors declare: no competing financial interests and that all material taken from other sources (including their own published works) is clearly cited and that appropriate permits are obtained.

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